Outline

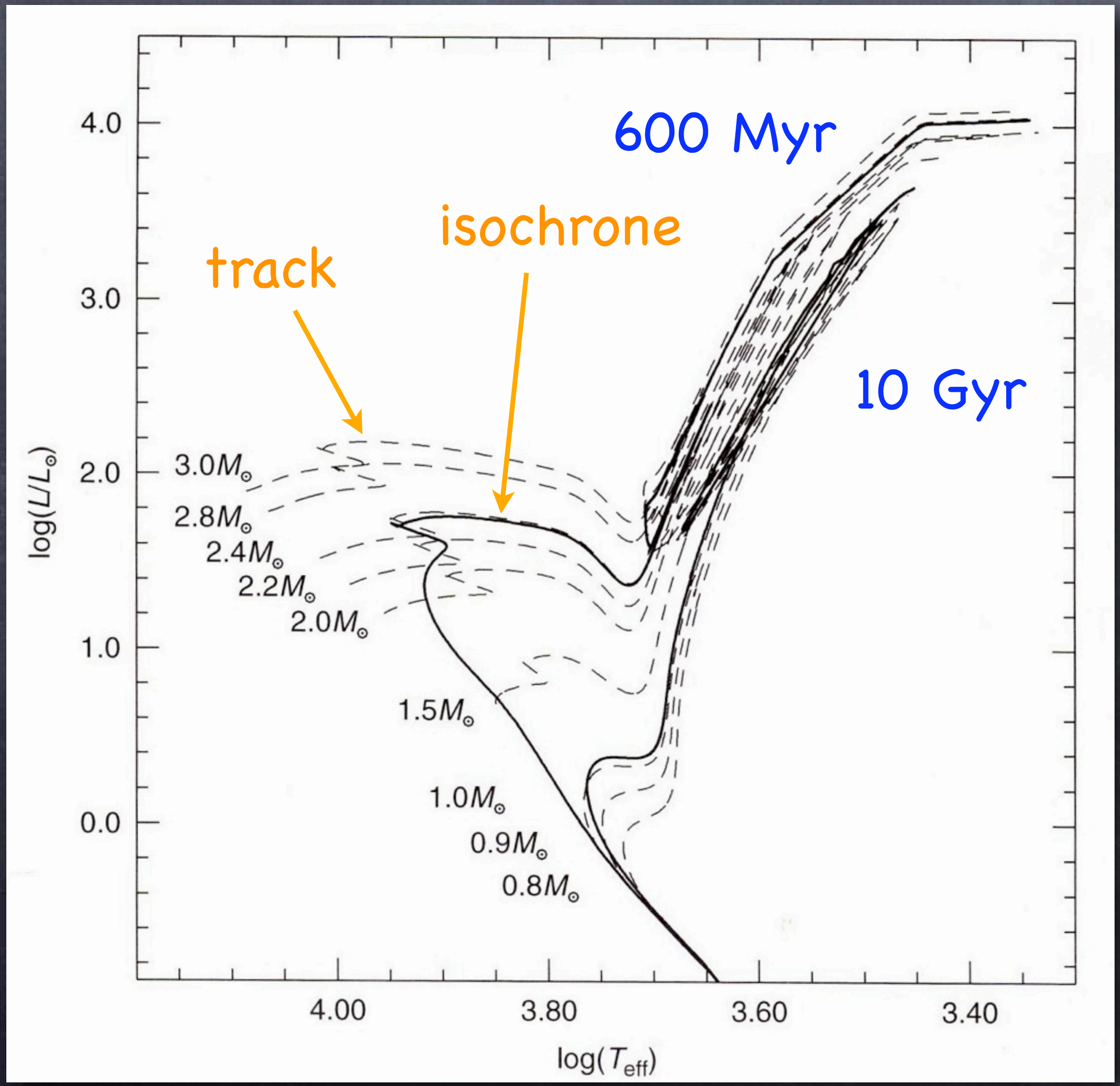
- Part I (Nov. 3rd - Monday): crash course on star formation, stellar structure and evolution, nucleosynthesis, simple stellar populations

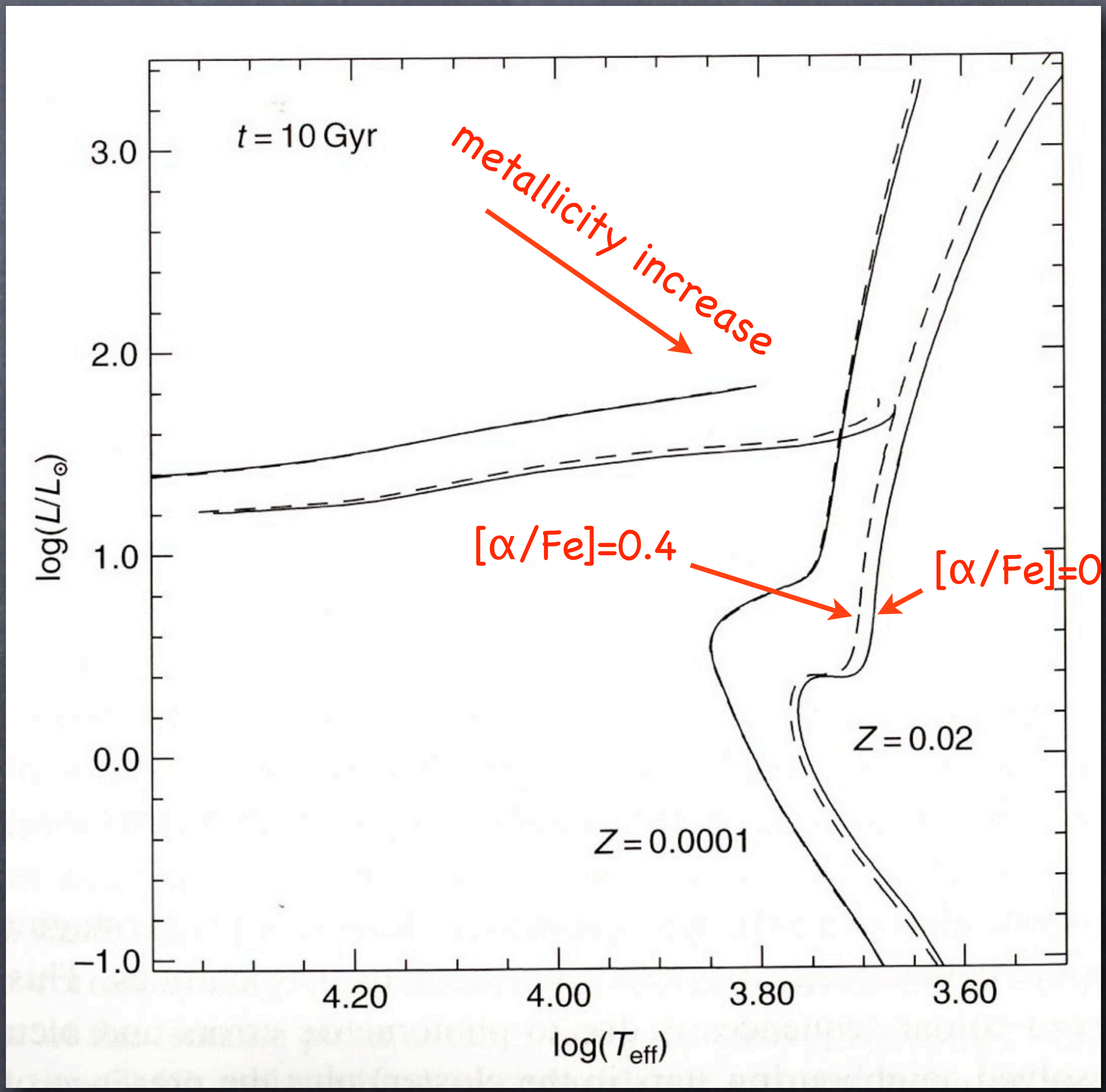
- Part II (Nov. 6th - Thursday): population synthesis models, photometric and spectroscopic stellar population diagnostics, galaxy formation, SN rates in galaxies, star-formation histories, etc.

Evolution of Simple Stellar Populations

Stars of a given mass evolve along an evolutionary track at various speeds. At any given time during the evolution of an SSP we can take a snapshot of the H-R diagram and derive the "isochrone".

Isochrone: L-T relation for stars of a defined age and chemical mix.





Evolution of Simple Stellar Populations

Isochrone: L-T relation for stars of a defined age and chemical mix.

Consider a monotonic, linear new coordinate along each isochrone

$$p = p(t, m, X_i)$$

One can associate p with a particular stellar evolutionary phase on the isochrone at the time

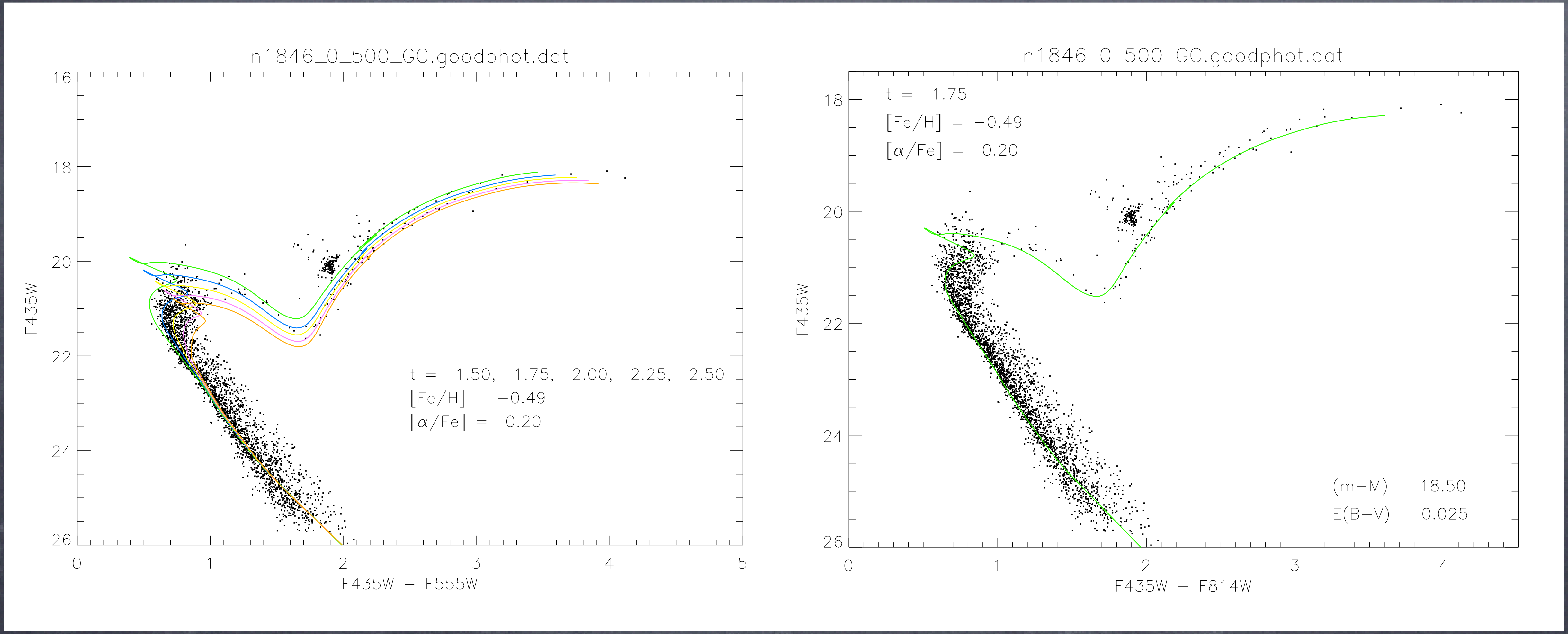
$$dt(p, m, X_i) = \left.\frac{dt}{dp}\right|_{t,X_i} dp + \left.\frac{dt}{dm}\right|_{p,X_i} dm + \left.\frac{dt}{dX_i}\right|_{p,m} dX_i$$

At a given age (i.e. dt=0) and assuming no change in chemical composition (i.e. dX=0) over time we obtain

$$\left.\frac{dm}{dp}\right|_{t,X_i} = - \underbrace{\left.\frac{dm}{dt}\right|_{p,X_i}}_{\text{mass-loss}} \underbrace{\left.\frac{dt}{dp}\right|_{m,X_i}}_{\text{inverse. evolutionary flux}}$$

Simple Stellar Populations

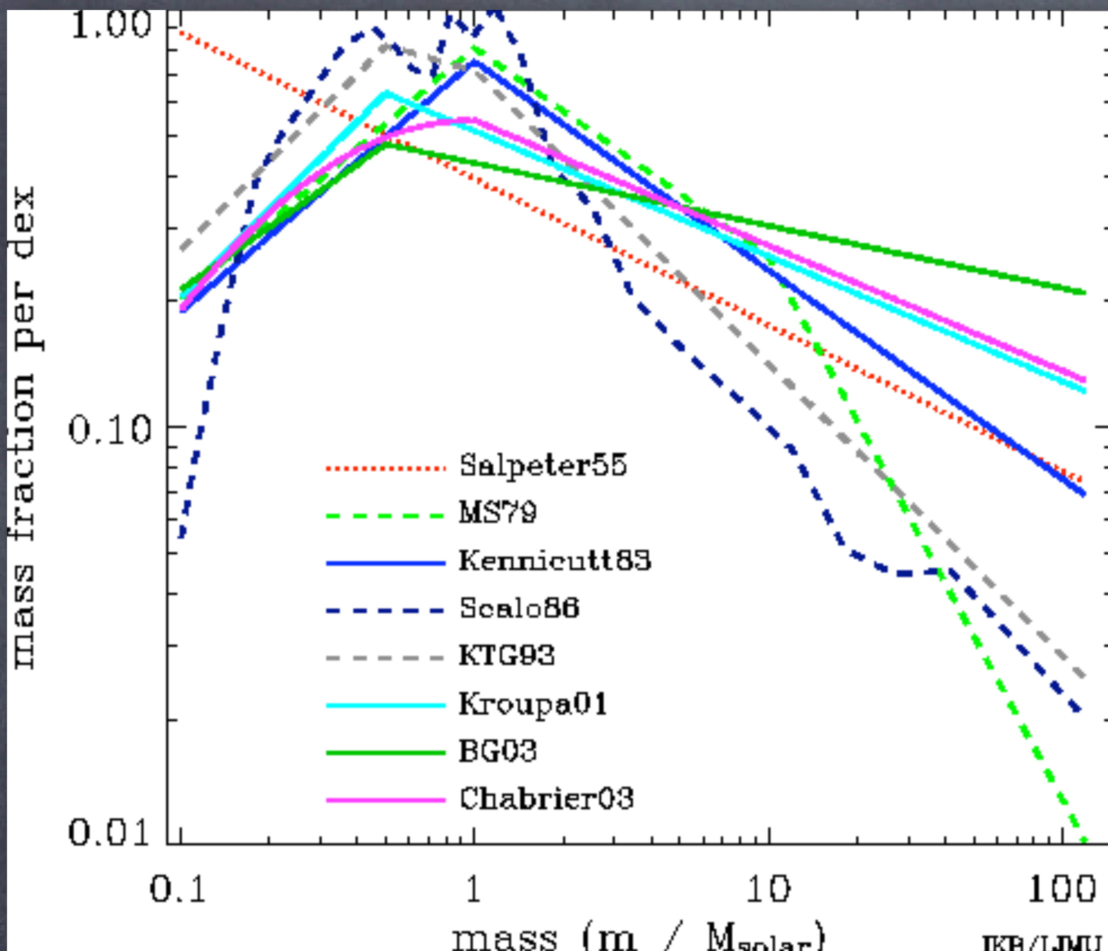
Isochrone: L-T relation for stars of a defined age and chemical mix.

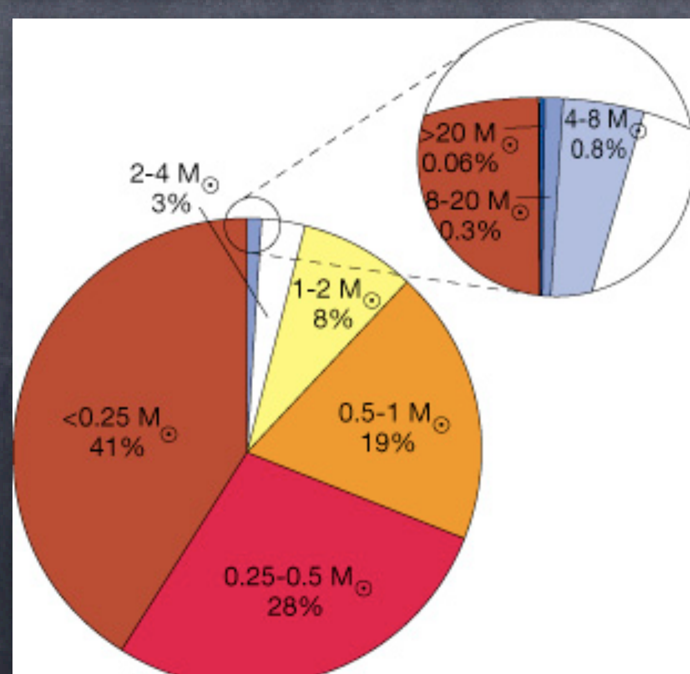


Goal: Find isochrone(s) that match data - watch degenracies!!

Create a synthetic stellar population based on set of evolutionary tracks and selection and error functions - build the isochrones by adopting a stellar mass function and binary spectrum.

Stellar Mass Function





The stellar mass function describes the number of stars per mass interval defined as

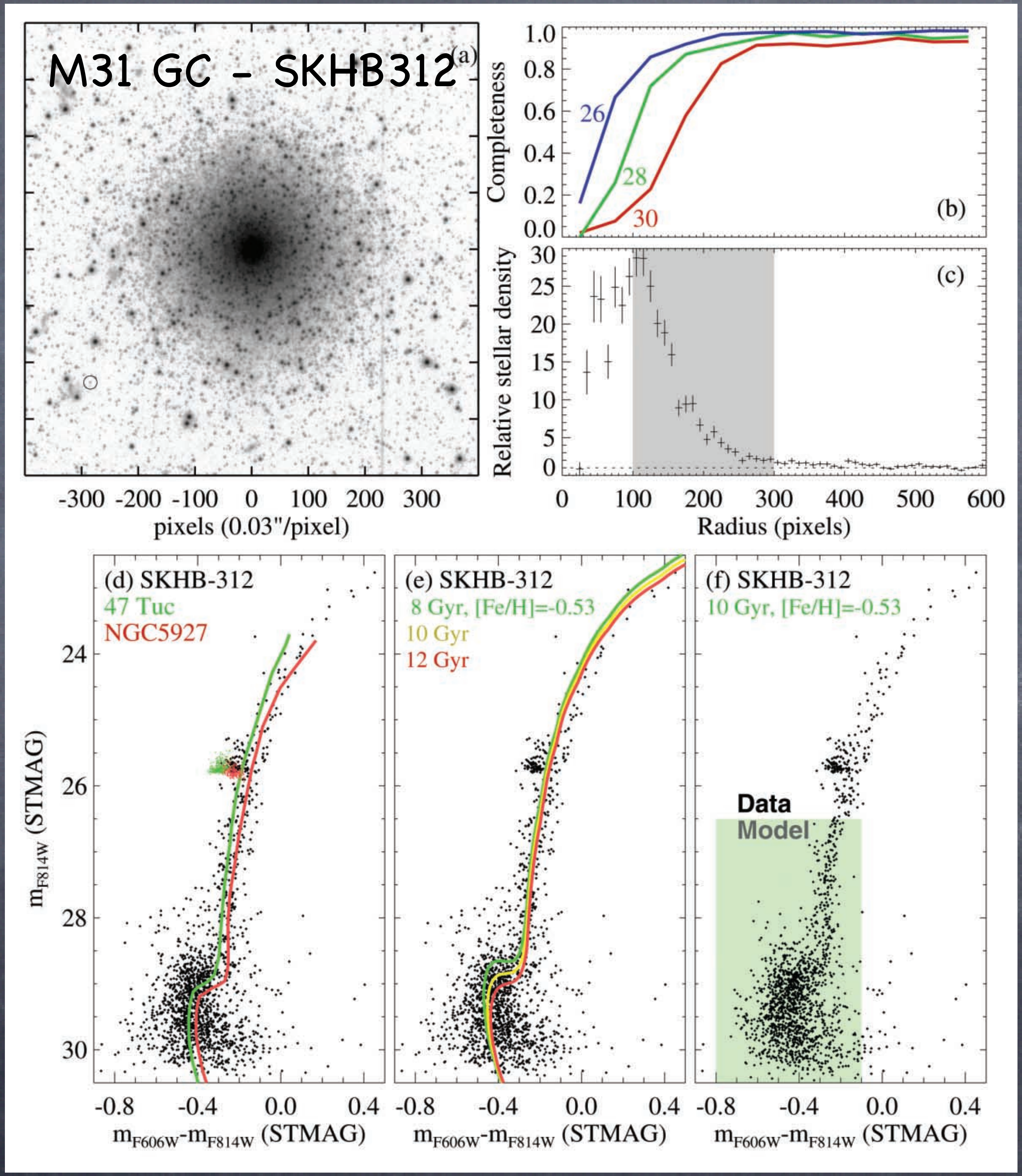
$$dN = cM^{-\gamma}dM$$

This function is often referred to as the "initial mass function", and is called Salpeter IMF when the exponent is $\gamma \approx 2.35$.

Other functional forms of the stellar mass distribution include a changing exponent for various mass ranges.

Note: for most IMFs the majority of stellar mass is in low-mass stars!

Simple Stellar Populations



Brown et al. (2003)

Composite Stellar Populations

The goal of population synthesis models is to predict the time evolution of the spectral energy distribution of a composite stellar population with defined star formation history + IMF(s), age-Z relations, etc.

