

CHAPTER 18

SPECTROSCOPIC BINARY STARS

18.1 *Introduction*

There are many binary stars whose angular separation is so small that we cannot distinguish the two components even with a large telescope – but we can detect the fact that there are two stars from their spectra. In favourable circumstances, two distinct spectra can be seen. It might be that the spectral types of the two components are very different – perhaps a hot A-type star and a cool K-type star, and it is easy to recognize that there must be two stars there. But it is not necessary that the two spectral types should be different; a system consisting of two stars of identical spectral type can still be recognized as a binary pair. As the two components orbit around each other (or, rather, around their mutual centre of mass) the radial components of their velocities with respect to the observer periodically change. This results in a periodic change in the measured wavelengths of the spectra of the two components. By measuring the change in wavelengths of the two sets of spectrum lines over a period of time, we can construct a *radial velocity curve* (i.e. a graph of radial velocity versus time) and from this it is possible to deduce some of the orbital characteristics. Often one component may be significantly brighter than the other, with the consequence that we can see only one spectrum, but the periodic Doppler shift of that one spectrum tells us that we are observing one component of a spectroscopic binary system. Thus we have to distinguish between a *double-lined spectroscopic binary* system and a *single-lined spectroscopic binary* system.

As with all topics in this series of notes, there has accumulated over centuries a vast body of experience, knowledge and technical facility, and this chapter is intended only as a first introduction to the basic principles. But all of us, whether beginners or experienced practitioners, have to know these!

The orbital elements of a binary star system are described in Chapter 17, and are a , e , i , Ω , ω and T . However, on thinking about the meaning of the element Ω , the position angle of the ascending node, the reader will probably agree that we cannot tell the position angle of either node from radial velocity measurements of an unresolved binary star. We have no difficulty, however, in determining which component is receding from the observer and which is approaching, and therefore we can determine which node is ascending and which is descending, and the sign of the inclination. Thus we can determine some things for a spectroscopic binary that we cannot determine for a visual binary, and *vice versa*. If a binary star is both spectroscopic and visual (by which I mean that we can see the two components separately, and we can detect the periodic changes in radial velocity from the spectra of each), then we can determine almost anything we wish about the orbits without ambiguity. But such systems are rare – and valuable. Usually (unless the system is very close to us) the linear separation between the pairs of a visual binary is very large (that's why we can see them separately) and so the speeds of the stars

in their orbits are too slow for us to measure the changes in radial velocity. Typically, orbital periods of visual binary stars are of the order of years – perhaps many years. Stars whose binarity is detected spectroscopically are necessarily moving fast (typically their orbital periods are of the order of days), which means they are close together – too close to be detected as visual binaries.

Of course, in addition to the periodic variations in radial velocity, which give rise to periodic Doppler shifts in the spectra, the system as a whole may have a radial velocity towards or away from the Sun. The radial velocity of the system – or its centre of mass – relative to the Sun is called, naturally, the *systemic velocity*, and is one of the things we should be able to determine from spectroscopic observations. I shall be using the symbol V_0 for the systemic velocity, though I have seen some authors use the symbol γ and even refer to it as the “gamma velocity”. [By the way have you noticed the annoying tendency of the semi-educated these days to use technical words that they don’t know the meaning of? An annoying example is that people often talk of “systemic discrimination”, presumably because they think that the word “systemic” sounds scientific, when they really mean “systematic discrimination”.] We must also bear in mind that the actual observations of the star are made not from the Sun, but from Earth, and therefore corrections must be made to the observed radial velocity for the motion of Earth around the Sun as well as for the rotation of Earth around its axis.

18.2 *The Velocity Curve from the Elements*

In this section, we calculate the velocity curve (i.e. how the radial velocity varies with time) to be expected from a star with given orbital elements. Of course, the practical situation is quite the opposite: we observe the velocity curve, and from it, we wish to determine the elements. We’ll deal with that later.

