7 Tidal heating

Note: Previous "Maths Notes" have highlighted maths skills of use in the course and assignments. This particular section will not form part of any assessment as the required maths goes beyond the level expected in the course. However, it is still presented as it is of value in appreciating the mathematical background behind the concept of tidal heating.

The tidal force refers to the difference in gravitational force experienced by the near and far sides of a satellite orbiting a parent body. We will consider the changing tidal force experienced by Io as it orbits Jupiter to estimate the amount of tidal energy available to heat Io's interior.

The gravitational force between two massive bodies (M_J and M_{Io}) separated by a distance r is

$$F = -\frac{GM_J M_{Io}}{r^2}$$

where G is the gravitational constant.

Let's say that Io has a radius R_{Io}. One method of computing the tidal force is just to compute the difference between the near and far side gravitational force, e.g. $F_{tidal} = F(r - R_{Io}) - F(r + R_{Io})$. Instead we will start with the formal definition of tidal force as the gradient of the gravitational force, e.g.

$$F_{tidal} = \frac{\mathrm{d}F}{\mathrm{d}r} = \frac{2GM_J M_{Io}}{r^3}.$$

The total force experienced across Io is equal to the tidal force (force per unit distance) multiplied by the diameter of Io (distance), i.e. $F_{total} = F_{tidal} \times 2R_{Io}$.

In the case of Io, it is the change in the tidal force experienced as Io proceeds about its slightly eccentric orbit that results in the stretching and contraction of its interior. We can compute this force by calculating the difference in the tidal force experienced at perijove (closest to Jupiter) and apojove (furthest from Jupiter), i.e.

$$\Delta F = [F_{tidal}(r_{peri}) - F_{tidal}(r_{apo})] \times 2\ R_{Io} = 9 \times 10^{20}\text{N}$$

This difference in force results in the movement (stretching and contraction) of Io's interior structure. The work done (energy expended) by this movement of rock layers is $W = F \times d$, where d is the distance by which the rock layer moves. Measurements of the surface distortion of Io via satellite radar altimeter mapping indicate that the surface rises and falls by up to 100m during one-half orbit. Only the surface layers will move by this amount. Interior layers within Io will move by a smaller amount and we assume that, on average, the enrire mass of Io is moved through 50m. The average power (rate of energy expended) is equal to the work done on the rock during one-half orbit divided by half of the orbital period (t_{Io}), i.e.

$$\bar{P} = \frac{\Delta F \times d}{0.5 \times t_{Io}} = \frac{9 \times 10^{20}\text{N} \times 50\text{m}}{0.5 \times 146880\text{s}} = 6 \times 10^{17}\text{W}.$$

We can compare this value to the total power radiated by Io. Io radiates as a blackbody, i.e. the total power output depends upon the surface temperature and the surface area, and infrared observations of Io determine the surface temperature. Io's total heat output is approximately 1×10^{14}W. Therefore, under 1% of the tidal energy is converted to heat in Io's interior – most of the energy is spent moving the rock. A more detailed calculation (mainly a more detailed treatment of Io's interior) would certainly refine this answer. However, the above analysis is useful to obtain an order of magnitude understanding of the tidal heating of Io.