Mock Lab Write-Up

Objective:

To use several pendulums to determine the mass of the Earth.

Introduction:

The Earth is one of eight planets that makes up our Solar System. Located third from the Sun, the Earth is the largest of the terrestrial planets but far smaller and less massive than the gas giants located further away (1). All the planets together still do not match the mass of the Sun, which dominates the physics in the Solar System.

Mass is an important property. A planet (or star) of greater mass has a greater gravitational pull on its surroundings. This will affect the type of atmosphere, the number of moons, and the influence on passing bodies. For example, Jupiter is the most massive planet and has shattered passing comets with its gravitational influence (2). Jupiter has a complement of $\sim$ 60 moons (3) and a very thick gaseous atmosphere, whereas the Earth has a single moon and a much thinner atmosphere.

Although the Earth is not the most massive planet in the Solar System, it is still difficult to leave its surface. A rocket must be traveling $\sim$ 12 km/s (43200 km/hour) to escape Earth's gravitational pull. The strength of this gravitational pull depends to Earth's mass and size, thus we can determine the Earth's mass from this quantity (1).

One method for measuring the gravitational pull (or acceleration) is to use a pendulum. A pendulum is a simple device with a string fixed at one point and a weight attached at the other end (see Figure 1). When displaced from equilibrium, the pendulum will swing under the influence of gravity such that the time for one full cycle (the period, T) depends only on the length of string (L) and the gravitational pull (g).

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{1}$$

The gravitational pull depends on the Earth's mass such that,

$$M_{Earth} = 2.41\text{x}10^{25}\frac{L}{m_p T^2} \tag{2}$$

where L is the length of the pendulum in metres, m_p is the mass of the pendulum in kilograms and T is the time in seconds for one cycle (4). This gives us the mass of the Earth in kilograms. By measuring the time required for a full cycle of the pendulum, we can determine the mass of the Earth.

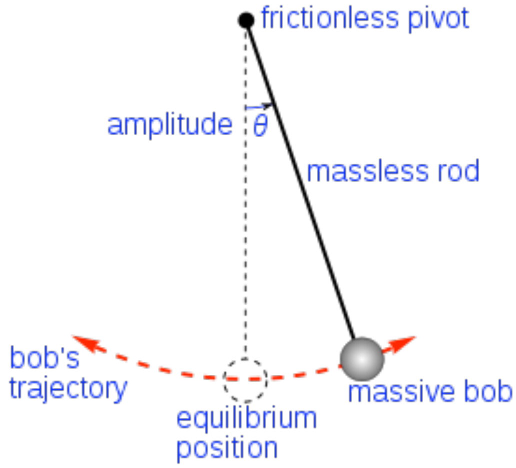

Figure 1: Schematic of a simple pendulum moving under the influence of gravity. The pendulum consists of a string fixed at one end (top) and attached to a mass at the other end (bottom). When displaced from the equilibrium position (or rest position), the pendulum will swing back and forth across the equilibrium position. Image from http://en.wikipedia.org/wiki/Pendulum.

Equipment:

- 6 pendulums (masses 0.2 kg, 0.4 kg, 1 kg, 1.2 kg, 1.5 kg, 2 kg)
- metre stick
- hook
- stopwatch

Procedure:

1. The length of each pendulum was measured and recorded.

2. The 0.2 kg pendulum was hung from the desk using the hook.

3. The 0.2 kg pendulum was moved an angle of $\sim 60\,\text{deg}$, let go, and the time for 10 full cycles were recorded using the stopwatch.

4. Steps 2 and 3 were repeated for the remaining 5 pendulums. For the heavier pendulums, the pendulum was hung from the ceiling.

Observations:

Table 1 provides are measurements for the length of each pendulum and the length of time for 10 oscillations.

Table 1: Measurements for the Pendulums

Pendulum	Mass (kg)	Length (cm)	Time (s)
1	0.2	20	25
2	0.4	50	24
3	1.0	98	21
4	1.2	120	21
5	1.5	150	20
6	2.0	150	17

Calculations:

For Equation (2), we require that L be in metres, T be in seconds, and m_p be in kilograms. For our first pendulum,

- $L = 20$ cm $= 0.20$ m

- The time for 10 cycles was 28 seconds, thus time for 1 cycle is 28 seconds / 10 cycles = 2.8 seconds.