I’m going to use the convenient phrase “plane of the sky” to mean a plane tangent to the celestial sphere, or normal to the line of sight from observer to the centre of mass of the system. The centre of mass C of the system, then, is stationary in the plane of the sky. The plane of the orbits of the two stars around their centre of mass is inclined at an angle i to the plane of the sky. I am going to follow the adventures of star 1 about the centre of mass C. And I am going to assume that Chapter 9 is all fresh in your mind!

The semi major axis of the orbit of star 1 about C is a_1 , and the semi latus rectum $l_1 = a_1(1 - e^2)$. The angular momentum per unit mass of star 1 about C is $r_1^2 \dot{v} = \sqrt{GMl_1}$, where v is the true anomaly and $M = m_2^3 / (m_1 + m_2)^2$. The orbital period P is given by $P^2 = \frac{4\pi^2}{GM} a_1^3$. The mean motion n is $2\pi/P$, and hence $n^2 a_1^3 = GM$.

Therefore the angular momentum per unit mass is

$$r_1^2 \dot{v} = n a_1^2 \sqrt{1 - e^2}. \quad 18.2.1$$

In figure XVIII.1 we see the star 1 (labelled S) in orbit around C, and at some time the *argument of latitude* of S is θ , and its distance from C is r_1 . Its distance above the plane of the sky is z , and $z = r_1 \sin \beta$. The inclination of the plane of the orbit to the plane of the sky is i , and, in order to find an expression for β in terms of the argument of latitude and the inclination, I'm just going to draw, in figure VIII.2, these angles on the surface of a sphere. The sphere is centred at C, and is of arbitrary radius.