Generally, the following assumptions apply:

1. The stellar models accurately predict the observed properties of stars of different masses as a function of their age and metallicity.
2. The stellar mass function, either independent of age and chemical mix or variable, is a realistic counterpart to the true IMF
3. The observational errors can be accurately measured and modeled
4. The theoretical stellar populations represent all the populations present in the observed composite stellar population

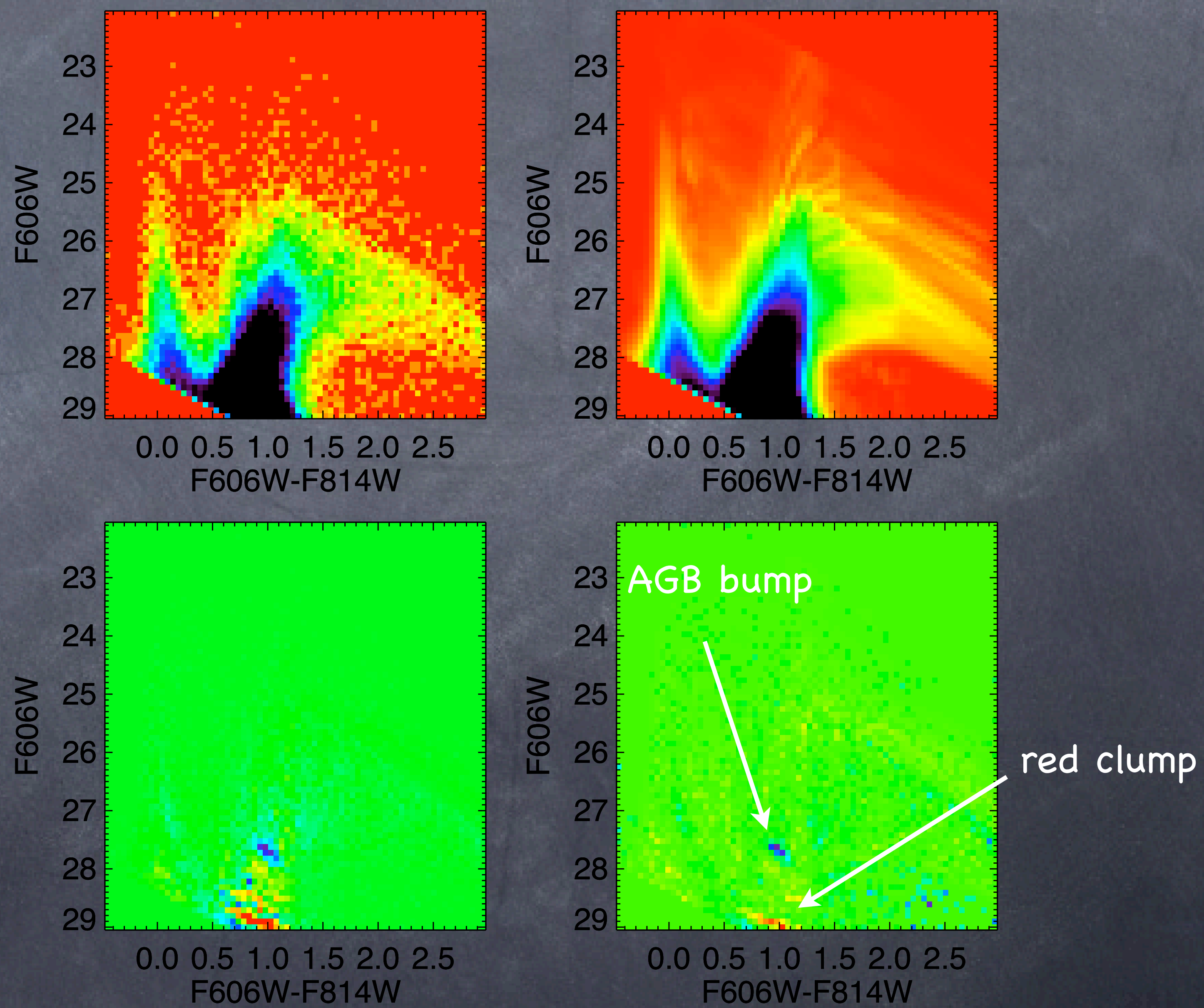
Resolved Composite Stellar Population

At any given time the star formation rate includes the integral over a given stellar mass function for each SSP. These contributions are then summed up over time and modulated by the age-metallicity relation to derive the final star formation history.

$$\Upsilon(t, Z) = \int_0^t \Xi(t)\Psi(t)dt$$

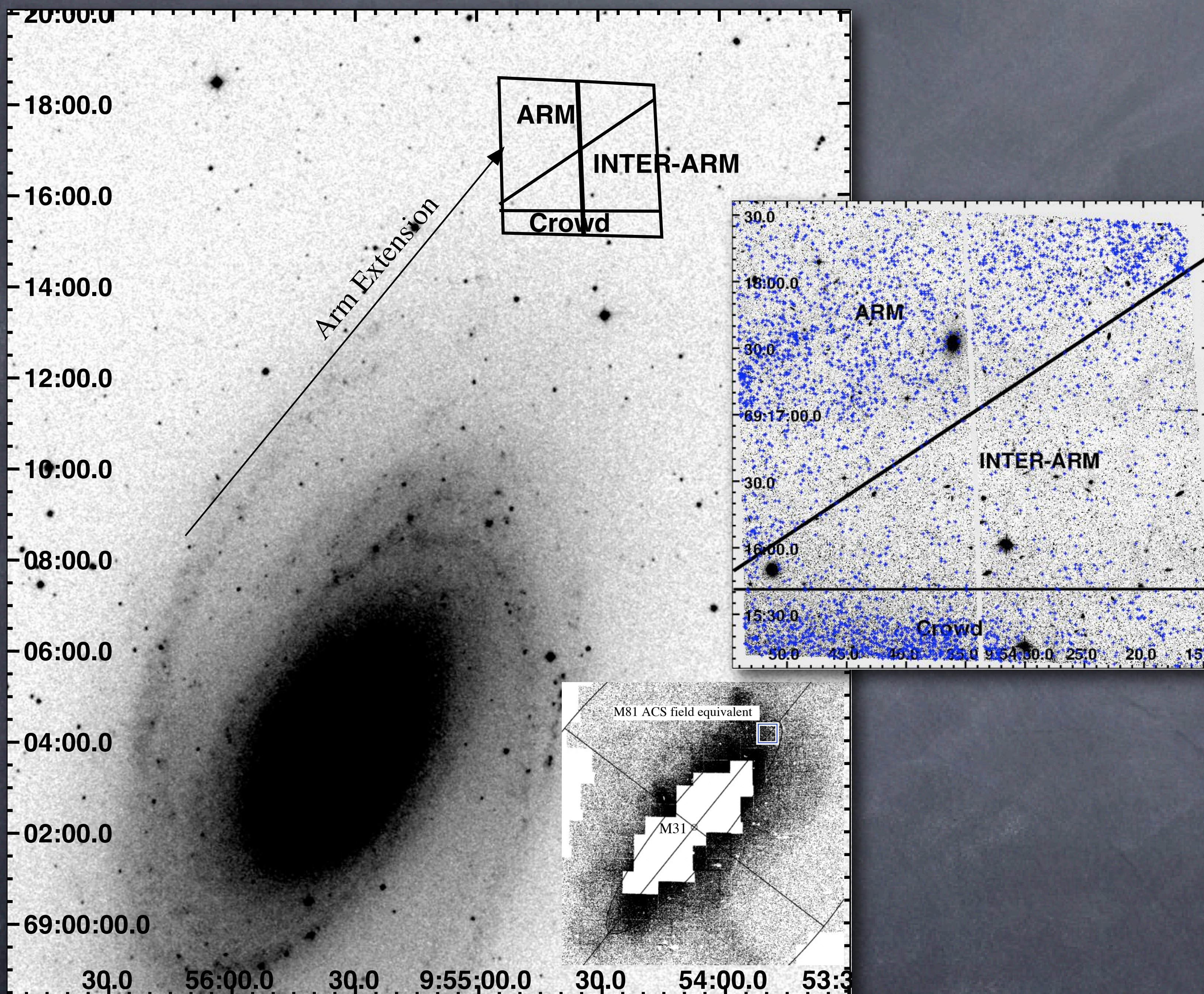
Age-Z Relation

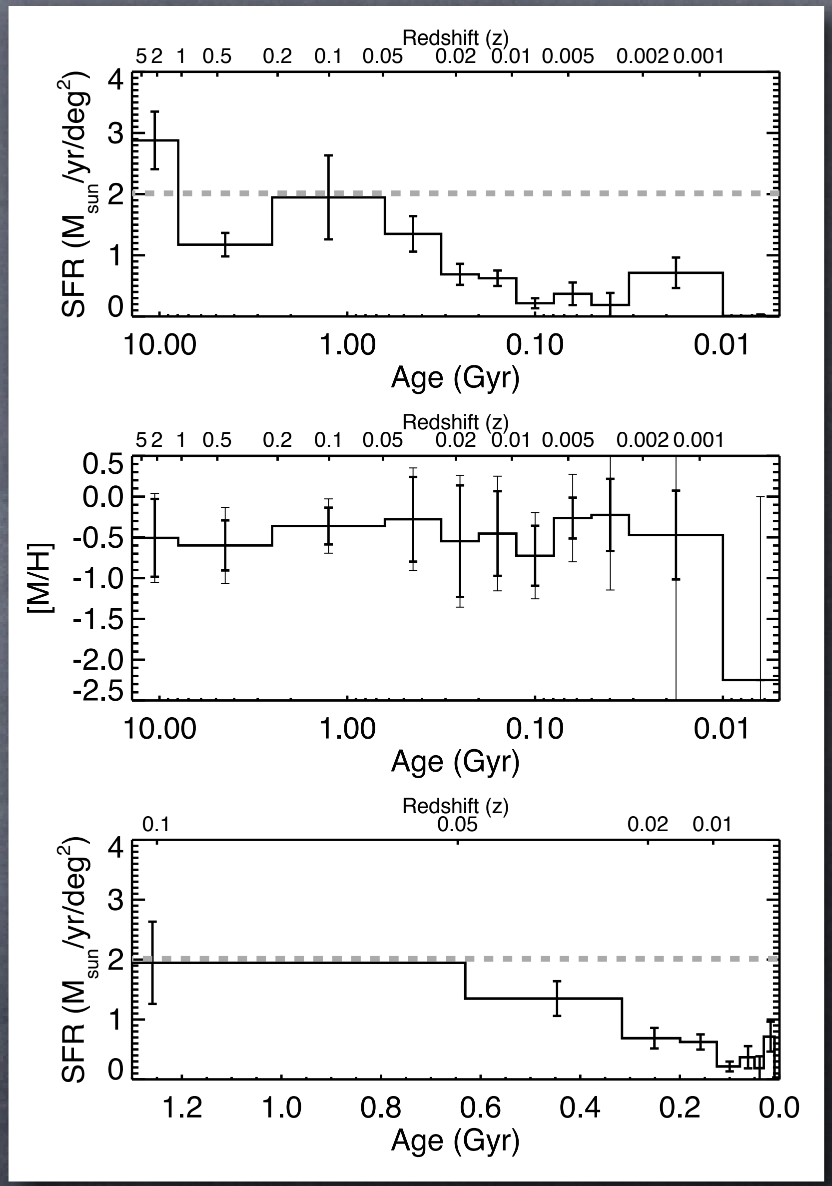
SFR



Williams et al. (2008)

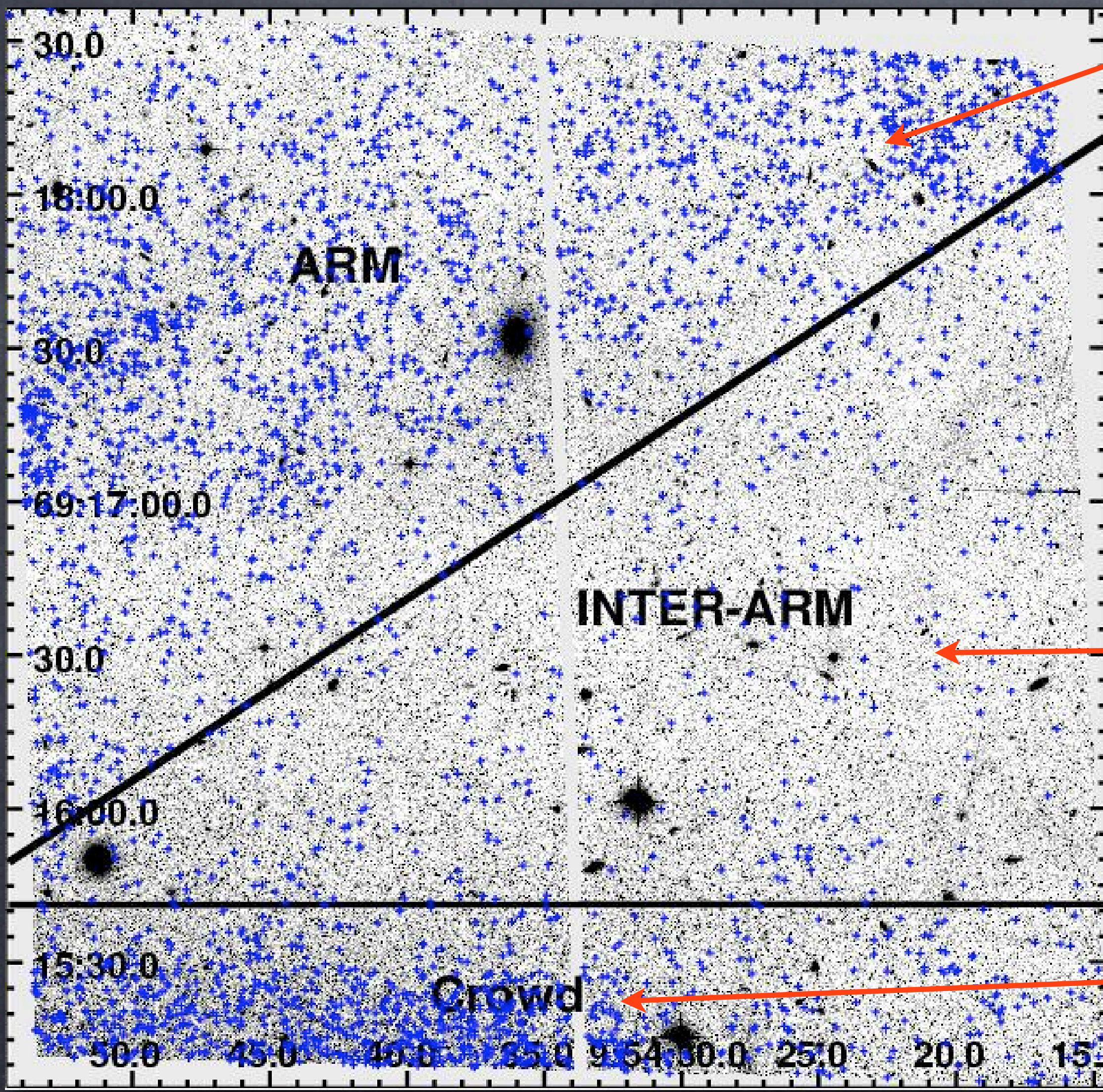
Resolved Composite Stellar Population

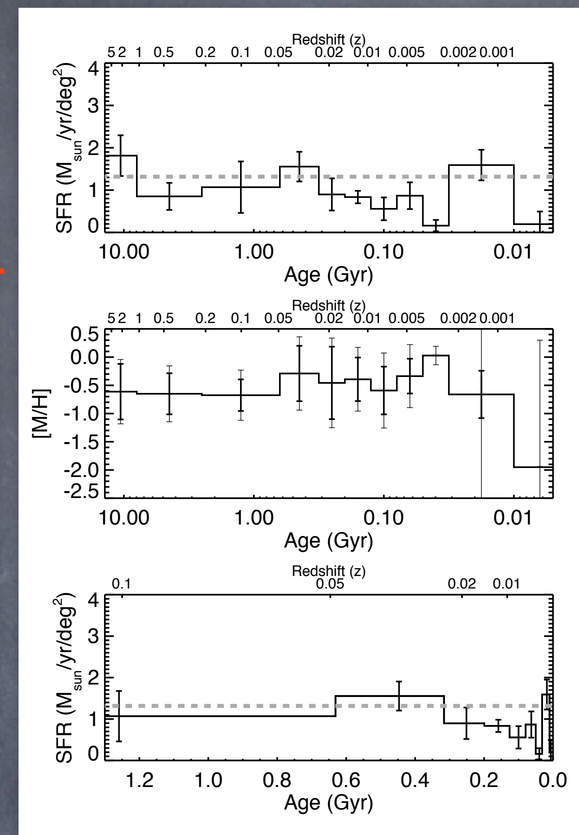


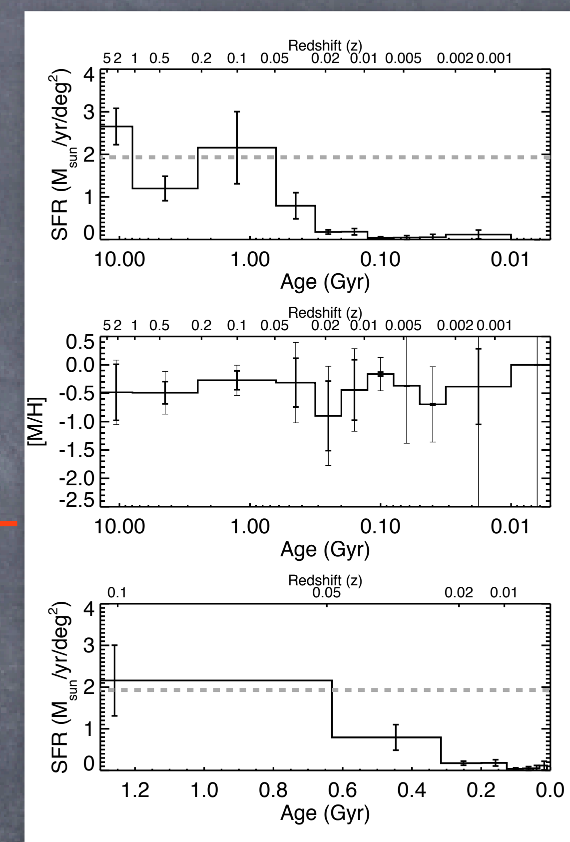


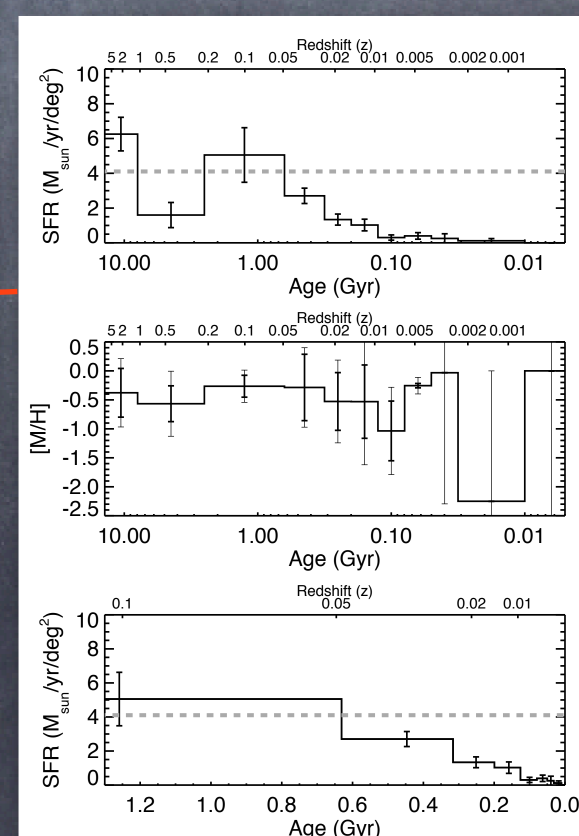
Williams et al. (2008)