Plugging $L = 0.20$ m, $T = 2.5$ s, and $m_p = 0.2$ kg into Equation 2:

$$\begin{aligned} M_{Earth} &= 2.41\text{x}10^{25}\left(\frac{L}{m_p T^2}\right) \\ M_{Earth} &= 2.41\text{x}10^{25}\left(\frac{0.2}{0.2 \cdot (2.5)^2}\right) \\ M_{Earth} &= 3.86\text{x}10^{24}\text{kg} \end{aligned}$$

Therefore, the mass of the Earth from the first pendulum is 3.86x10^{24} kg. Table 2 provides the results for the rest of the pendulums.

The average mass can be found from summing the mass measurement from each pendulum and dividing by the number of pendulums (see Table 2),

$$\begin{aligned} M_{ave} &= \frac{1}{6}\Big(3.86\text{x}10^{24}\text{kg} + 5.23\text{x}10^{24}\text{kg} + 5.36\text{x}10^{24}\text{kg} + 5.46\text{x}10^{24}\text{kg} + 6.02\text{x}10^{24}\text{kg} \\ &\quad +6.25\text{x}10^{24}\text{kg}\Big) \\ M_{ave} &= 5.36 \text{ x } 10^{24} \text{ kg} \end{aligned}$$

Table 2: Measurements for the Earth's Mass

Pendulum	Pendulum Mass (kg)	Length (m)	Period (s)	Earth's Mass (kg)
1	0.20	0.20	2.5	3.86 $x10^{24}$
2	0.40	0.50	2.4	5.23 $x10^{24}$
3	1.0	0.98	2.1	5.36 $x10^{24}$
4	1.2	1.2	2.1	5.46 $x10^{24}$
5	1.5	1.5	2.0	6.02 $x10^{24}$
6	2.0	1.5	1.7	6.25 $x10^{24}$

To estimate the uncertainty, the greatest deviation between six mass measurements and the average mass is $5.36 \mathrm{x} 10^{24}$ kg - $3.86 \mathrm{x} 10^{24}$ kg = $1.5 \mathrm{x} 10^{24}$ kg.

Results:

We obtained an average mass estimate for the Earth of $(5.36 \pm 1.5) \mathrm{x} 10^{24}$ kg. The accepted value for the mass of the Earth is $5.98 \mathrm{x} 10^{24}$ kg (4), which is quite close to what we measured. The least massive pendulum, however, has a rather large deviation from the rest. This could be related to errors in the experiment.

Our error of $1.5 \mathrm{x} 10^{24}$ kg is quite large. There are a number of possible effects that could cause errors in our measurements. Using a stopwatch to time our results is difficult because each person has a different “reaction” time when starting or stopping the watch. As well, it's not entire obvious when a pendulum cycle is compete, making the timings more difficult. Another problem is with swinging the pendulum itself. It was difficult to get a nice, smooth set of oscillations without any wobbling. Wobbles would affect the rate of each cycle as well as our ability to count 10 cycles. This was particularly problematic for the least massive pendulum. It is possible that it wasn't massive enough to have stable oscillations (maybe air resistance caused the wobbles).

Conclusions:

In this lab, we measured the mass of the Earth by measuring the periodic motions of 6 different pendulums. Our final result of $(5.36 \pm 1.5) \mathrm{x} 10^{24}$ kg agrees with the accepted value of $5.98 \mathrm{x} 10^{24}$ kg, though our least massive pendulum might be a poor instrument in measuring the mass of the planet (due to its light weight, additional factors like air resistance might be influencing it). In spite of our errors, we have successfully shown that the mass of a planet can be measured using the oscillations of a pendulum. With more trials and better weighted pendulums, we would be able to measure the Earth's mass more accurately!

References:

1: Bradley Carroll & Dale Ostlie, *An Introduction to Modern Astrophysics*, 2006

2: David Levy, *Impact Jupiter - The Crash of Comet Shoemaker-Levy 9*, 2003

3: http://solarsystem.nasa.gov/planets/profile.cfm?Display=Moons&Object=Jupiter, 2008

4: David Halliday, Robert Resnick, & Jearl Walker, *The Fundamentals of Physics - 7th Ed.*, 2004