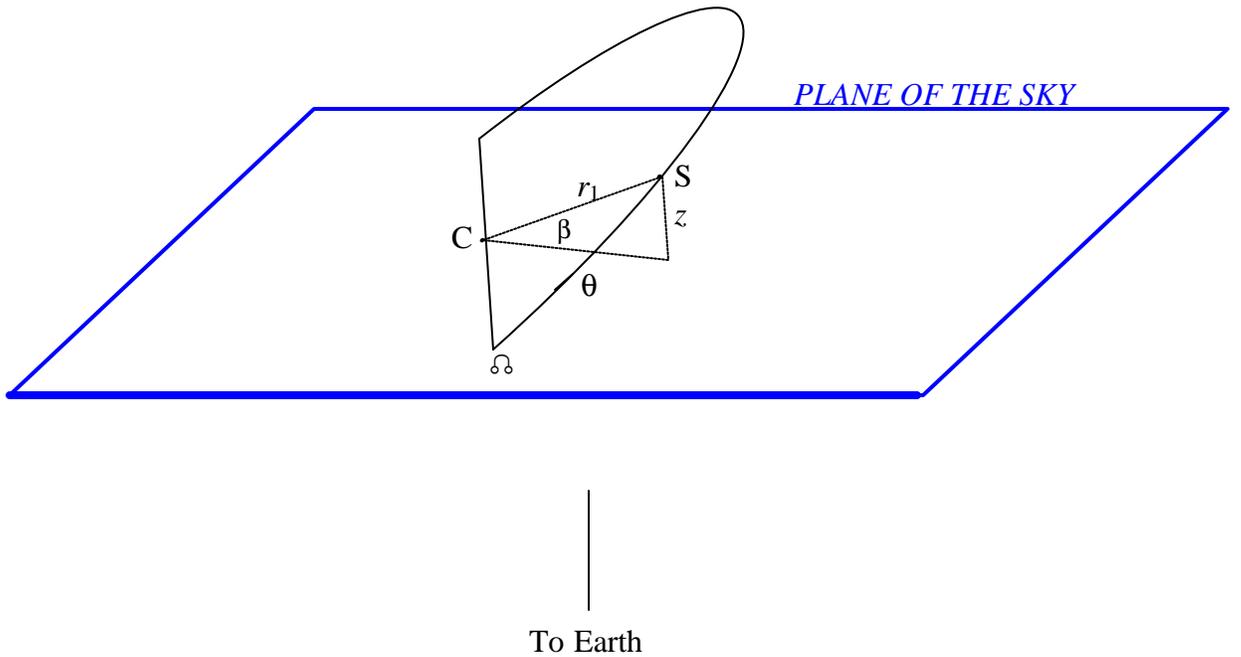


FIGURE XVIII.1

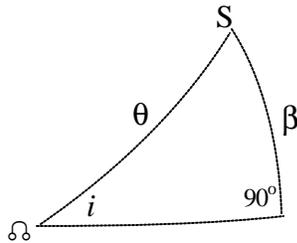


FIGURE XVIII.2

We see from this triangle that $\sin \beta = \sin i \sin \theta$. Also, the argument of latitude $\theta = \omega + v$, (ω = argument of periastron, v = true anomaly), and so

$$z = r_1 \sin i \sin(\omega + v). \quad 18.2.2$$

At this moment, the radial velocity V of star 1 relative to the Sun is given by

$$V = V_0 + \dot{z}, \quad 18.2.3$$

where V_0 is the radial velocity of the centre of mass C, or the systemic velocity. Differentiation of equation 18.2.1 with respect to time gives

$$\dot{z} = \sin i [\dot{r}_1 \sin(\omega + v) + r_1 \dot{v} \cos(\omega + v)]. \quad 18.2.4$$

I would like to express this entirely in terms of the true anomaly v instead of v and r_1 . The equation to the ellipse is

$$r_1 = \frac{l_1}{1 + e \cos v} = \frac{a_1(1 - e^2)}{1 + e \cos v}, \quad 18.2.5$$

where l_1 is the semi latus rectum, and so

$$\dot{r}_1 = \frac{l_1 e \dot{v} \sin v}{(1 + e \cos v)^2} = \frac{r_1 e \dot{v} \sin v}{1 + e \cos v}. \quad 18.2.6$$

which helps a bit. Thus we have

$$\frac{\dot{z}}{\sin i} = r_1 \dot{v} \left(\frac{e \sin v \sin(\omega + v)}{1 + e \cos v} + \cos(\omega + v) \right). \quad 18.2.7$$

We can also make use of equation 18.2.1, and, with some help from equation 18.2.5, we obtain

$$\frac{\dot{z}}{\sin i} = \frac{na_1(1 + e \cos v)}{\sqrt{1 - e^2}} \left(\frac{e \sin v \sin(\omega + v)}{1 + e \cos v} + \cos(\omega + v) \right) \quad 18.2.8$$

or

$$\frac{\dot{z}}{\sin i} = \frac{na_1}{\sqrt{1 - e^2}} (e \sin v \sin(\omega + v) + (1 + e \cos v) \cos(\omega + v)). \quad 18.2.9$$

Now $e \sin v \sin(\omega + v) + e \cos v \cos(\omega + v) = e \cos \omega$, so we are left with

$$\dot{z} = \frac{na_1 \sin i}{\sqrt{1-e^2}} (\cos(\omega + v) + e \cos \omega). \quad 18.2.10$$

The quantity $\frac{na_1 \sin i}{\sqrt{1-e^2}}$, which has the dimensions of speed, is generally given the symbol K_1 , so that

$$\dot{z} = K_1 (\cos(\omega + v) + e \cos \omega), \quad 18.2.11$$

and so the radial velocity (including the systemic velocity) as a function of the true anomaly and the elements is given by

$$V = V_0 + K_1 (\cos(\omega + v) + e \cos \omega). \quad 18.2.12$$

You can see that \dot{z} varies between $K_1(1 + e \cos \omega)$ and $-K_1(1 - e \cos \omega)$, and that K_1 is the *semi-amplitude of the radial velocity curve*.