Resolved Composite Stellar Population







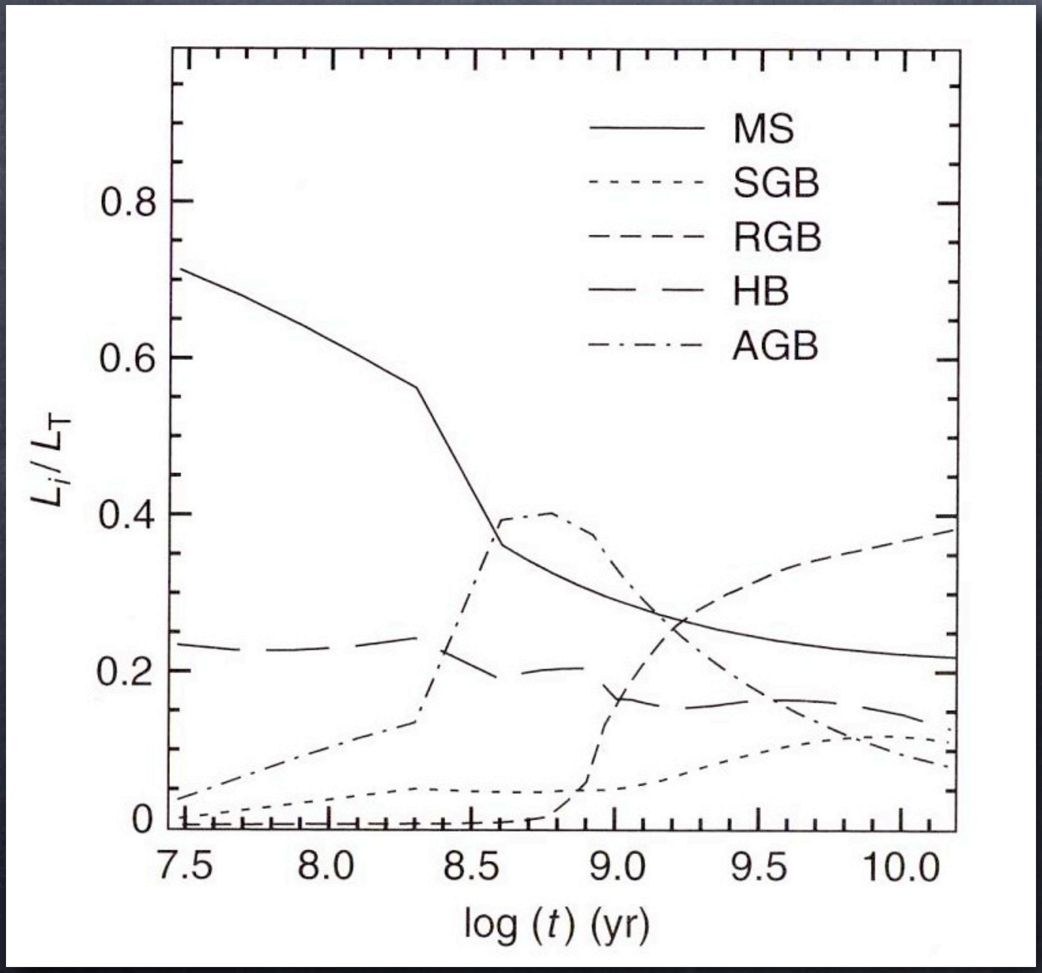


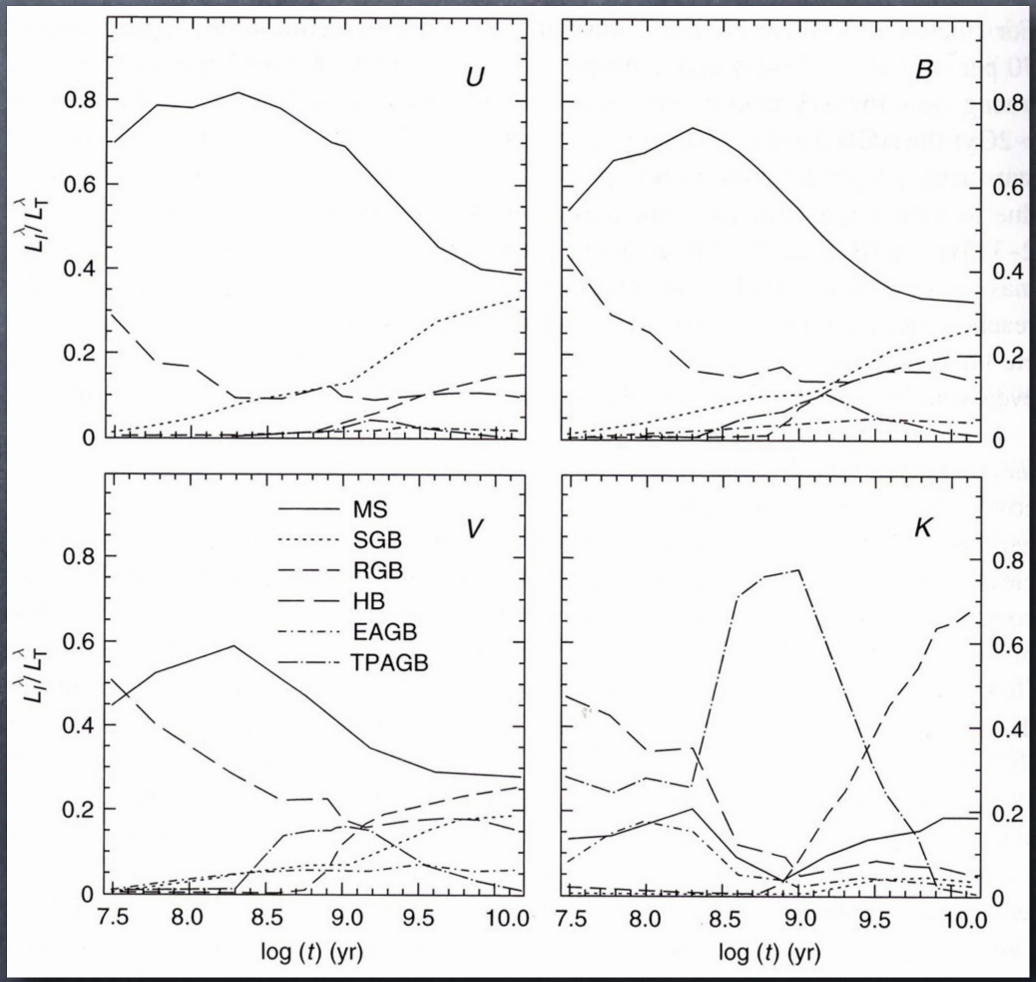
Williams et al. (2008)

Unresolved Simple Stellar Population

At a given wavelength, the integrated light of an unresolved stellar population is composed of the various contributions $f_\lambda(m,t,Z)$ from stars of a given mass and metallicity, integrated over the IMF $\Phi(m)$.

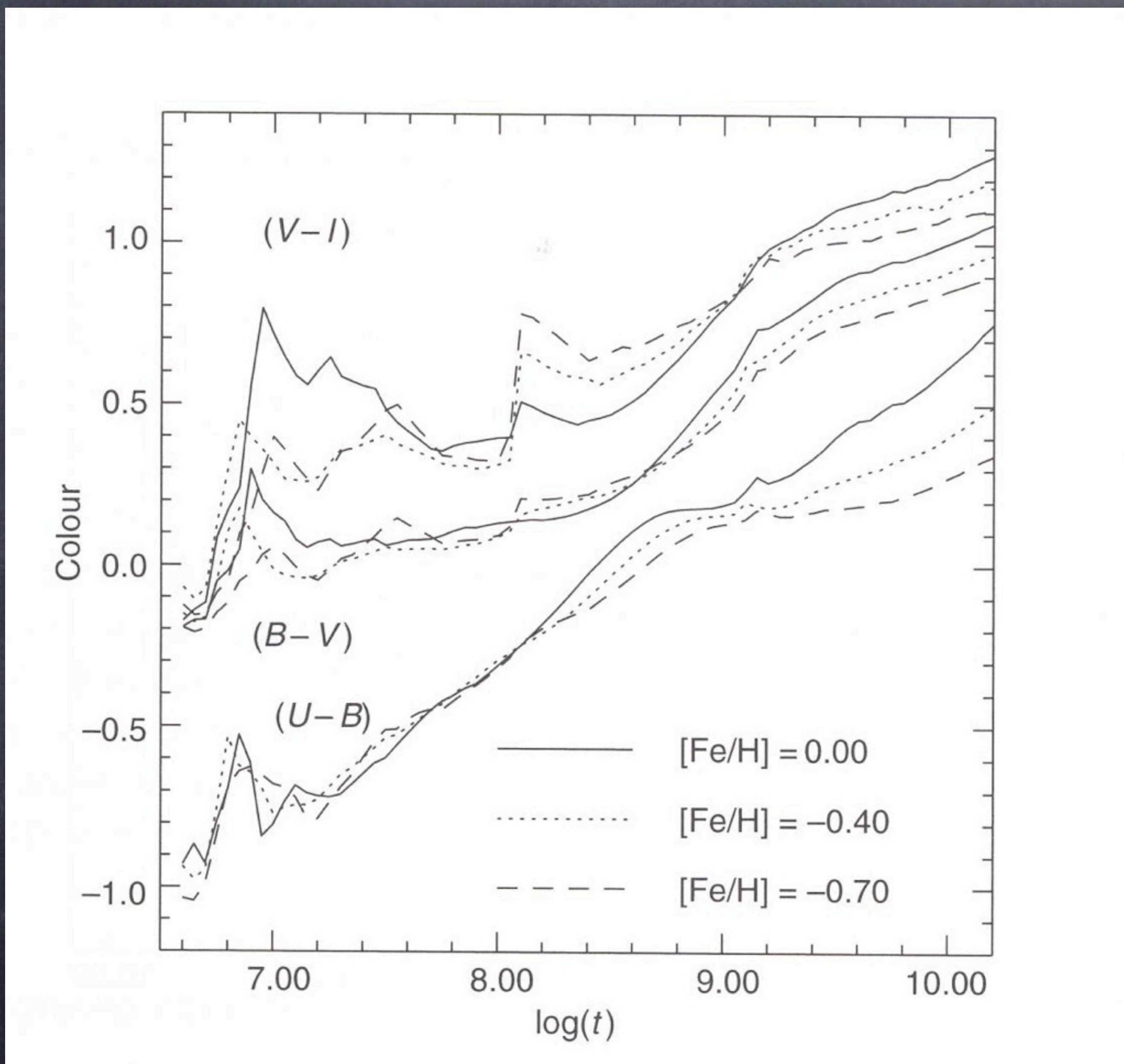
$$F_\lambda(t,Z) = \int_{m_1}^{m_2} f_\lambda(m,t,Z)\Phi(m)dm$$

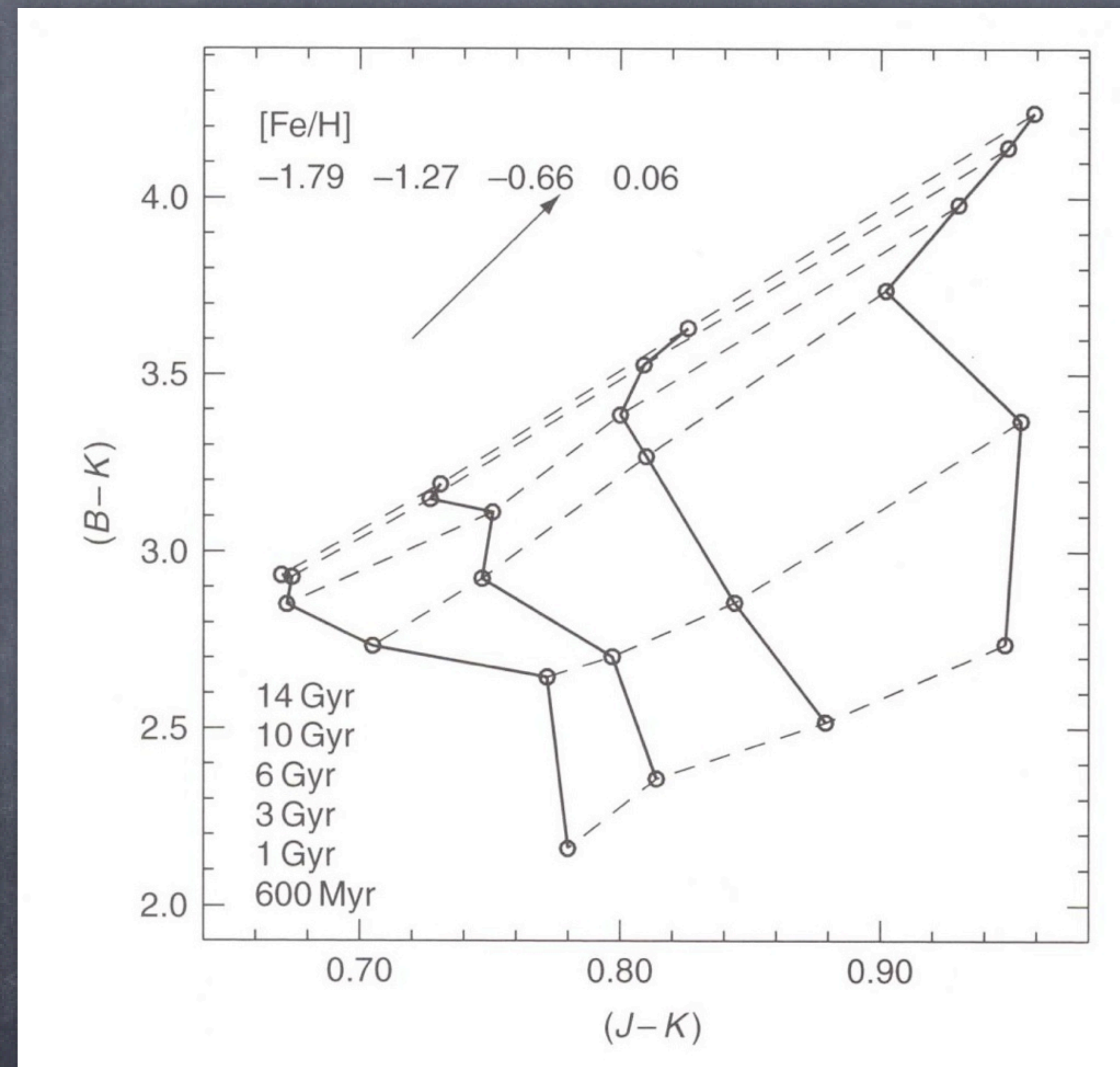




Unresolved Simple Stellar Population

The evolution of broad-band photometric colors can be modeled and the best combination to break the age-metallicity degeneracy can be found.





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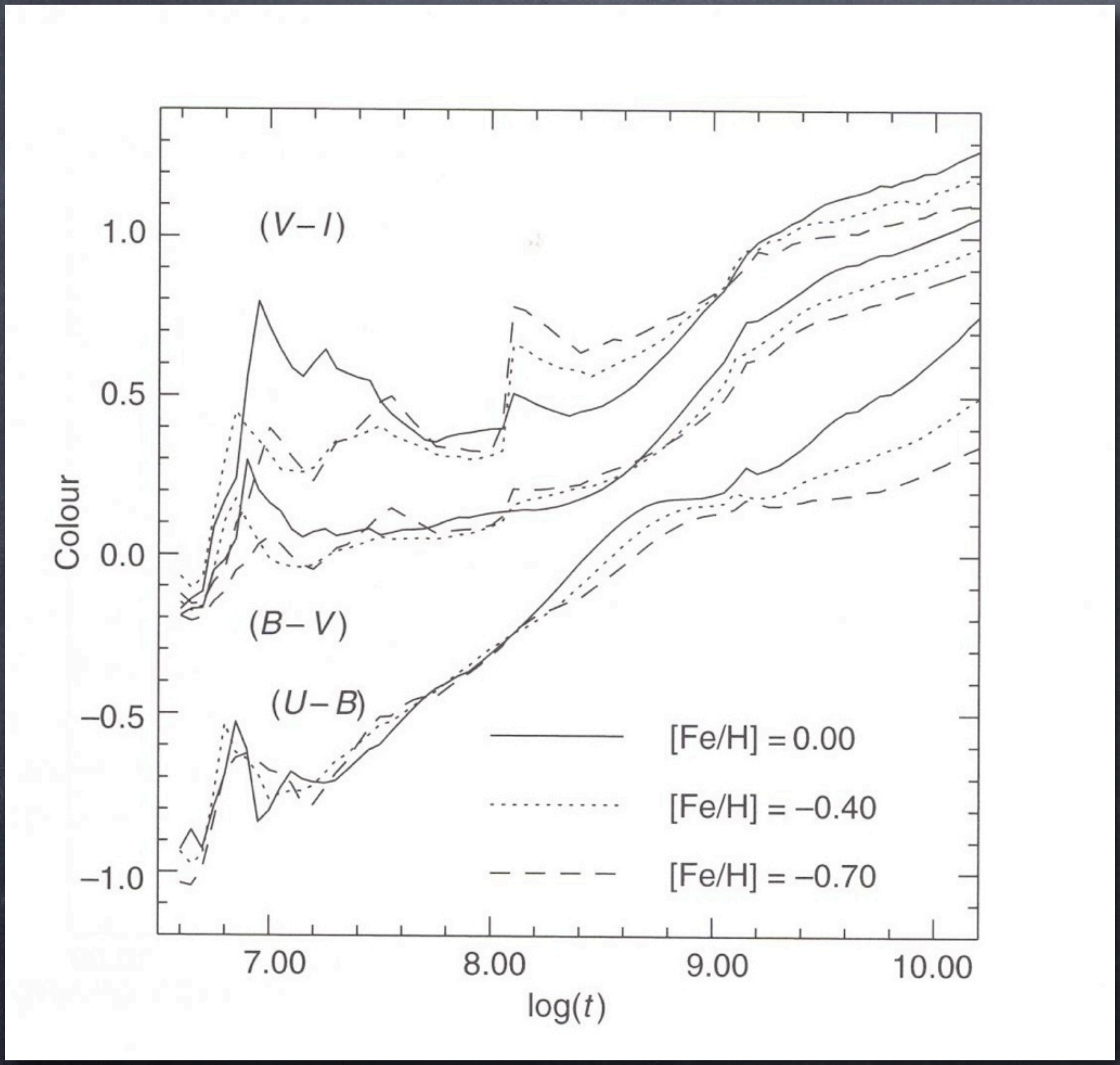
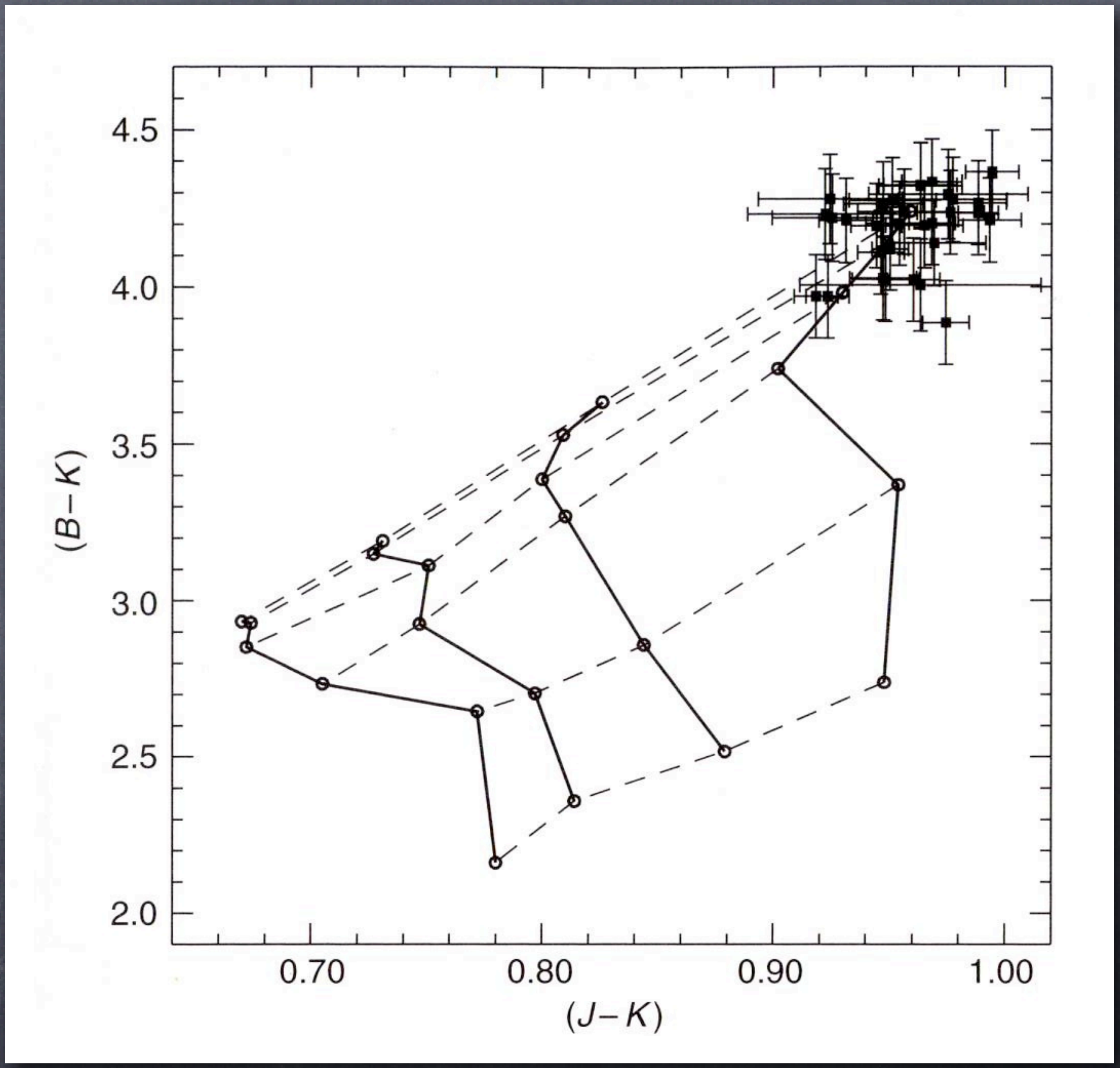
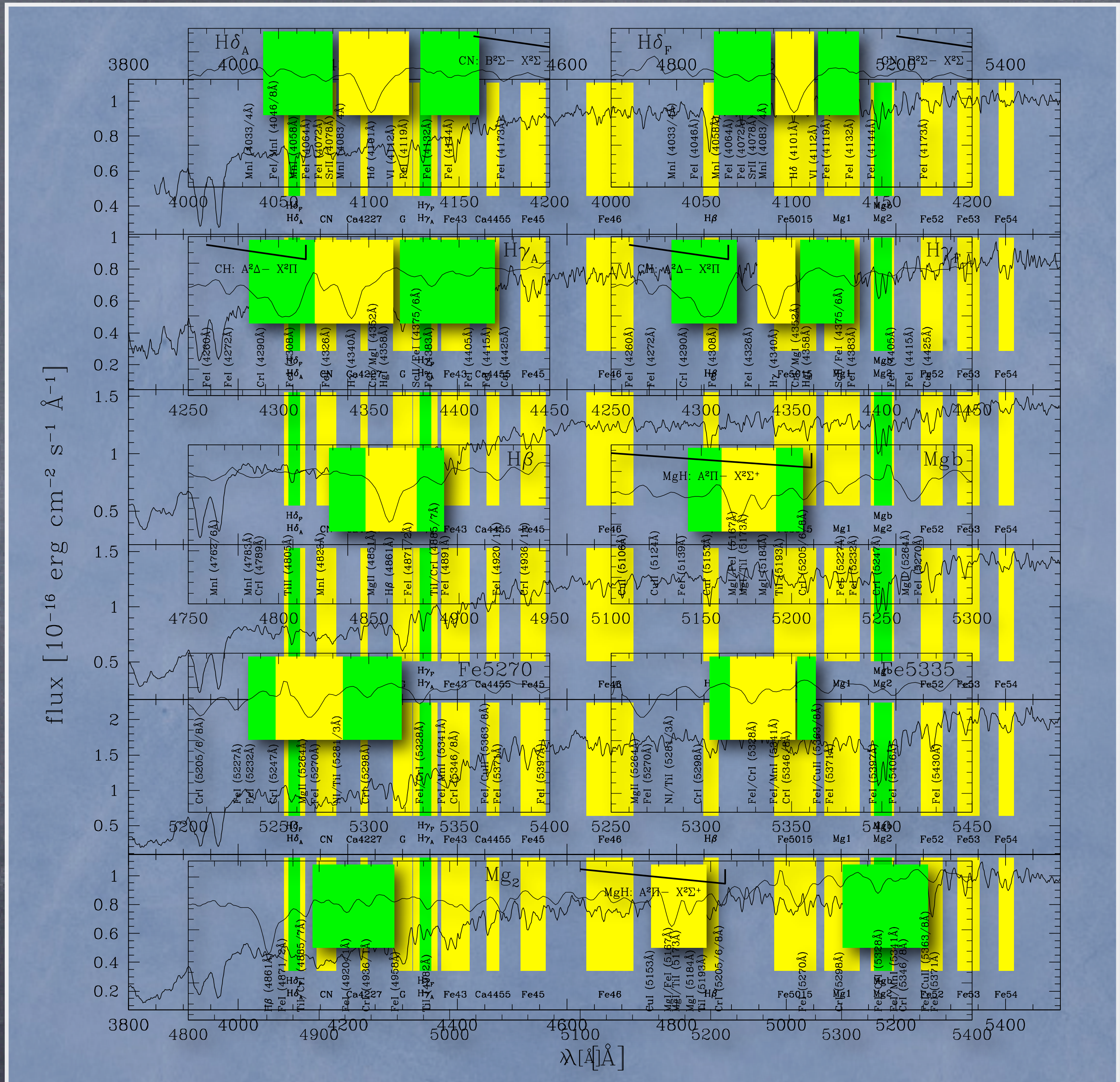