Equation 18.2.12 gives the radial velocity as a function of the true anomaly. But we really want the radial velocity as a function of the time. This is easy, or at least straightforward, because we already know how to calculate the true anomaly as a function of time. I give here the relevant equations. I have retained their original numbering, so that you can locate them in the earlier chapters.

$$M = \frac{2\pi}{P}(t-T). \quad 9.6.4$$

$$M = E - e \sin E. \quad 9.6.5$$

$$\cos v = \frac{\cos E - e}{1 - e \cos E}. \quad 2.3.16$$

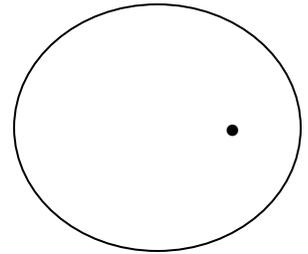
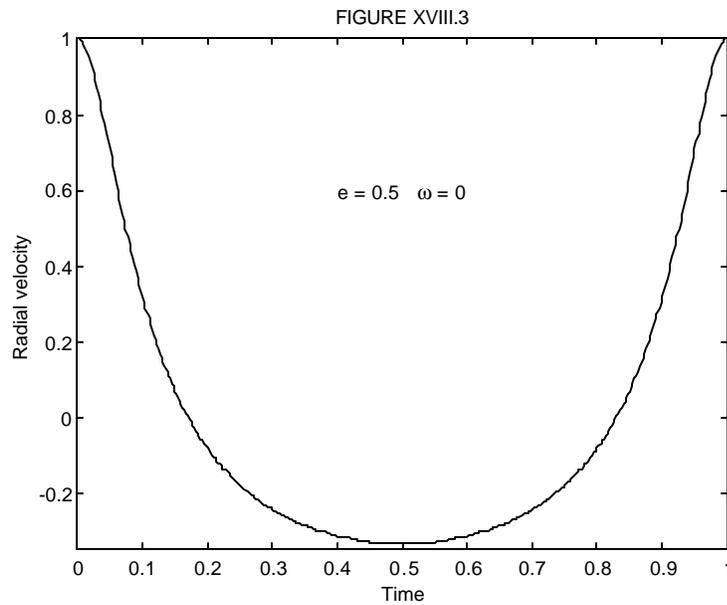
From trigonometric identities, this can also be written

$$\sin v = \frac{\sqrt{1-e^2} \sin E}{1 - e \cos E}, \quad 2.3.17a$$

or
$$\tan v = \frac{\sqrt{1-e^2} \sin E}{\cos E - e} \quad 2.3.17b$$

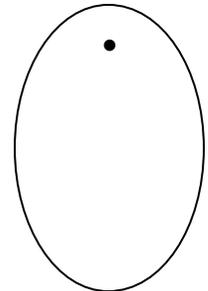
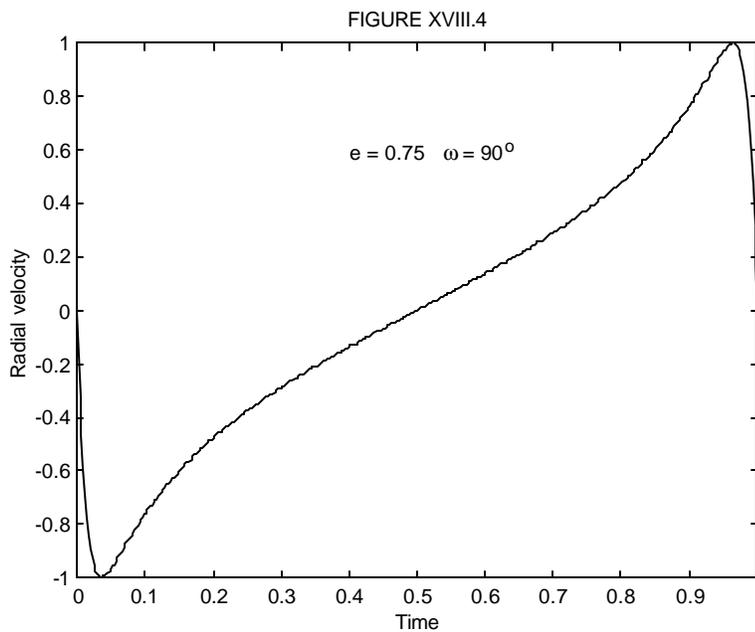
or
$$\tan \frac{1}{2}v = \sqrt{\frac{1+e}{1-e}} \tan \frac{1}{2}E. \quad 2.3.17c$$

I show in figures XIII.3 and XIII.4 two examples of velocity curves. Figure XIII.3 is computed for $e = 0.5$, $\omega = 0^\circ$. Figure XIII.4 is computed for $e = 0.75$, $\omega = 90^\circ$.



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To Earth



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To Earth

In order to draw these two figures, it will correctly be guessed that I have written a computer program that will calculate equations 9.6.4, 9.6.5, 2.3.16 and 18.2.12 in order, for the chosen values of e and ω . This is perfectly straightforward except that equation 9.6.5, Kepler's equation, requires some iteration. The solution of Kepler's equation was discussed in Section 9.6. If I were seriously going to be interested in computing the orbits of spectroscopic binary stars I would at this stage use this program to generate and print out 360 radial velocity curves for 36 values of ω going from 0° to 350° and ten

value of e going from 0.0 to 0.9. Then, when I had a real radial velocity curve of a real spectroscopic binary star to analyse, I would be able to compare it with my set of theoretical curves and hence be able to get a least a rough first approximation to the eccentricity and argument of periastron.

I have drawn figures XVIII.3 and 4 for a *systemic velocity* V_0 of zero. A real star will not have a zero systemic velocity and indeed one of the aims must be to determine the systemic velocity.

Thus in figure XVIII.5 I have drawn a radial velocity curve (I'm not saying what the values of ω and e are), but this time I have not assumed a zero systemic radial velocity. It will be noticed that the observed star spends much longer moving towards us than away from us. If we draw a horizontal line *Radial Velocity* = V_0 across the figure, this line must be drawn such that the area between it and the radial velocity curve above it is equal to the area between it and the radial velocity curve below it. How to position this line? That is a good question. If nothing else, you can count squares on graph paper. That at least will give you a first rough idea of what the systemic velocity is.