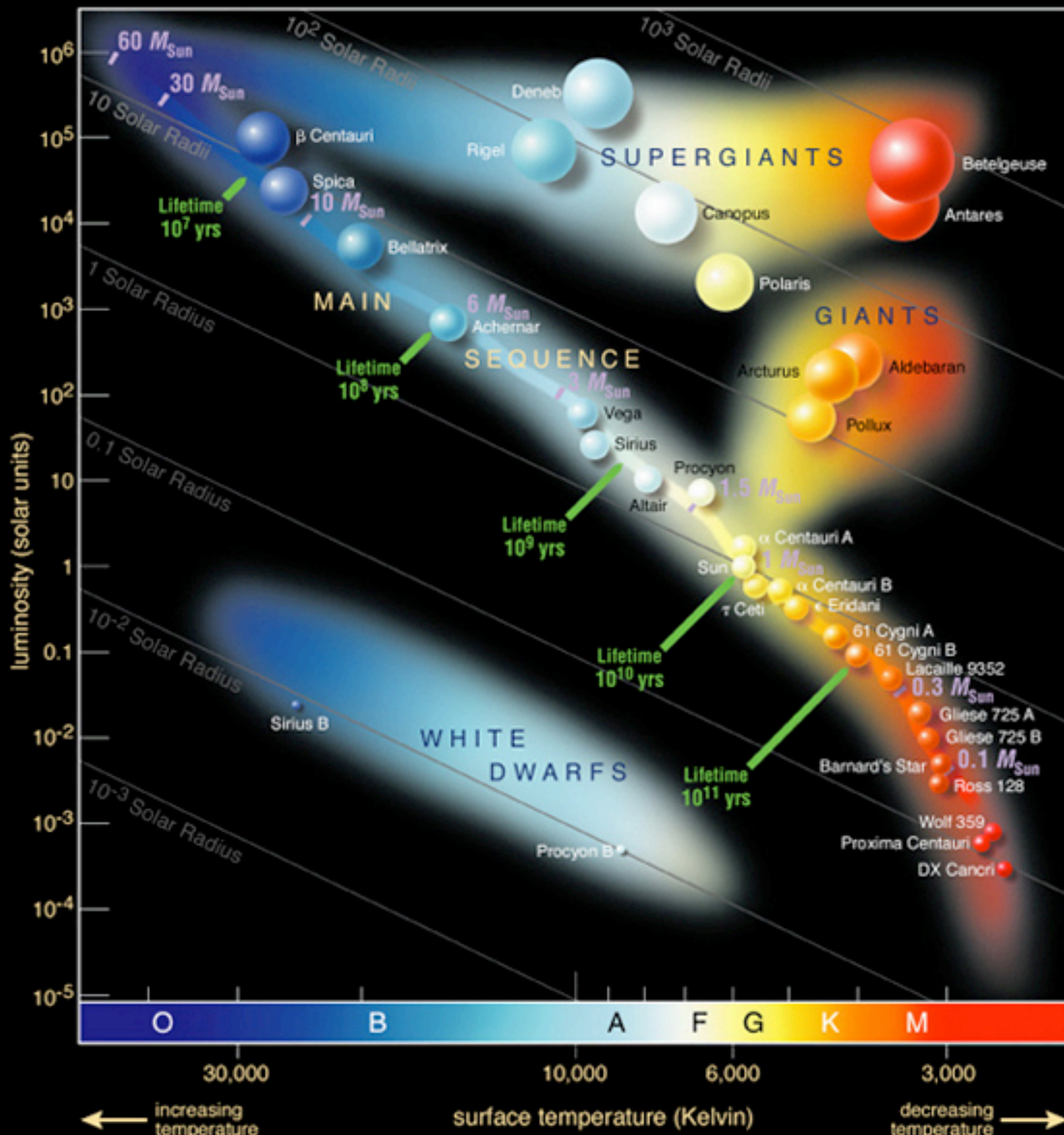


GCS spectroscopy: Lick index system

- defined in the 80s by the Lick group (Burstein, Faber, Worthey et al.)
- 25+ indices that cover 4100-6400 Å
- Lick system provides “simple” means to calculate theoretical index predictions
- designed to investigate stellar populations of giant elliptical galaxies ⇒ 8-12 Å resolution



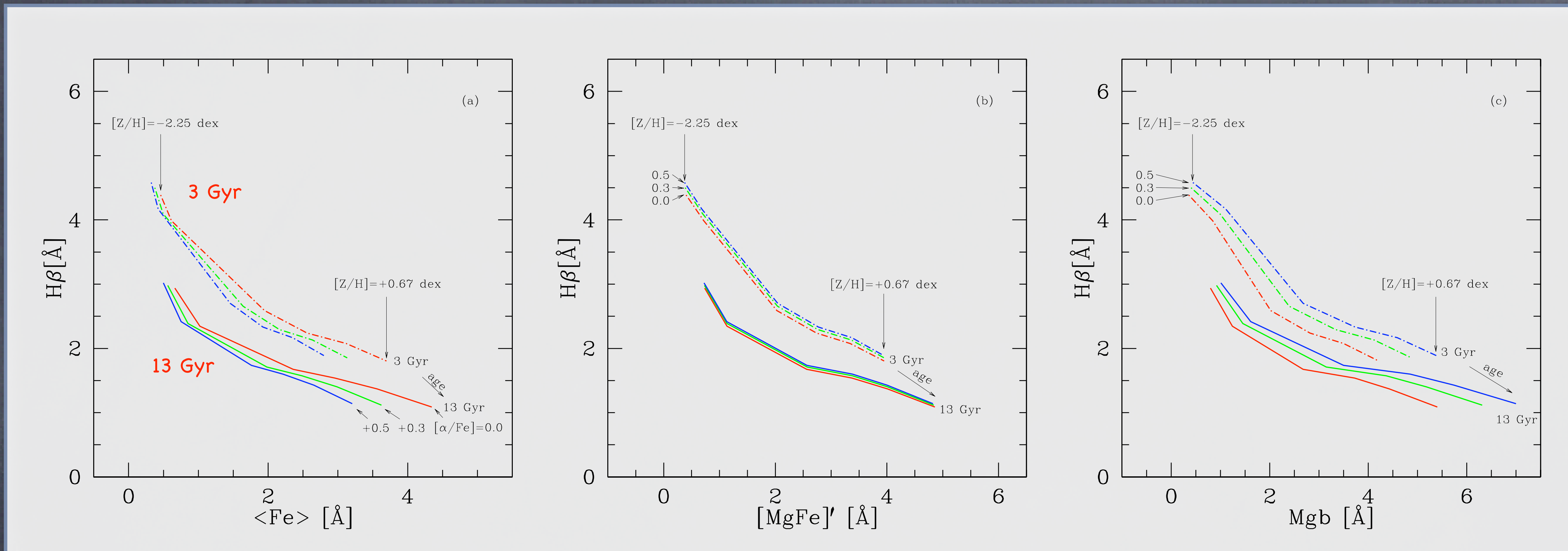
⇒ metallicities, chemical compositions, ages



60 M_Sun
30 M_Sun
10 M_Sun
6 M_Sun
3 M_Sun
1.5 M_Sun
1 M_Sun
0.3 M_Sun
0.1 M_Sun
10^2 Solar Radii
10^3 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^-2 Solar Radius
10^-3 Solar Radius
Deneb
Rigel
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Procyon
Altair
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
Canopus
Polaris
Betelgeuse
Antares
Arcturus
Aldebaran
Pollux
SUPERGIANTS
GIANTS
MAIN
SEQUENCE
WHITE
DWARFS
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^10 yrs
Lifetime 10^11 yrs
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^-2
10^-3
10^-4
10^-5
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature

Age-Metallicity Diagnostic Plots

age indicator



metallicity indicator

Puzia et al. (2005)

best metallicity indicator:

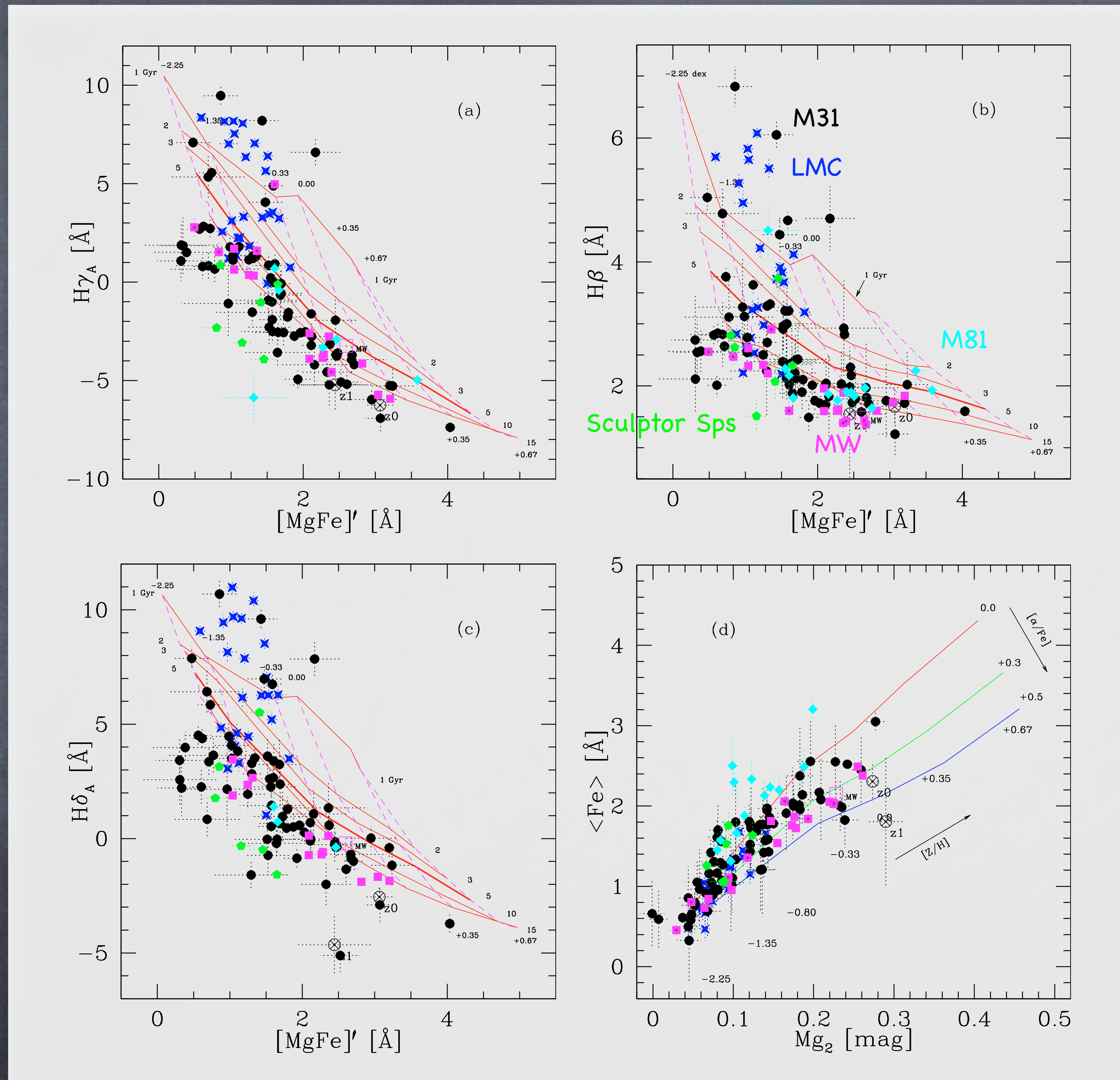
$$[\mathrm{MgFe}]' = \sqrt{\mathrm{Mg}b \cdot (0.72\,\mathrm{Fe5270} + 0.28\,\mathrm{Fe5335})}$$

best age indicator:

$$H'(t) = \sum_i^n c_i H_i(t, Z, [\alpha/\mathrm{Fe}])$$

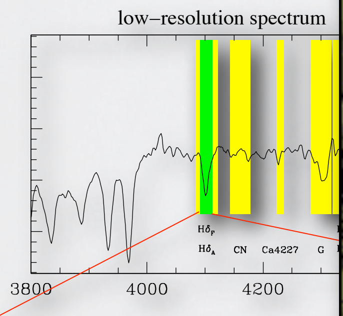
SSP models: Thomas et al. (2003)

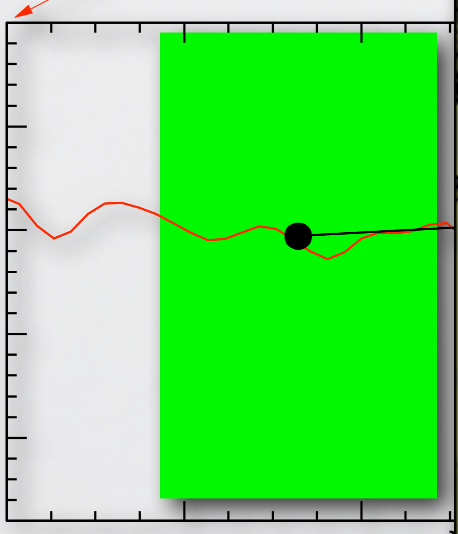
M31 GC ages and chemical compositions

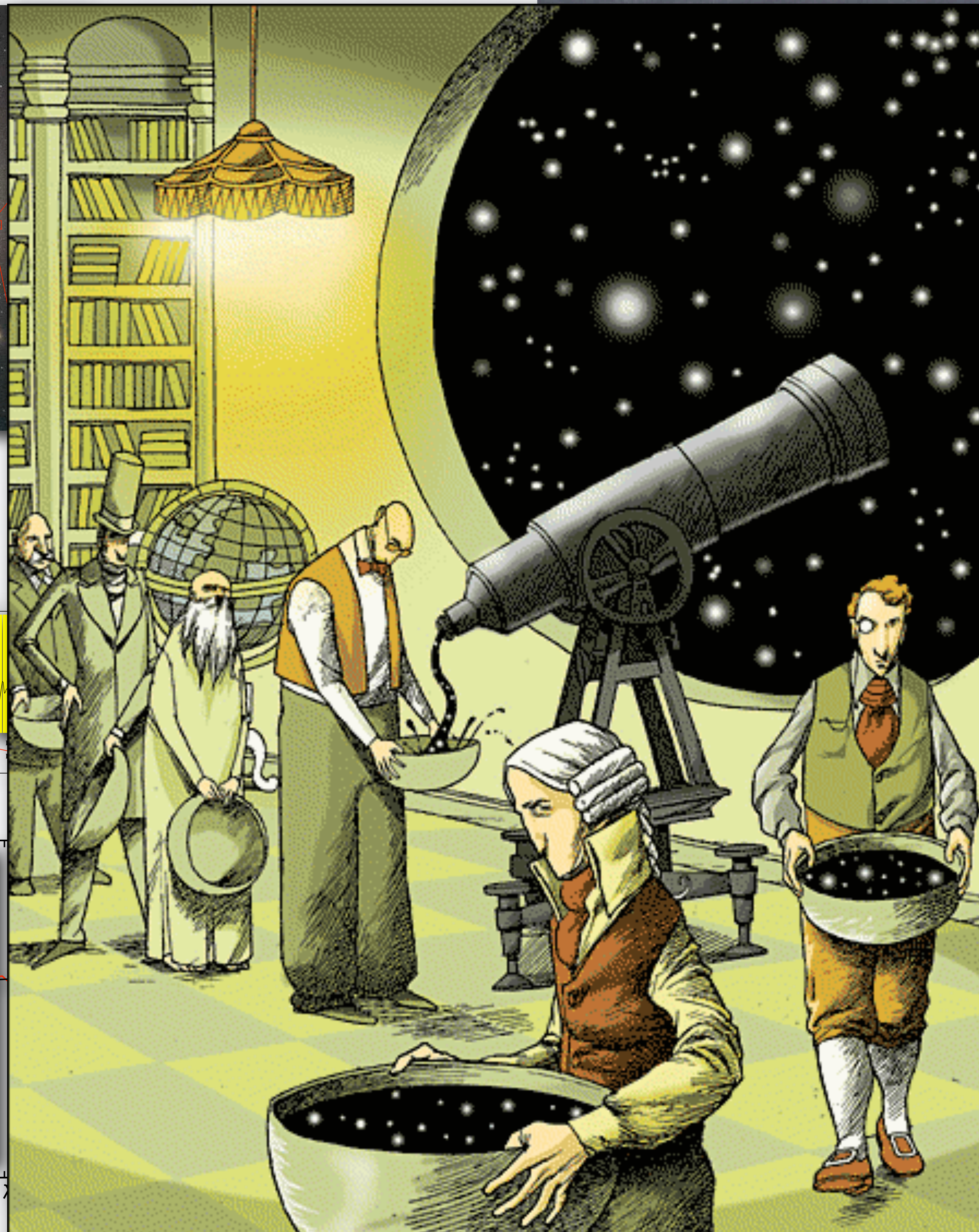


Puzia, Perrett, Bridges (2005)