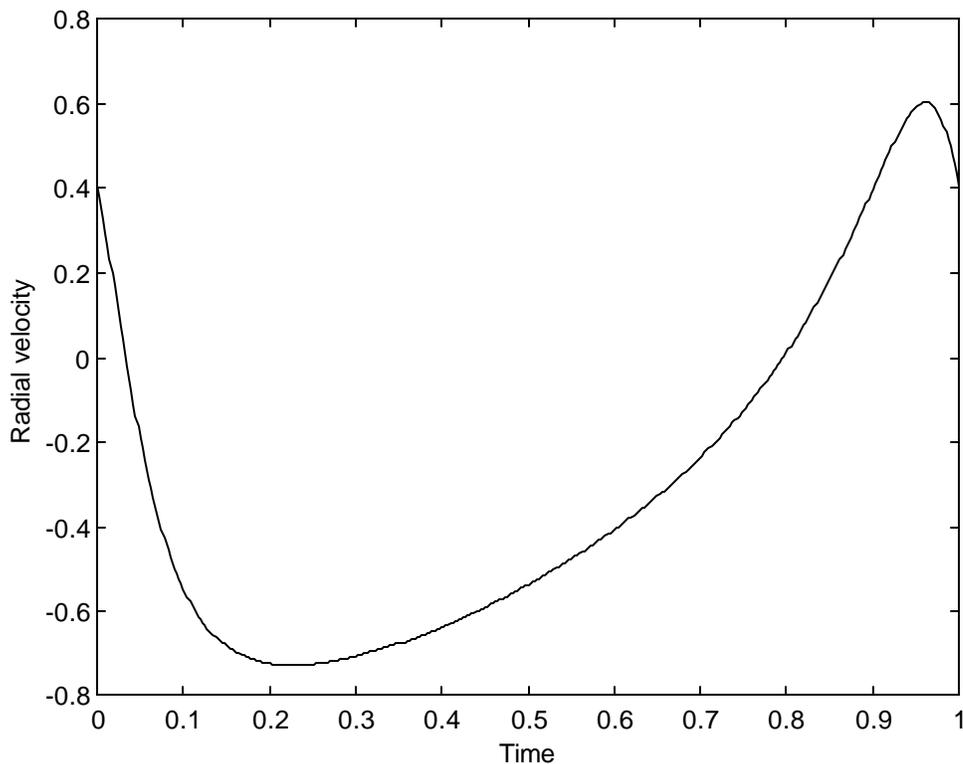


FIGURE XVIII.5

If you have a *double-lined* binary, you will have two radial velocity curves. They are not quite mirror images of each other; the semiamplitude of each component is inversely proportional to its mass. But the systemic velocity is then easy, because the two curves cross when the radial velocity of each is equal to the radial velocity of the system.

18.3 Preliminary Elements from the Velocity Curve

We have seen in the previous section how to calculate the velocity curve given the elements. The more practical problem is the inverse: In this section, we assume that we have obtained a velocity curve observationally, and we want to determine the elements. The assumption that we have obtained a precise radial velocity curve is, of course, rather a large one; but, for the present, let us assume that this has been done and we are trying to determine what we can about the orbit. We limit ourselves in this section to determining from the curve only very rough first estimates of the elements. This will also serve the purpose of establishing what information *is obtainable in principle* from the velocity curve. A later section will deal with refining our estimates and obtaining precise values.

The assumption that we have already obtained the radial velocity curve implies that we already know the period P of the orbit.

The radial velocity curve is given by equation 18.2.12:

$$V = V_0 + K_1(\cos(\omega + v) + e \cos \omega). \quad 18.2.12$$

Here $v = v(t, T, e)$. Thus, from the radial velocity curve, we should be able to determine V_0 , K_1 , e , ω and T . We shall remind ourselves a little later of the meaning of K_1 , but in the meantime we can note that the radial velocity varies between a maximum of $V_{\max} = V_0 + K_1(e \cos \omega + 1)$ and a minimum of $V_{\min} = V_0 + K_1(e \cos \omega - 1)$. The difference between these two is $2K_1$. Thus K_1 is the *semiamplitude of the radial velocity curve*, regardless of the shape of the curve and the values of ω and e , and so (again assuming that we have a well-determined radial velocity curve) K_1 can be readily determined.

The systemic velocity V_0 is such that the area under the radial velocity curve above it is equal to the area above the radial velocity curve below it. Thus at least a rough preliminary estimate can be made of V_0 , regardless of the shape of the curve and of the values of ω and e .

The shape of the radial velocity curve (as distinct from its amplitude and phase) is determined by ω and e . As suggested in the previous section, we can prepare a set of, say, 360 theoretical curves covering 36 values of ω from 0 to 350° and 10 values of e from 0.0 to 0.9. (By making use of symmetries, one need cover ω only from 0 to 90°, but computers are so fast today that one might as well go from 0 to 350°) By comparing the observed curve with these theoretical curves, we get a first estimate of ω and e . We

could then I suppose, take advantage of today's fast computers and prepare a set of velocity curves with much finer intervals around one's first estimate. This would not, of course, allow us to calculate definitive precise values of ω and e , but it would give us a pretty good first guess.