M87, a giant elliptical galaxy

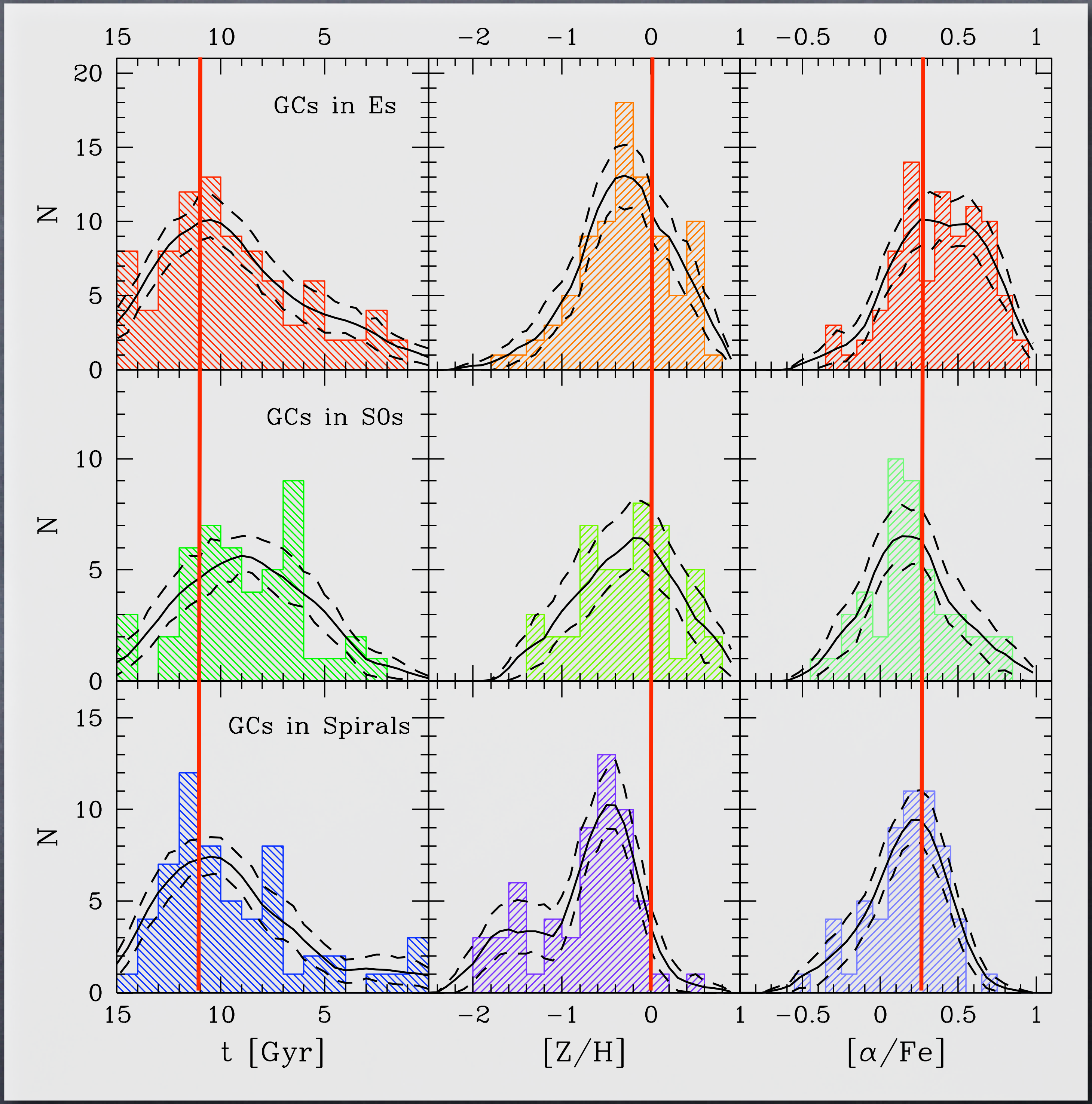






GC ages, metallicities and [α/Fe] ratios

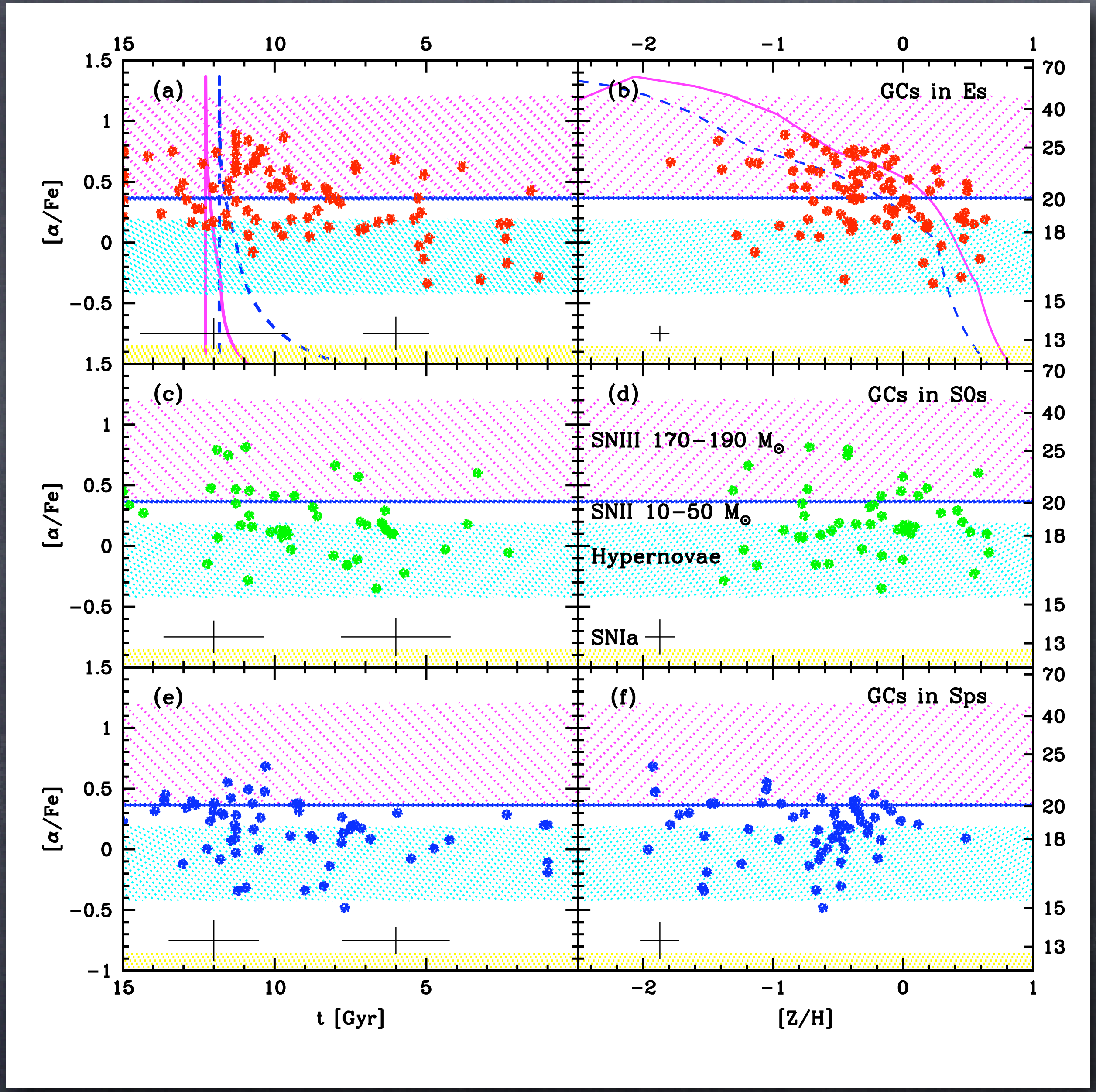
- GCs in Es and Spirals have similar mean age
- GCs in E/Sp are on average older than GCs in S0s
- GCs in Es and S0s reach higher [Z/H] than in spirals
- GCs in Es have highest mean [α/Fe] ratios



Puzia et al. (2006)

Comparison with SN ejecta models

monolithic models are solid (10^{11} M$_\odot$) and dashed lines (10^{12} M$_\odot$)



pair-inst. SNe (170-190 M$_\odot$)
taken from Heger & Woosley (2002)

type-II SNe (13-70 M$_\odot$)
taken from Nomoto et al. (1997)

hypernovae (>10^{52} erg)
taken from Nakamura et al. (2001)

type-Ia SNe
taken from Nomoto et al. (1997)

Puzia, Kissler-Patig, Goudfrooij (2006)

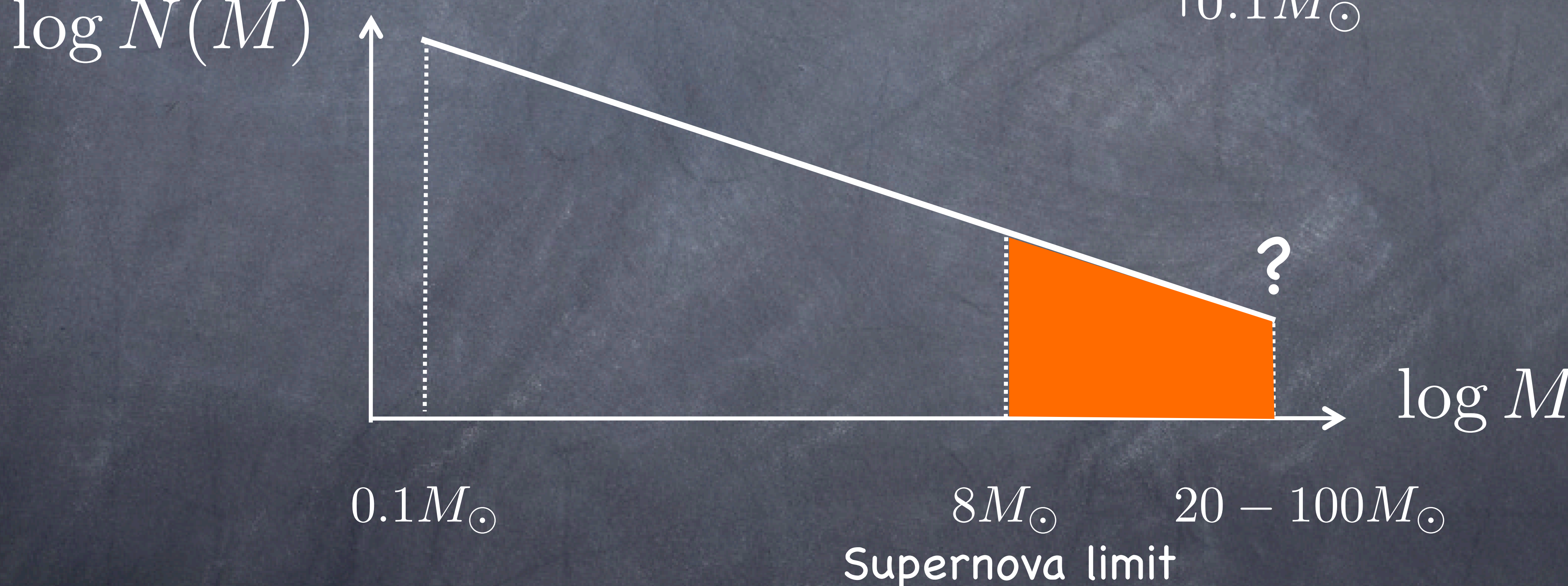
GCs in early-type galaxies show signatures of massive type-II and/or PI-SNe

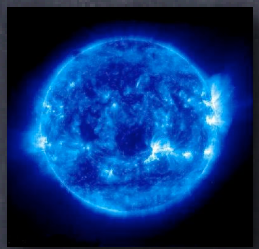
Stellar Mass Function and SN Rates

How many supernovae per year does a stellar population produce?

Use power-law IMF, Salpeter slope -2.35 ≈ -7/3

$$N(M) \propto M^{-7/3} \Big|_{0.1M_\odot}^{20-100M_\odot}$$





Stellar Mass Function and SN Rates

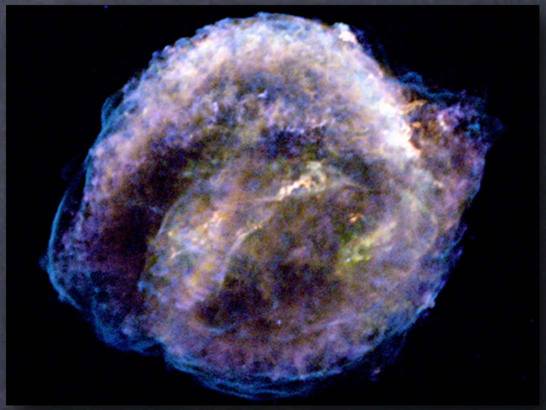
Number of stars

$$\int \Phi(M)dM = c\int_{m_l}^{m_u} M^{-\gamma}dM = \frac{c}{1-\gamma}M^{1-\gamma}\Big|_{m_l}^{m_u} \quad \text{if } \gamma \neq 1$$

Fraction of stars with $20M_\odot > M > 8M_\odot$ over the IMF is then

$$f_N = \frac{\int_8^{20} M^{-7/3}dM}{\int_{0.1}^{20} M^{-7/3}dM} = \frac{\#\text{supernovae}}{\#\text{stars}}$$

$$f_N = \frac{M^{-4/3}\ |_8^{20}}{M^{-4/3}\ |_{0.1}^{20}} = \frac{0.0184 - 0.0625}{0.0184 - 21.544}$$



$f_N = 0.002 = 0.2\% \Rightarrow$ 500 stars make 1 supernova!

Stellar Mass Function and SN Rates

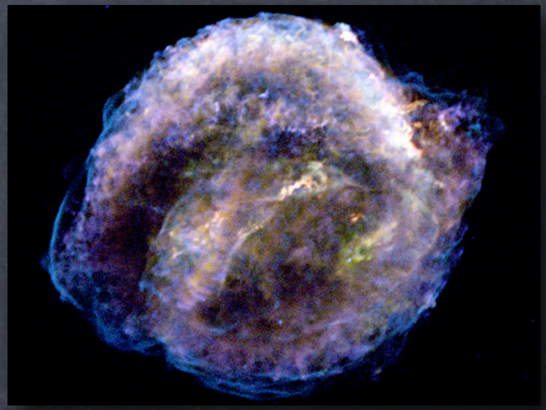
SNe are rare (depending on the slope of the IMF), but each is very massive.

What fraction of the mass goes into SNe?

$$f_M = \frac{\int_8^{20} M \cdot M^{-7/3} dM}{\int_{0.1}^{20} M \cdot M^{-7/3} dM}$$

$$f_M = \frac{M^{-1/3} \mid_8^{20}}{M^{-1/3} \mid_{0.1}^{20}} = \frac{0.368 - 0.5}{0.368 - 2.154}$$

supernova mass fraction: $f_M = 0.0738 = 7.4\%$

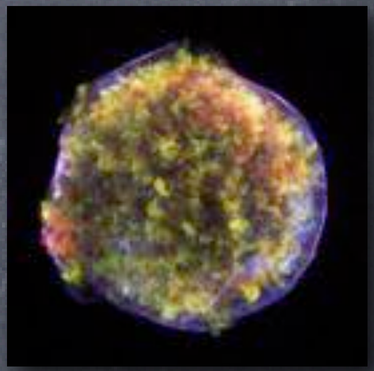


Stellar Mass Function and SN Rates

What is the mean SN mass?

$$\langle M \rangle = \frac{\int\limits_{8}^{20} M \cdot M^{-7/3} dM}{\int\limits_{8}^{20} M^{-7/3} dM} = \frac{\frac{1}{-1/3} M^{-1/3} \Big|_{8}^{20}}{\frac{1}{-4/3} M^{-4/3} \Big|_{8}^{20}}$$

$$= \frac{4(0.368 - 0.5)}{0.0184 - 0.0625} = 11.97 \approx 12$$



median SN mass:

$$\frac{1}{2} = \frac{\int\limits_{8}^{\bar{M}_{\rm SN}} M \cdot M^{-7/3} dM}{\int\limits_{8}^{20} M \cdot M^{-7/3} dM} = \frac{\bar{M}_{\rm SN}^{-1/3} - 0.5}{0.368 - 0.5} \Rightarrow \bar{M}_{\rm SN} \simeq 12.23$$

SN Rates in Galaxies

We can now calculate the SN rate in a spiral galaxy with a typical star formation rate

$$SFR = 8 M_\odot \mathrm{yr}^{-1}$$

where 7.4% of the stellar mass detonate in SN explosions. That implies

$$8 \times 0.074 \simeq 0.6 M_\odot \mathrm{yr}^{-1}$$

go into SN explosions!

Hence the SN rate is:

$$\frac{0.6 M_\odot \mathrm{yr}^{-1}}{\bar{M}_{\mathrm{SN}}} = \frac{0.6 M_\odot \mathrm{yr}^{-1}}{12.23 M_\odot} = 0.048 \mathrm{yr}^{-1} \approx 1 \mathrm{SN}/20 \mathrm{yr}$$

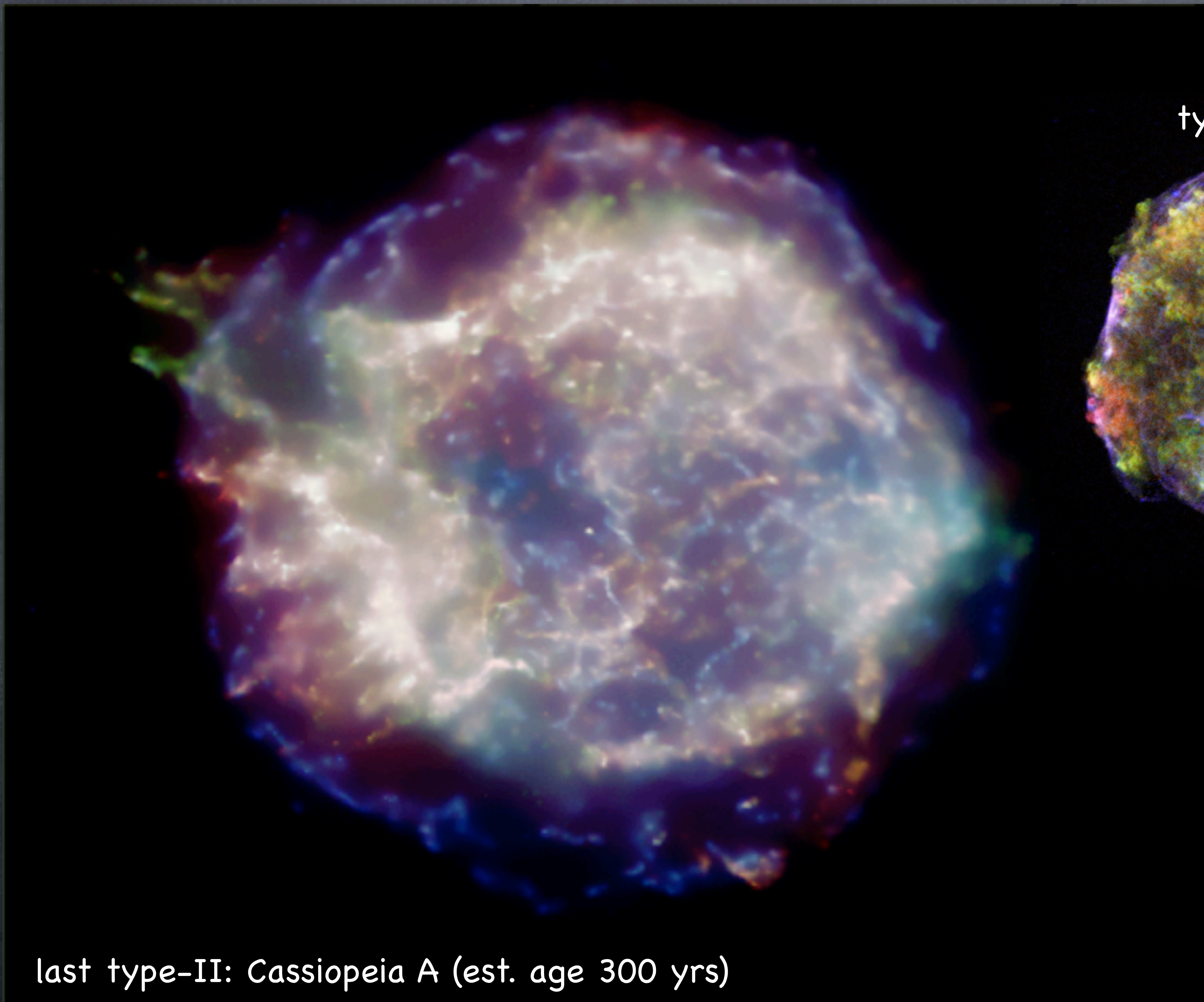
SN Rates in Galaxies

The star formation rate drives the chemical enrichment history of a galaxy. Let's compare the mean star formation rates and the resulting SN rates for various galaxy types:

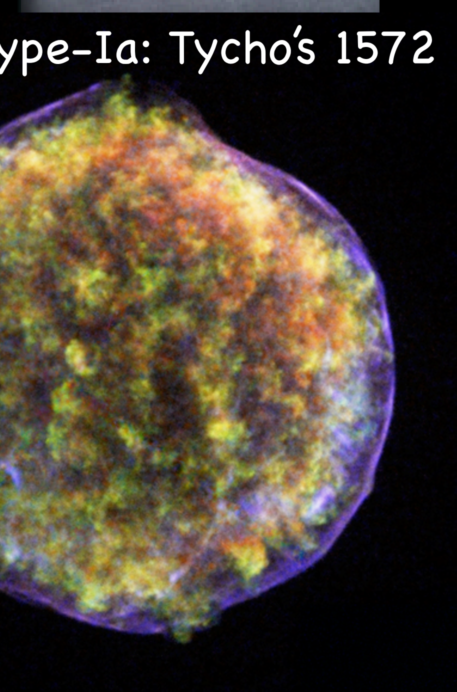
	star formation rate	supernova rate
Spiral Galaxy:	$\sim 8 M_\odot \mathrm{yr}^{-1}$	$\sim 1\mathrm{SN}/20\mathrm{yr}$
Elliptical Galaxy:	$\sim 5 \cdot 10^{-3} M_\odot \mathrm{yr}^{-1}$	$\sim 6\ \mathrm{SN}/10^6\mathrm{yr}$
Irregular Galaxy:	$\sim 0 - 100 M_\odot \mathrm{yr}^{-1}$	$\sim 60\ \mathrm{SN}/\mathrm{yr}$

SNe in the Milky Way

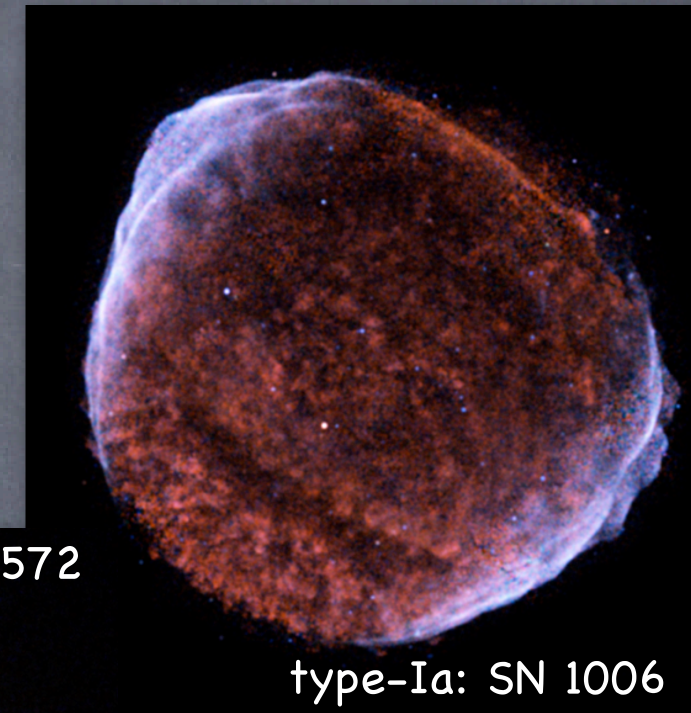
Last SN explosions in the Milky Way - the next one is overdue!