I have already pointed out that

$$V_{\max} = V_0 + K_1(e \cos \omega + 1) \quad 18.3.1$$

and
$$V_{\min} = V_0 + K_1(e \cos \omega - 1). \quad 18.3.2$$

From these we see that

$$e \cos \omega = \frac{V_{\max} + V_{\min}}{2K_1}. \quad 18.3.3$$

This allows us to determine $e \cos \omega$ without reference to the slightly uncertain V_0 , and we will want to see that our estimates of e and ω from the shape of the curve are consistent with equation 18.3.3.

The velocity curve also allows us to determine T , the time of periastron passage. For example, the sample theoretical velocity curves I have drawn in figures XIII.3, 4 and 5 all start at periastron at the left hand limit of each curve.

Note that we have been able to determine K_1 , which is $\frac{na_1 \sin i}{\sqrt{1-e^2}}$, and we can determine e

and n , which is $2\pi/P$. This means that we can determine $a_1 \sin i$, but that is as far as we can go without additional information; we cannot separate a_1 from i .

18.4 *Masses*

In Section 18.3 we saw that we could obtain approximate values of P , V_0 , K_1 , e , ω and T . But, apart from its being the semi-amplitude of the velocity curve, we have forgotten the meaning of K_1 . We remind ourselves. It was defined just after equation 18.2.10 as

$$K_1 = \frac{na_1 \sin i}{\sqrt{1-e^2}}. \quad 18.4.1$$

Here n is the mean motion $2\pi/P$. Thus, since we know P (hence n), e and K_1 , we can determine $a_1 \sin i$ – but we cannot determine a_1 or i separately.

Now the mean motion n is given just before equation 18.2.1 as

$$n^2 a_1^3 = GM, \quad 18.4.2$$

where
$$M = m_2^3 / (m_1 + m_2)^2. \quad 18.4.3$$

(A reminder: The subscript 1 refers, for a single-lined binary, to the star whose spectrum we *can* observe, and the subscript 2 refers to the star that we *cannot* observe.) All of this put together amounts to

$$K_1 = \frac{G}{(1 - e^2)a_1 \sin i} \times \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}. \quad 18.4.4$$

Thus we can determine the *mass function* $\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}$. We cannot determine the separate masses, or their ratio or sum, or the inclination.

In recent years, it has become possible to measure very small radial velocities of the order of a few metres per second, and a number of single-lined binary stars have been detected with very small values of K_1 ; that is to say, very small radial velocity amplitudes. These could, of course, refer to stars with small orbital inclinations, so that the plane of the orbit is almost perpendicular to the line of sight. It has been held, however, (on grounds that are not entirely clear to me) that many of these single-lined binary stars with small radial velocity variations are actually single stars with a planet (or planets) in orbit around them. The mass of the star that we can observe (m_1) is very much larger than the mass of the planet, which we cannot observe (m_2). To emphasize this, I shall use the symbol M instead of m_1 for the star, and m instead of m_2 for the planet. The mass function that can be determined is, then

$$\frac{m^3 \sin^3 i}{(M + m)^2}.$$

If m (the mass of the unseen body – the supposed planet) is very much smaller than the star (of mass M) whose radial velocity curve has been determined, then the mass function (which we can determine) is just

$$\frac{m^3 \sin^3 i}{M^2}.$$

And if, further, we have a reasonable idea of the mass M of the star (we know its spectral type and luminosity class from its spectrum, and we can suppose that it obeys the well-established relation between mass and luminosity of main-sequence stars), then we can determine $m^3 \sin^3 i$ and hence, of course $m \sin i$. It is generally recognized that we cannot determine i for a spectroscopic binary star, and so it is conceded that the mass of the unseen body (the supposed planet) is uncertain by the unknown factor $\sin i$.

However, the entire argument, it seems to me, is fundamentally and rather blatantly unsound, since, in order to arrive at $m \sin i$ and to hence to claim that m is of typically planetary rather than stellar mass, *the assumption that m is small and i isn't has already been made in approximating the mass function by $\frac{m^3 \sin^3 i}{M^2}$* . Unless there is *additional evidence of a different kind*, the observation of a velocity curve of small amplitude is not sufficient to indicate the presence of an unseen companion of planetary mass. Equally well (without additional evidence) the unseen companion could be of stellar mass and the orbital inclination could be small.