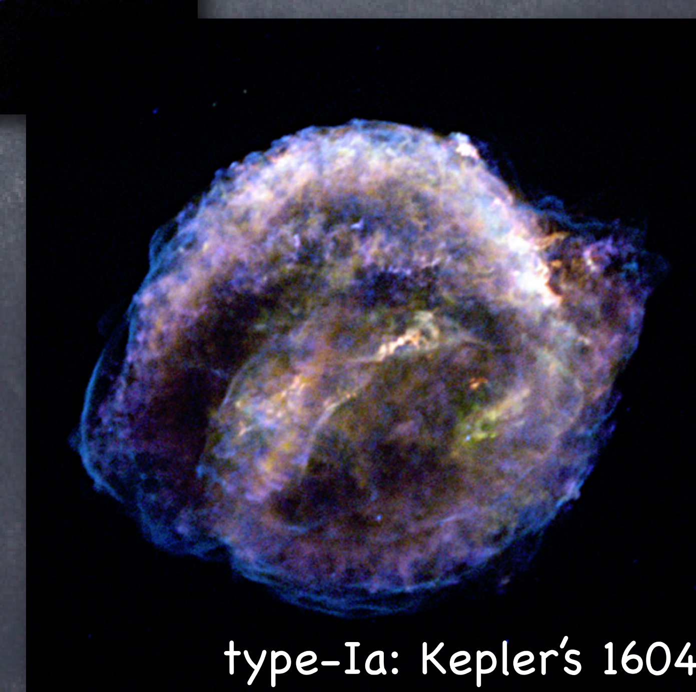
last type-II: Cassiopeia A (est. age 300 yrs)



type-Ia: Tycho's 1572



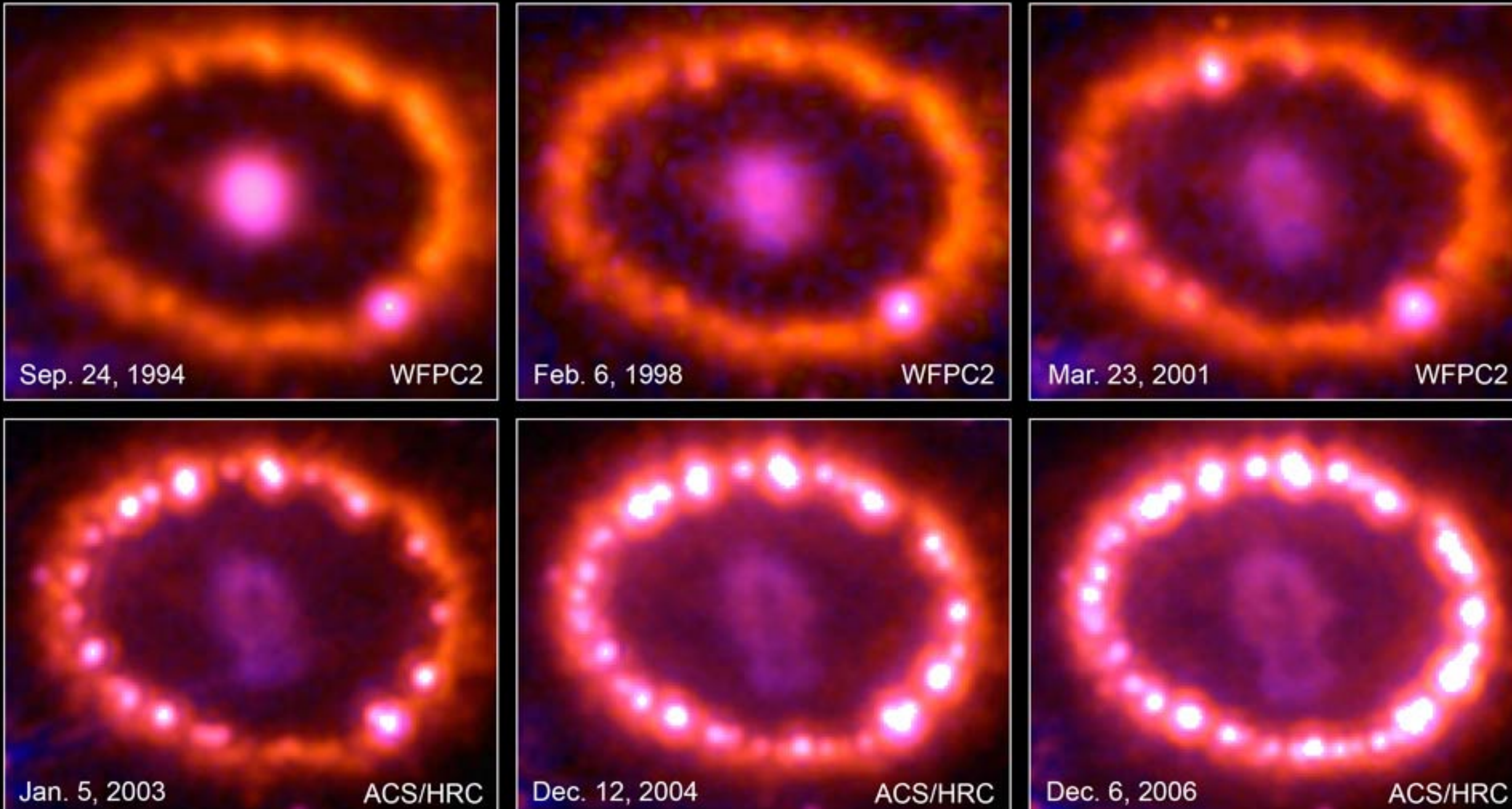
type-Ia: SN 1006



type-Ia: Kepler's 1604

all images are from Chandra X-ray observatory

Progenitor star visible: ~20 M_{sol} blue supergiant



Supernova 1987A • 1994-2006

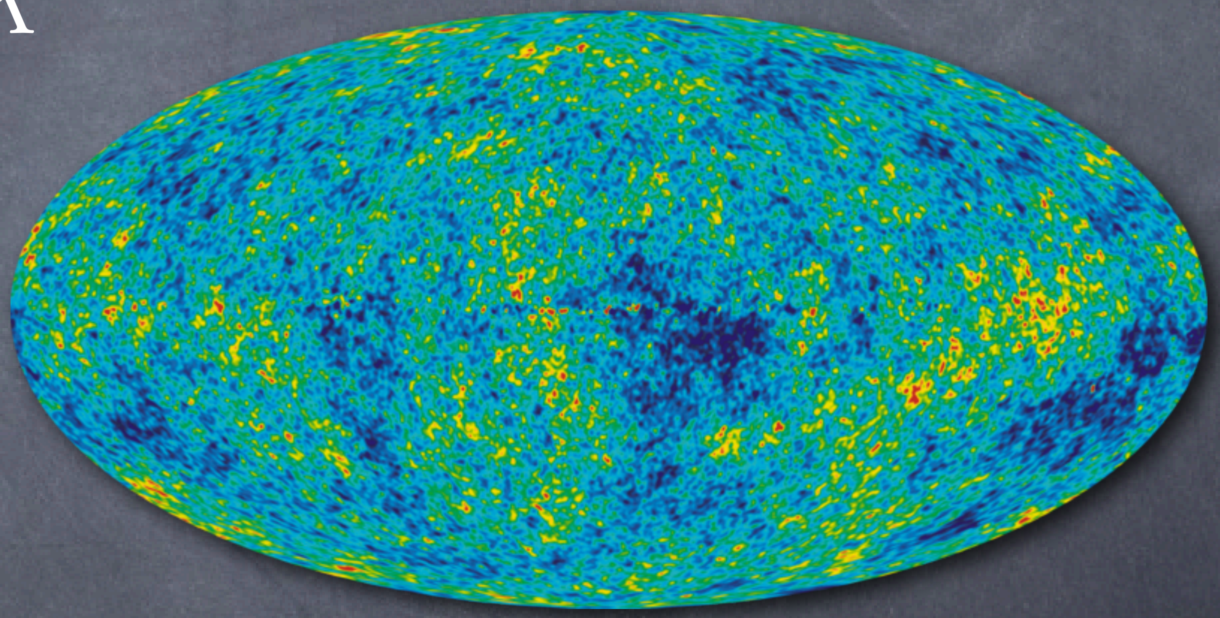
Hubble Space Telescope • WFPC2 • ACS

NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics) STScI-PRC07-10b

Galaxy Formation

Galaxy formation starts with the ripples in the density field of the early Universe. We find anisotropies in the CMB temperature

$$\frac{\Delta\rho}{\rho} \sim \frac{\Delta T}{T} \approx 10^{-5} K$$



at the time of decoupling.

These fluctuations are the seeds for halo formation

that grow governed by dark matter to form today's galaxies.

How do we study the formation and evolution of galaxies?

1. By looking at galaxy formation directly:
   high-z star-forming galaxies: LyBG, Sub-m., EROs, etc.

2. By looking at large galaxy samples over a redshift range:
   statistical analysis of galaxy parameters: evolution of Luminosity (Mass) Function and Scaling Relations

3. By looking at “fossil records” in nearby galaxies:
   Stellar Remnants, X-ray Halos, and star clusters

Galaxy Formation

The three key galaxy formation models:

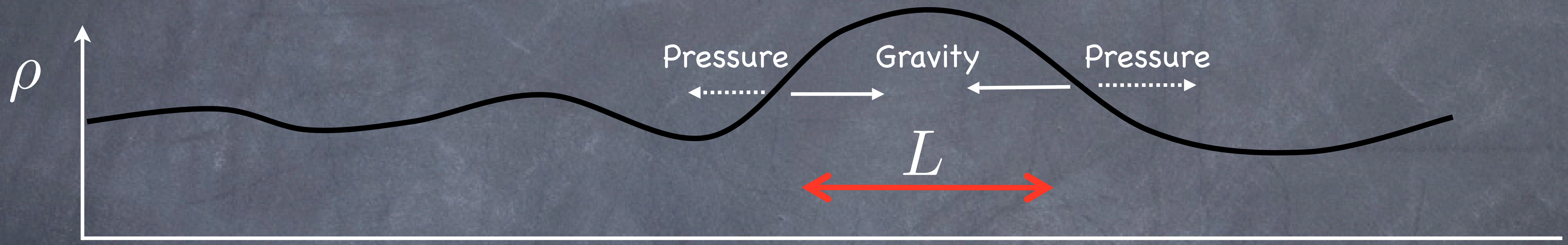
1. Monolithic Collapse (top down): collapse of individual gas clouds early in the history of the Universe.

2. Hierarchical Merging (bottom-up): formation of galaxies through the merging and accretion of many smaller ones.

3. Secular Evolution: formation as a result of internal processes, such as the actions of spiral arms and bars.

Jean's criterion for gravitational instability

Which ripples will collapse ?



Gravity pulls matter in. Pressure pushes it back out.

When pressure wins -> stable oscillations (sound waves).

When gravity wins -> collapse.

Cooling lowers pressure, triggers collapse.

Applies to both Star Formation and Galaxy Formation.

When does Gravity win?

Let's consider N molecules of mass m, in a sphere of size R, at Temp. T

Gravitational Energy: $$E_G \propto \frac{GMM}{R^2}$$

Thermal Energy: $$E_T \propto N k_B T$$ $$M = Nm \propto R^3 \rho$$

Ratio: $$\frac{E_G}{E_T} \propto \frac{GM^2}{RNk_BT} \propto \frac{G\rho R^3 m}{Rk_BT}$$

Jeans Length: $$R_J = \sqrt{\frac{k_BT}{G\rho m}}$$

Gravity wins when $R > R_J$.

When does Gravity win?

Let's consider N molecules of mass m, in a sphere of size R, at Temp. T

Gravitational Energy: $$E_G \propto \frac{GMM}{R^2}$$

Thermal Energy: $$E_T \propto N k_B T \qquad M = Nm \propto R^3 \rho$$

Ratio: $$\frac{E_G}{E_T} \propto \frac{GM^2}{RNk_BT} \propto \frac{G\rho R^3 m}{Rk_BT}$$

Jeans Length: $$R_J = \sqrt{\frac{k_B T}{G\rho m}}$$

Gravity wins when $R > R_J$.

Jean's instability - recap.

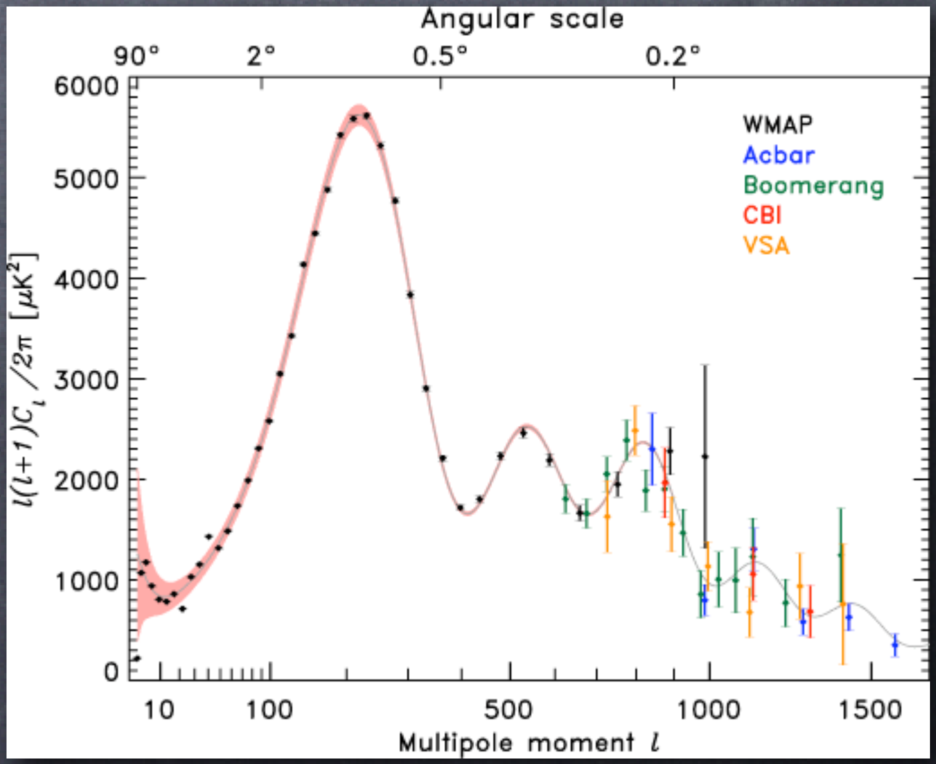
Gravity tries to pull material in.

Pressure tries to push it out.

Gravity wins for $R > R_J$ ----> large regions collapse.

Pressure wins for $R < R_J$ ----> small regions oscillate.

Jeans Length: $$R_J = \sqrt{\frac{k_B T}{G \rho m}}$$



Ergo: Large cool dense regions collapse!

Collapse Timescale

Ignore Pressure!

Gravitational acceleration: $$g \propto \frac{GM}{R^2} \propto \frac{R}{t^2} \qquad M \propto \rho R^3$$

Time to collapse: $$t_G \propto \sqrt{\frac{R}{g}} \propto \sqrt{\frac{R^3}{GM}} \propto \frac{1}{\sqrt{G\rho}}$$

This is the gravitational timescale, or dynamical timescale.

Note: denser regions collapse faster.

collapse is independent of size.

Oscillation Timescale

Ignore Gravity!

Pressure waves travel at sound speed: $c_S \propto \sqrt{\frac{P}{\rho}} \propto \sqrt{\frac{k_B T}{m}}$

Sound crossing time: $t_S \propto \frac{R}{c_S} \propto R\sqrt{\frac{m}{k_B T}}$

Ergo: Small hot regions oscillate more rapidly!

Note that before decoupling $P_{\mathrm{rad}} >> P_{\mathrm{gas}}$ and $c_S \sim \sqrt{3}c$.

Ratio of Timescales

Collapse time:

$$t_G \propto \frac{1}{\sqrt{\rho G}}$$

Sound crossing time:

$$t_S \propto \frac{R}{c_S} \text{ where } c_S \propto \sqrt{\frac{k_B T}{m}}$$

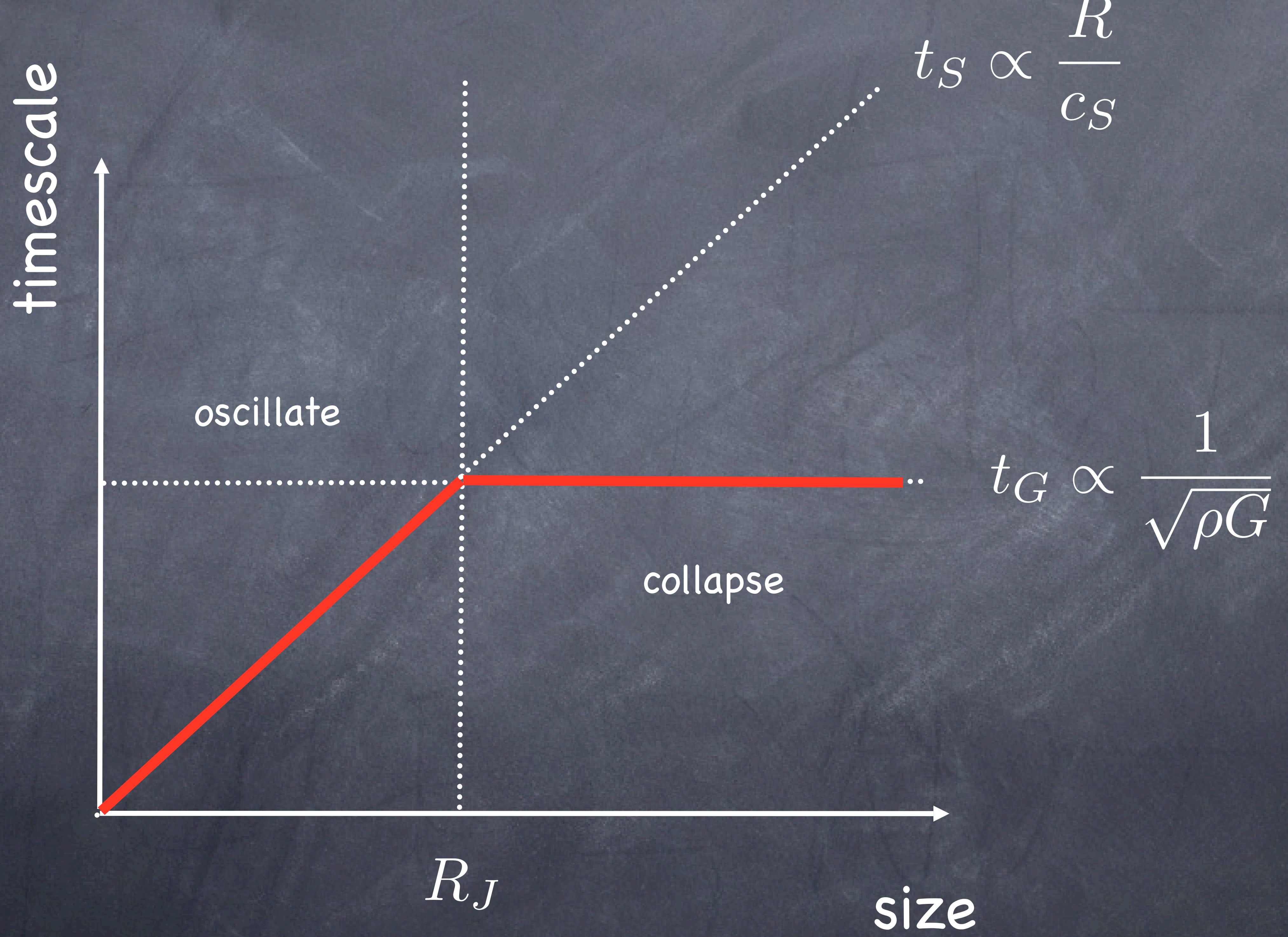
Ratio of timescales:

$$\frac{t_S}{t_G} \propto \frac{R\sqrt{G\rho}}{c_S} \propto R\sqrt{\frac{G\rho m}{k_B T}} \propto \frac{R}{R_J}$$

Jeans length (again!)

$$R_J \propto \frac{c_S}{\sqrt{G\rho}}$$

Sizes and Timescales



Jeans Mass and Length

Jeans Length : (smallest size that collapses)

$$R_J \propto \sqrt{\frac{k_B T}{G \rho m}}$$

Jeans Mass: (smallest mass that collapses)

$$M_J \propto \rho R_J^3 \propto \rho \left(\frac{k_B T}{G \rho m} \right)^{3/2} \propto T^{3/2} \rho^{-1/2}$$

It requires cool dense regions to collapse stars, but galaxy-mass/size regions can collapse sooner.

Conditions at Decoupling

Today: $T_0 = 2.7K$ $\rho_0 = 10^{-28} kg/m^3$

In an expanding Universe: $T \propto R^{-1}$ $\rho \propto R^{-3} \propto T^3$

At decoupling we have T=3000 K

$$\rho = 10^{-28} \left(\frac{3000}{2.7} \right)^3 = 1.4 \times 10^{-19} kg/m^3$$

That is about the density of $2\, M_\odot\, \mathrm{pc}^{-3}$

Size and Mass of first Galaxies

$$T = 3000K \qquad \rho = 1.4 \times 10^{-19}\mathrm{kg\,m}^{-3} \Rightarrow 2\,M_\odot\ \mathrm{pc}^{-3}$$

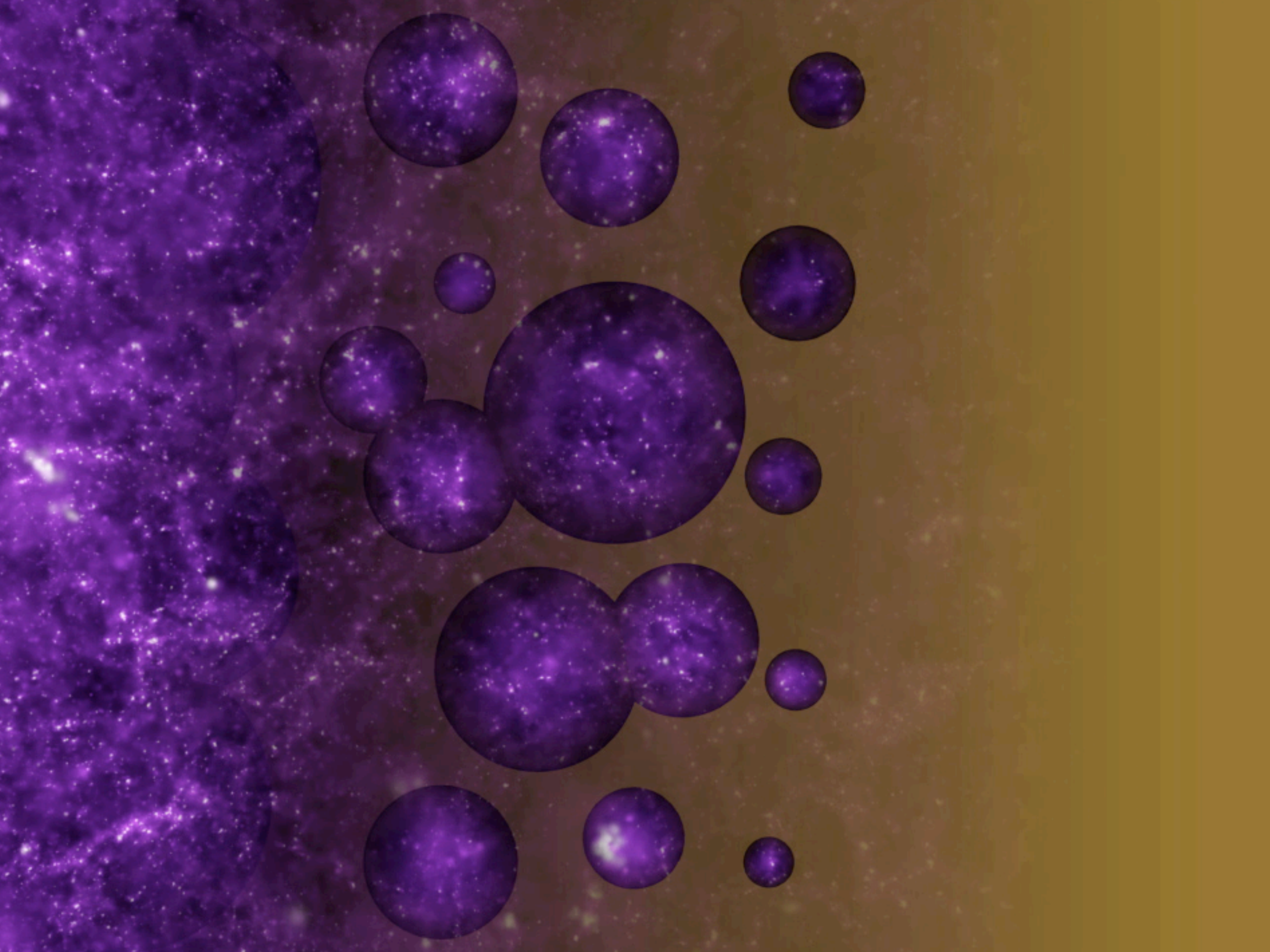
Jeans Length : $R_J \simeq \left(\frac{k_B T}{G\rho m}\right)^{1/2} = \frac{1.6 \times 10^{18} m}{3.2 \times 10^{16} m/pc} = 50\,pc$

Jeans Mass: $M_J \simeq \rho R_J^3 \simeq 2M_\odot \mathrm{pc}^{-3} \cdot (50\,pc)^3$

$$\simeq 3 \times 10^5 M_\odot$$

More than a star, but less than a galaxy,

close to a globular cluster mass.



Globular Clusters

47 Tuc with HST

3-D view of GCs in the Milky Way

Time to form first galaxies

At decoupling: $\rho = 1.4 \cdot 10^{-19} kg\, m^{-3}$

Collapse timescale: $t_G \simeq \dfrac{1}{\sqrt{\rho G}} = 3.3 \cdot 10^{14} s = 10^7 \mathrm{yr}$

Expect first galaxies to form 10^7 yr after decoupling - <u>for all sizes</u>!!

$$R_J \propto \left(\frac{k_B T}{G \rho m} \right)^{1/2} \approx 50\, pc \qquad\qquad M_J \propto \rho R_J^3 \approx 10^6 M_\odot$$

More small ripples than large waves.

--> Universe dominated by globular clusters (?!)

Caveats

Dimensional Analysis --> we left out dimensionless factors (factor ~ 10).

We also ignored: angular momentum

--> slows and can halt the collapse --> Spiral galaxies

cosmological expansion

--> delays collapse until: expansion time > collapse time

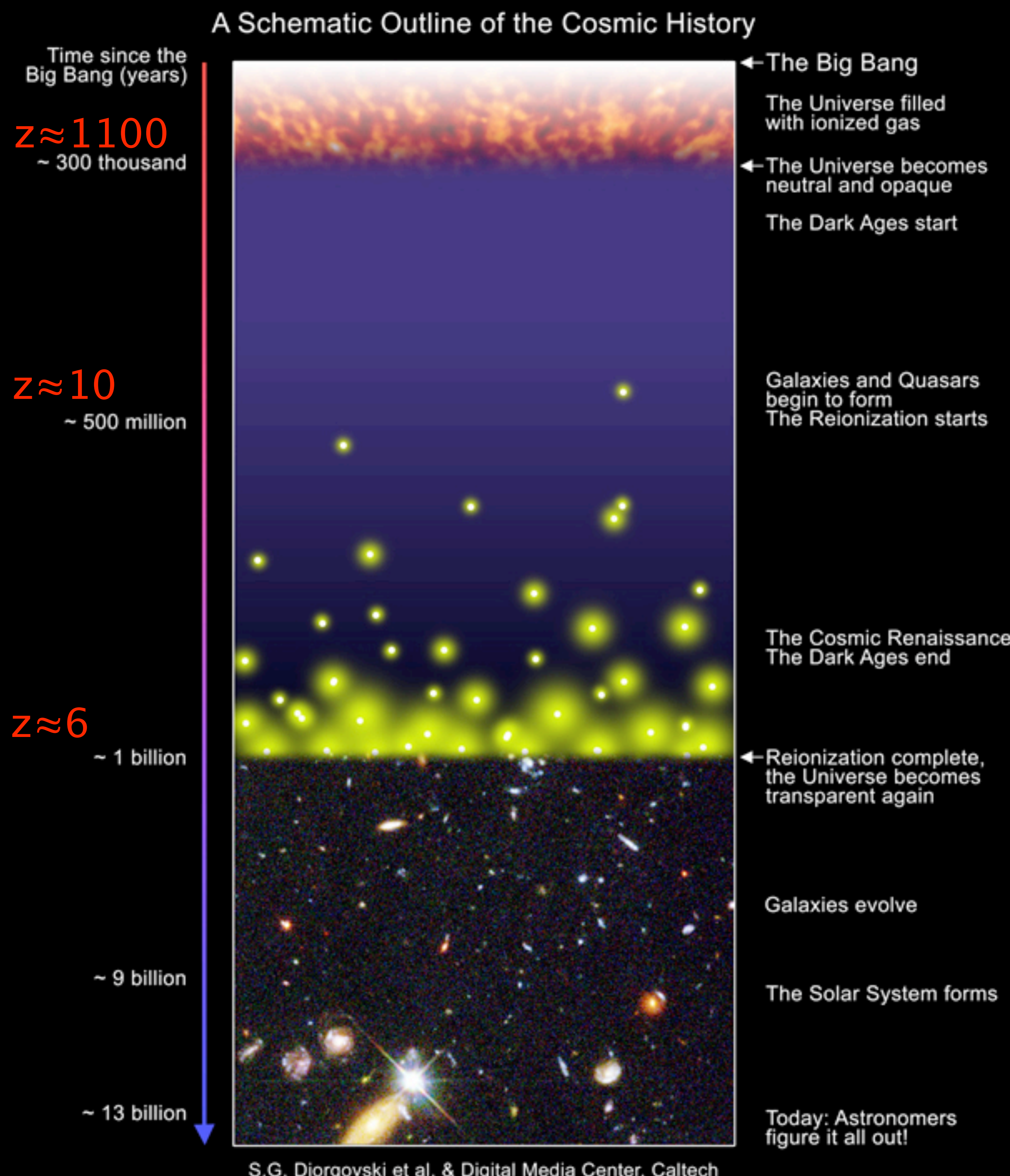
So, how did galaxies form?!

--> "Dark Matter halos" (collapse before!! decoupling) -> baryons follow.

If large enough, i.e. $R > R_J$ - and massive enough, i.e. $M > M_J$

Smallest halos that collapse: globular-cluster-ish

Tiny halo regions stable: can't form stars (yet!).



Uniform neutral IGM (Inter-Galactic Medium)

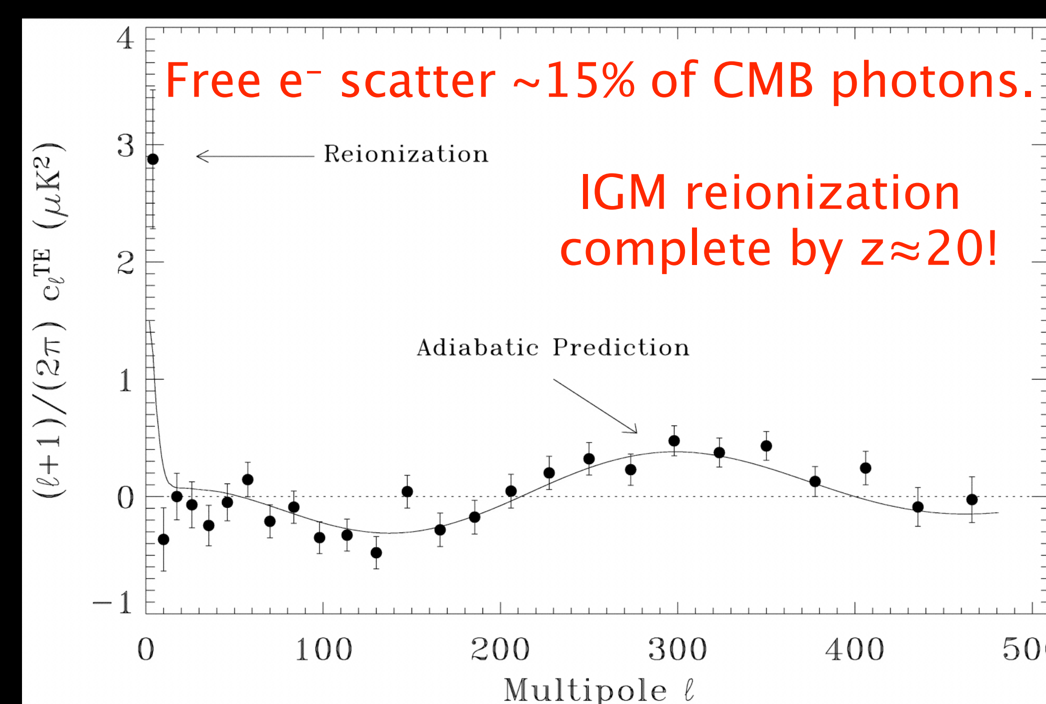
Proto-globular clusters

Rare larger objects:

- proto-galaxies
- proto-clusters

$T_{CMB}=2.7(1+z)$ K

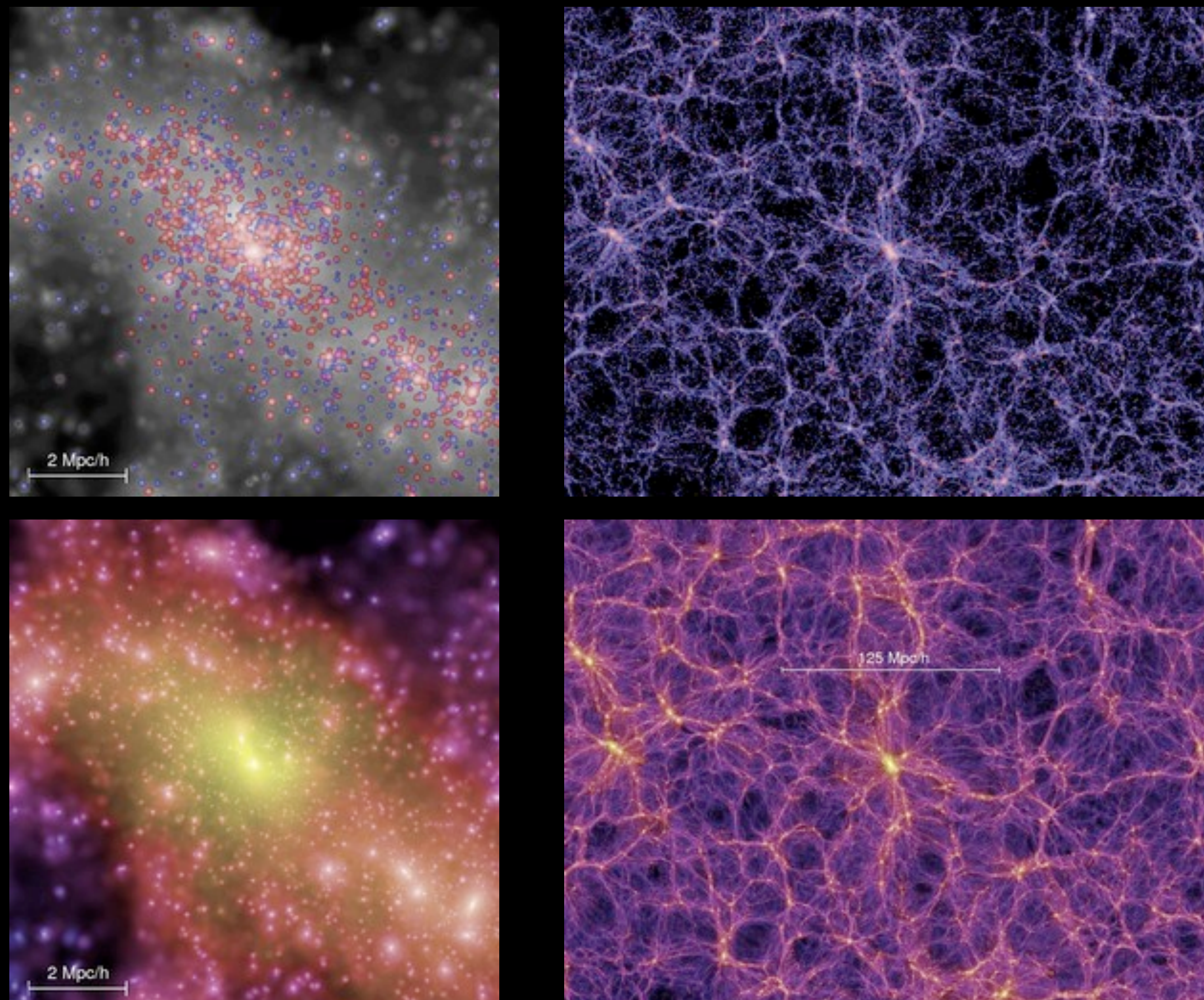
No stars!



CMB Polarization – Spergel et al. (2003)

Baryons

Dark Matter

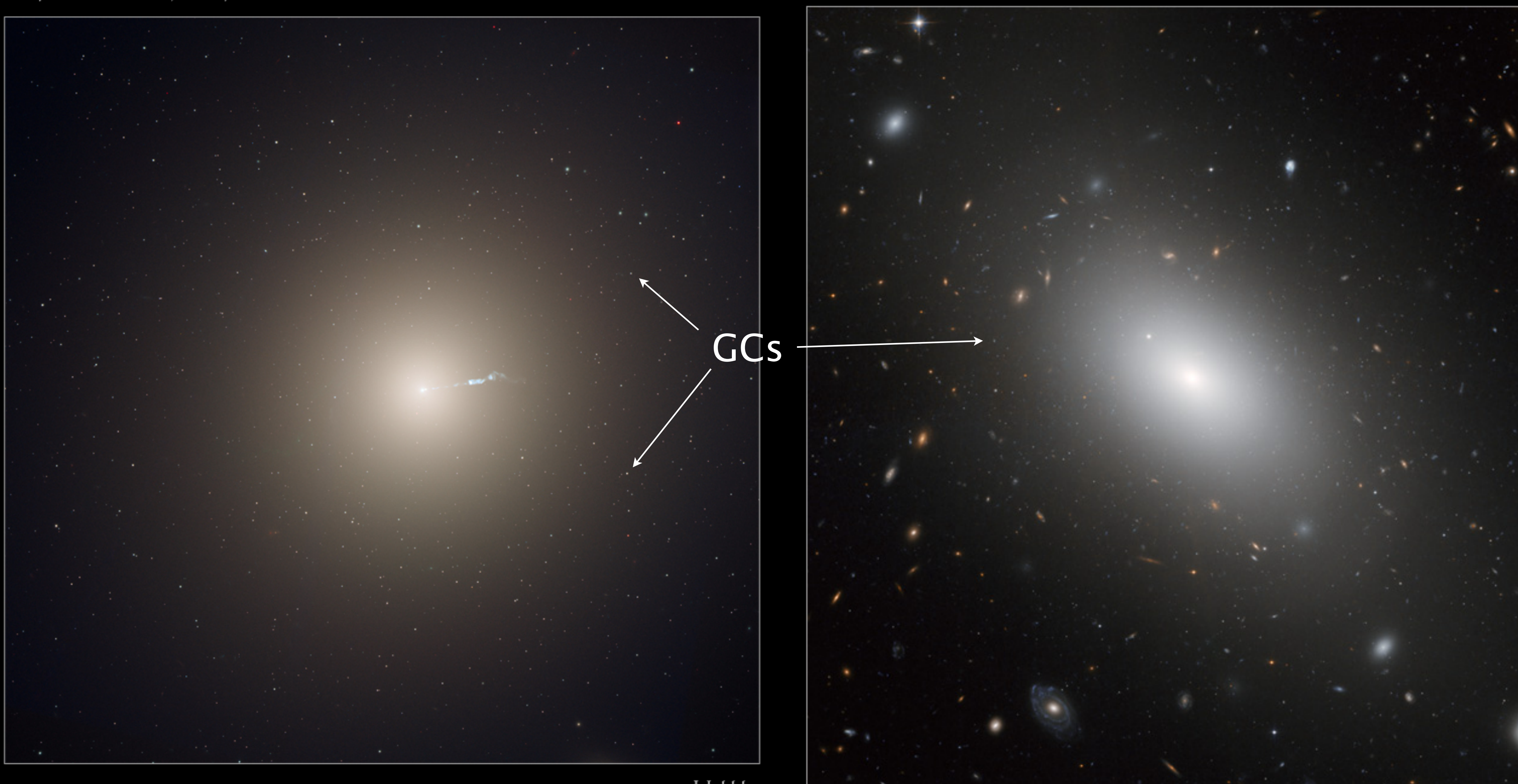


Millenium Simulation,
Springel et al. (2003)

Elliptical Galaxies

Elliptical Galaxy M87

Elliptical Galaxy NGC 1132



Hubble Heritage

NASA, ESA, and the Hubble Heritage (STScI/AURA)-ESA/Hubble Collaboration
Hubble Space Telescope ACS • STScI-PRC08-07

NASA, ESA, and the Hubble Heritage Team (STScI/AURA) • *Hubble Space Telescope* ACS • STScI-PRC-08-30b

Elliptical Galaxies

...in a nutshell.

Red ----------------> Old stars

Few emission lines ----------------> Low SFR

Little dust or gas ----------------> Gas converted to stars.

High surface brightness -----------> Form via mergers

No net rotation ----------------> with low net v/σ

Found in high density environment -> galaxy clusters

Have many GCs

Spiral Galaxies

Spiral Galaxy M81

Hubble Heritage

NASA, ESA, and The Hubble Heritage Team (STScI/AURA) • *Hubble Space Telescope* ACS • STScI-PRC07-19a

Spiral Galaxy NGC 4414

Hubble Heritage

PRC99-25 • Hubble Space Telescope WFPC2 • Hubble Heritage Team(AURA/STScI/NASA)

Barred Spiral Galaxy NGC 1300

Hubble Heritage

NASA, ESA and The Hubble Heritage Team (STScI/AURA) • Hubble Space Telescope ACS • STScI-PRC05-01

Spiral Galaxy NGC 3370

Hubble Heritage

NASA, The Hubble Heritage Team and A. Riess (STScI) • Hubble Space Telescope ACS • STScI-PRC03-24

Spiral Galaxies

...in a nutshell.

Red halo, blue disc ---------------> Old and young stars.

Emission & absorption lines -----------> Star formation
+ old stars

Dust lanes & HI ---------------> Gas available
to form stars

Moderate surface brightness --------> Form via collapse with
and Rotating disk high v/σ

In clusters & field ---------------> Can survive mergers.

Have fewer GCs

Irregular Galaxies

Dwarf Irregular Galaxy NGC 1705

Hubble Heritage

Sagittarius Dwarf Irregular Galaxy

Hubble Heritage

NASA, ESA, and The Hubble Heritage Team (STScI/AURA)
Hubble Space Telescope ACS • STScI-PRC04-31b

Irregular Galaxies

...in a nutshell.

Blue	--------------->	Young stars
Strong emission lines	--------------->	High SFR
Sub-mm signal	--------------->	Large gas reservoir
Rotating	--------------->	High v/σ
Mainly in field	--------------->	Easily disrupted.

Have few GCs

Star-Formation Rates (SFR)

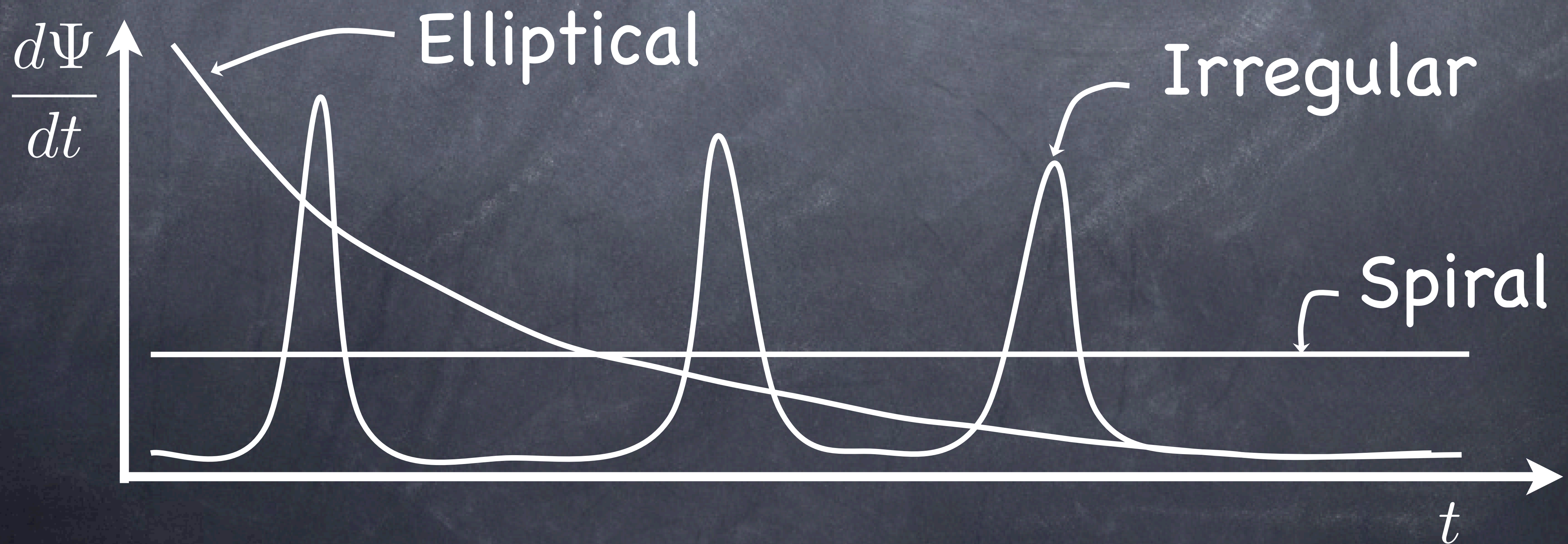
...in a nutshell.

Consider a condensation of primordial mix [X=0.75, Y=0.25, Z=0.0]

Total mass: M_{gas}

Star formation: M_{gas} ---> M_{stars}

How quickly? With what efficiency?



Closed Box Model

M_0 = initial gas mass

$M_G(t)$ = gas mass at time t

$M_S(t)$ = mass converted to stars

β = fraction of M_S returned to gas (SNe, stellar winds, PNe)

$$M_G = M_0 - M_S + \beta M_S$$

$$= M_0 - (1-\beta)M_S = M_0 - \alpha M_S$$

α = 1-β = fraction of M_S retained in stars

= star formation efficiency

in density speak: $\rho_G = \rho_0 - \alpha\rho_S$

Closed Box Model

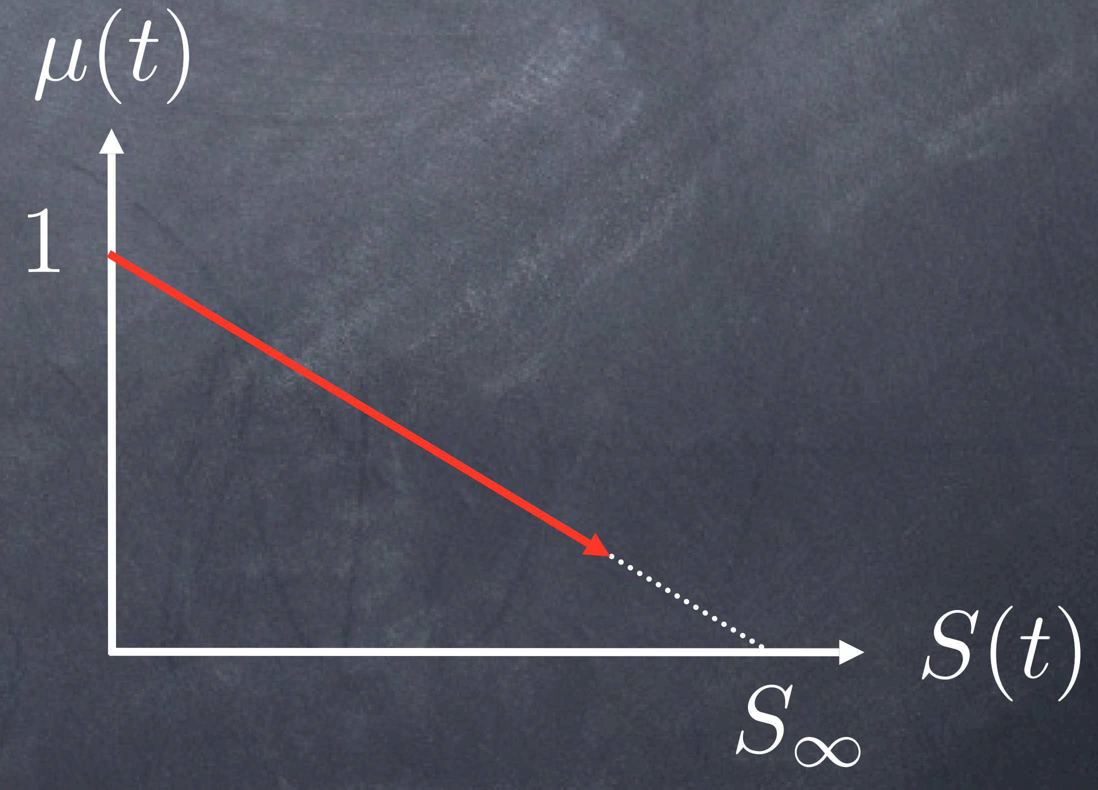
$$\mu(t) \equiv \frac{M_G(t)}{M_0}$$ = mass fraction of M_0 in gas

$$S(t) \equiv \frac{M_S(t)}{M_0}$$ = mass fraction of M_0 turned into stars

$$M_G = M_0 - \alpha M_S$$

$$\mu = 1 - \alpha S$$

Since $\alpha < 1,\ S(t) \rightarrow S_\infty > 1$



SFR in Ellipticals

Assume $dS(t)/dt \propto \mu$ which means more gas -> more stars form

$$\mu(t) = 1 - \alpha S(t)$$

$$\frac{d\mu}{dt} = -\alpha \frac{dS}{dt} = -\alpha C \mu \qquad \frac{dS}{dt} = C\mu$$

$$\frac{d\mu}{\mu} = -\alpha C dt = -\frac{dt}{t} \qquad C = \frac{1}{\alpha t_\star}$$

$$\ln \mu = -\frac{t}{t_\star} + A \qquad A = 0 \text{ gives } \mu(0) = 1$$

Gas: $\mu(t) = e^{-t/t_\star}$ Stars: $\alpha S(t) = 1 - e^{-t/t_\star}$

Star Formation Timescales

$t_\star$ = e-folding timescale

= time to turn mass M_0/e into stars.

Typically, $t_\star \approx 0.5 - 5\mathrm{Gyr}$

What is $t_\star = 2\,\mathrm{Gyr}$, how long would it take to turn 90% of gas into stars?

Consider:

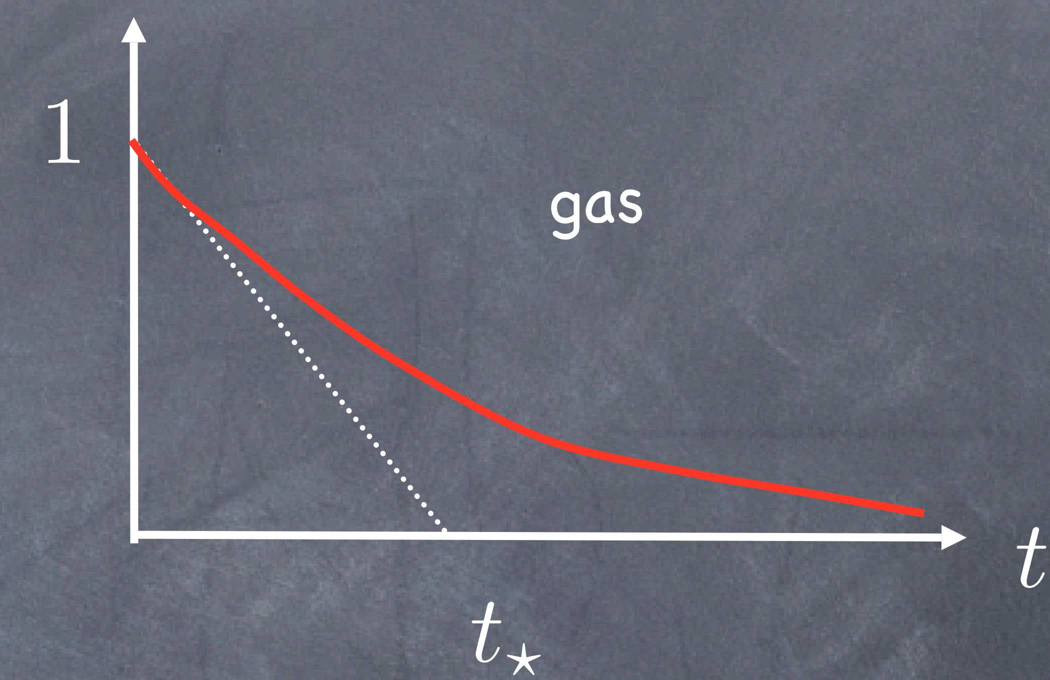
$$\mu(t) = e^{-t/t_\star} = 0.1$$

then

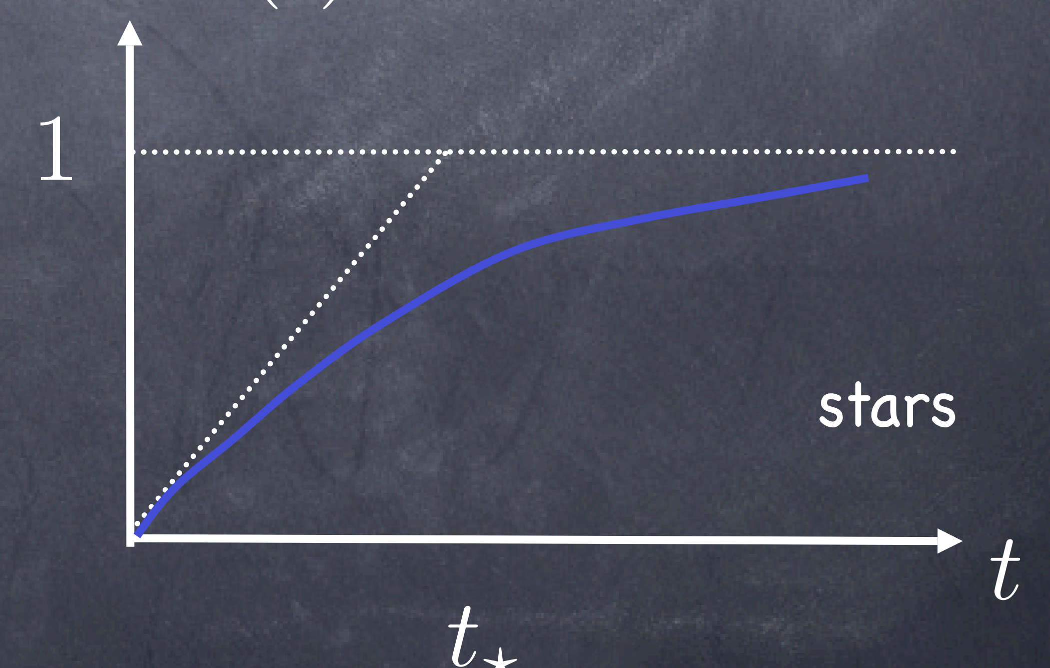
$$t = -t_\star \ln(\mu)$$

$$= -2\ln(0.1) = 4.6\,\mathrm{Gyr}$$

$$\mu(t) = e^{-t/t_\star}$$



$$\alpha S(t) = 1 - e^{-t/t_\star}$$



SFR in Spirals

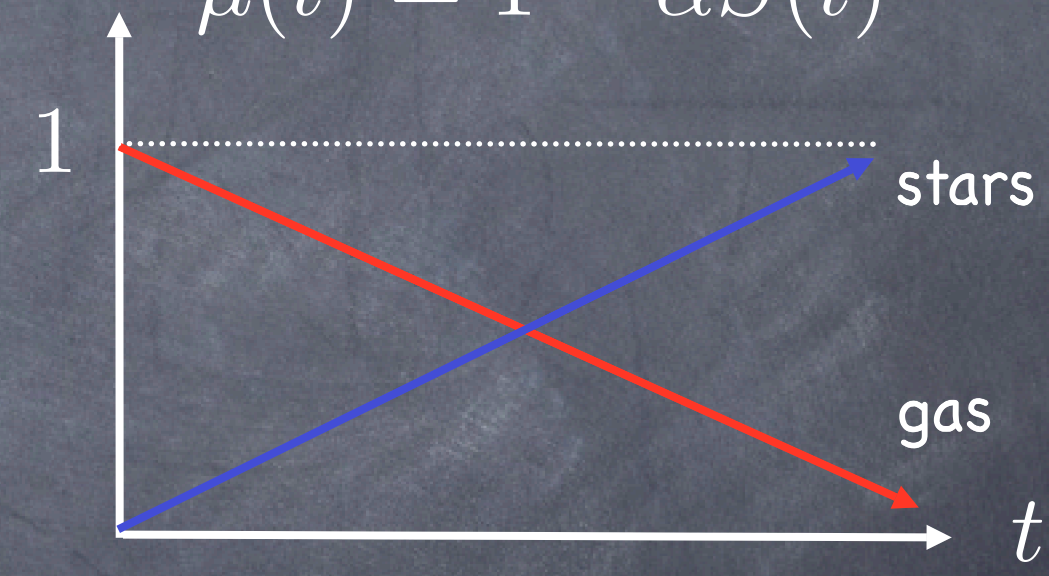
Assume SFR = constant

$\dot{M}$ = mass converted per year

$$\frac{dS}{dt} = \frac{\dot{M}}{M_0}$$

$$\frac{d\mu}{dt} = -\alpha\frac{dS}{dt} = -\alpha\frac{\dot{M}}{M_0}$$

$$\mu(t) = 1 - \alpha S(t)$$



Gas: $\mu(t) = 1 - \alpha\frac{\dot{M}}{M_0}t$

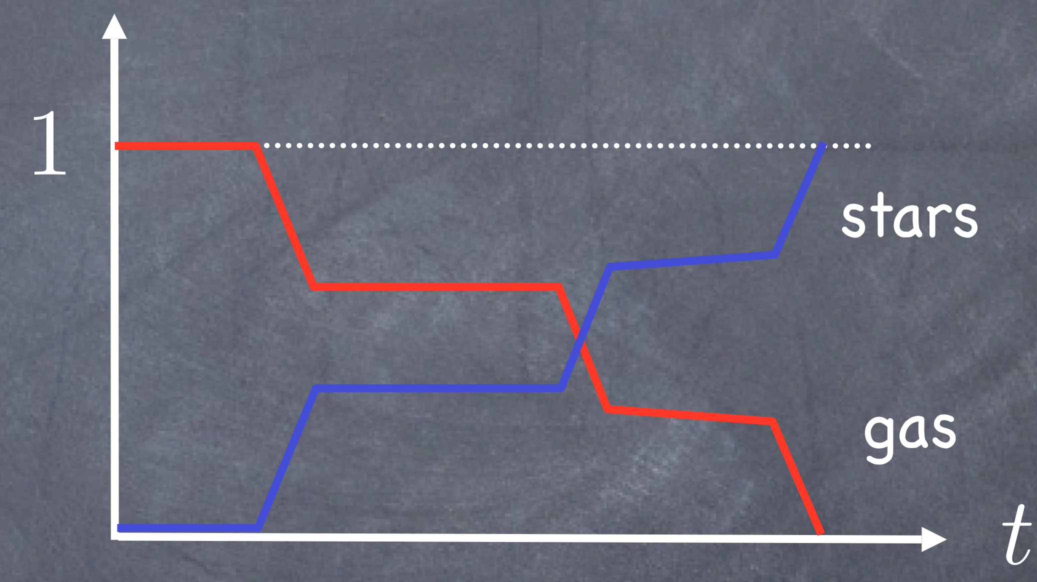
Stars: $S(t) = \frac{\dot{M}}{M_0}t$

SFR in Irregulars

Episodic SF: typically bursts of 100 M_{sol}/yr for 0.5 Gyr at intermittent intervals:

$$\frac{dS}{dt} = f\frac{\dot{M}}{M_0}$$

$$\mu(t) = 1 - \alpha S(t)$$



where f is the fraction of time

spent in starburst mode

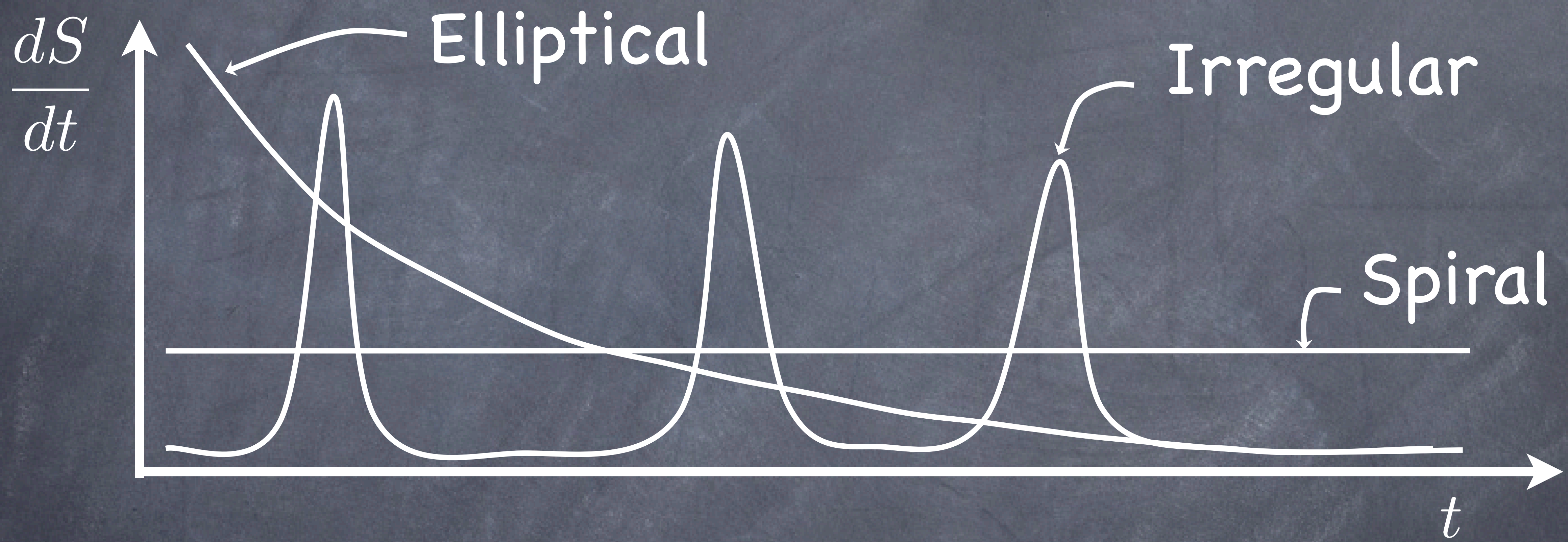
$$\frac{d\mu}{dt} = -\alpha f\frac{\dot{M}}{M_0}$$

Gas: $\mu(t) = 1 - \alpha f\frac{\dot{M}}{M_0}t$

Stars: $S(t) = f\frac{\dot{M}}{M_0}t$

Star-Formation Histories - recap.

...in a nutshell.



Ellipticals form most of their stars early on in short starburst. Their stars all roughly same age (co-eval); closest to SSP.

Spirals and Irregulars have prolonged, complex SF histories.

Star-Formation Histories - Summary

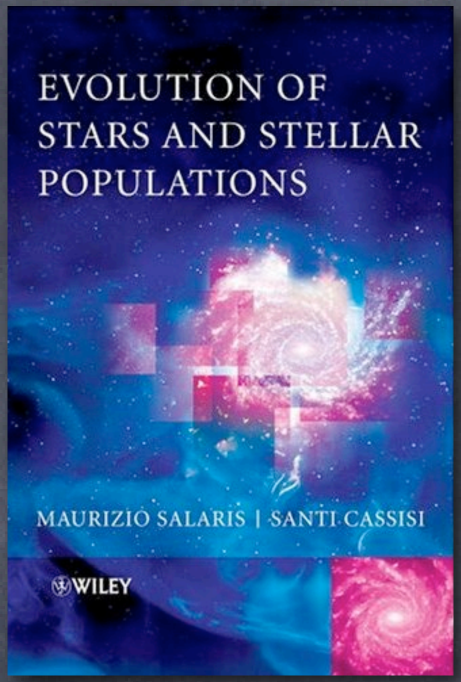
$\mu_{\rm ell} = e^{-t/t_\star}$ where $t_\star$ is the e-folding time

$\mu_{\rm sp} = 1 - \alpha \frac{\dot{M}}{M_0} t$ where α is the star-formation efficiency

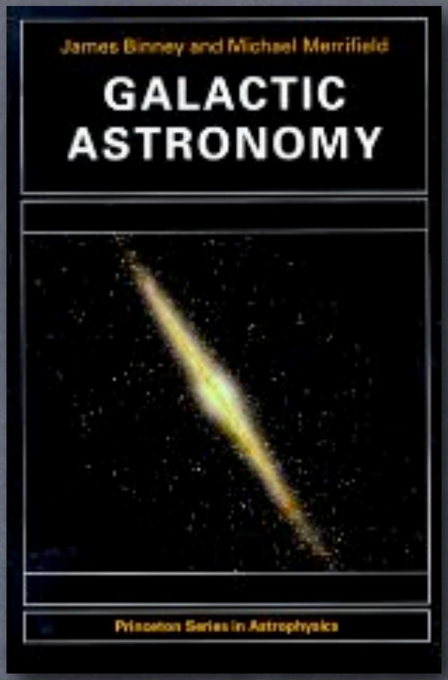
and $\dot{M}$ the gas mass conversion rate

$\mu_{\rm irr} = 1 - \alpha f \frac{\dot{M}}{M_0} t$ where f is the time in starburst mode

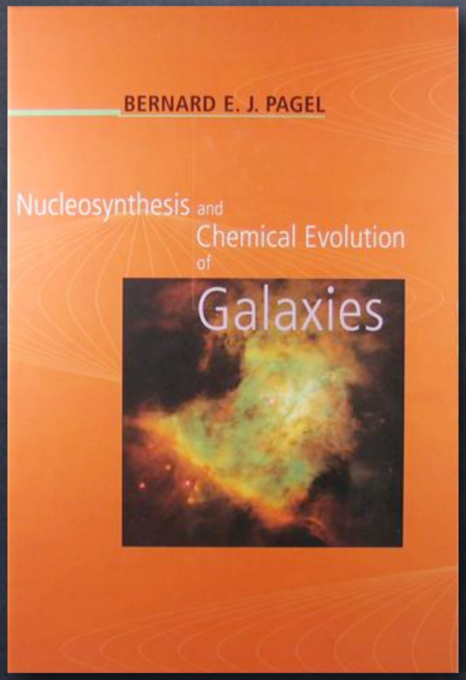
Further reading



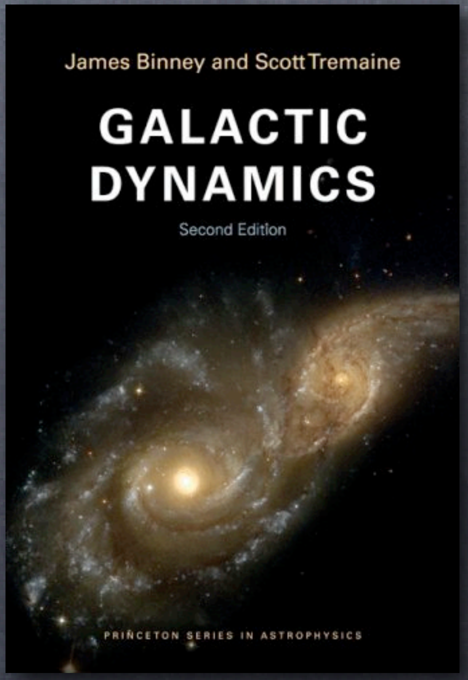
Evolution of Stars and Stellar Populations
M. Salaris & S. Cassisi
Wiley, 2005, $70.-



Galactic Astronomy
J. Binney & M. Merrifield
Princeton UP, 1998, $50.-



Nucleosynthesis and Chemical Evolution of Galaxies
B.E.J. Pagel
Cambridge UP, 1997, $110.-



Galactic Dynamics
J. Binney & S. Tremaine
Princeton UP, 2nd edition
2008, $50.-