If the system is a *double-lined* spectroscopic binary system, we can determine the mass function for each component. That is, we can determine $\frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2}$ and $\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}$. The reader should now convince him- or herself that, since we now know these two mass functions, we can determine the *mass ratio* and we can also determine $m_1 \sin^3 i$ and $m_2 \sin^3 i$ separately. But we cannot determine m_1 , m_2 or i .

18.5 Refinement of the Orbital Elements

By finding the best fit of the observational values of radial velocity to a set of theoretical radial velocity curves, we have by now determined, if only graphically, a preliminary estimate of the orbital elements. We now have to refine these estimates in order to obtain the best set of elements that we can from the data.

Let us remind ourselves of the theoretical equation (equation 18.2.12) that we developed for the radial velocity:

$$V = V_0 + K_1(\cos(\omega + v) + e \cos \omega). \quad 18.5.1$$

Here
$$K_1 = \frac{na_1 \sin i}{\sqrt{1-e^2}} \quad 18.5.2$$

and
$$n = 2\pi/P. \quad 18.5.3$$

Also v is a function of the time and the elements T and e , through equations 9.6.4, 9.6.5 and 2.3.16 cited in Section 18.2. Thus equation 18.5.1 expresses the radial velocity as a function of the time (hence true anomaly) and of the orbital elements V_0 , K_1 , ω , e , n and T :

$$V = V(t; V_0, K_1, \omega, e, n, T). \quad 18.5.4$$

For each observation (i.e for each time t), we can use our preliminary elements to calculate what the radial velocity should be at that time, and compare it with the observed radial velocity at that time. Our aim is going to be to adjust the orbital elements so that the sum of the squares of the differences $V_{\text{obs}} - V_{\text{calc}}$ is least.

If we were to change each of the elements of equation 18.4.4 by a little, the corresponding change in V would be, to first order,

$$\delta V = \frac{\partial V}{\partial V_0} \delta V_0 + \frac{\partial V}{\partial K_1} \delta K_1 + \frac{\partial V}{\partial \omega} \delta \omega + \frac{\partial V}{\partial e} \delta e + \frac{\partial V}{\partial n} \delta n + \frac{\partial V}{\partial T} \delta T. \quad 18.5.5$$

When the differentiations have been performed, this becomes

$$\begin{aligned} \delta V = & \delta V_0 + (\cos(v+\omega) + e \cos \omega) \delta K_1 - K_1 (\sin(v+\omega) + e \sin \omega) \delta \omega \\ & + K_1 \left(\cos \omega - \frac{(2 + e \cos v) \sin(v+\omega) \sin v}{1 - e^2} \right) \delta e \\ & - \frac{\sin(v+\omega)(1 + e \cos v)^2 K_1 (t - T)}{(1 - e^2)^{3/2}} \delta n + \frac{K_1 n \sin(v+\omega)(1 + e \cos v)^2}{(1 - e^2)^{3/2}} \delta T. \end{aligned} \quad 18.5.6$$

In this equation, δV is $V_{\text{obs}} - V_{\text{calc}}$. There will be one such equation for each observation, and hence, if there are N (> 6) observations there will be N *equations of condition*. From these, six *normal equations* will be formed in the manner described in Section 1.8 and solved for the increments in the orbital elements. These are then subtracted from the preliminary elements to form an improved set of elements, and the process can be repeated until there is no significant change.

This process can be highly automated by computer, but in practice the calculation is best overseen by an experienced human orbit computer. While a computer may produce a formal solution, there are a number of situations that may result in a solution that is unrealistic or even quite wrong. Much depends on the distribution of the observations, and on whether the observational errors are normally distributed. Also, if the system has been observed for a long time over many orbital periods, the period may be known to great precision, and the investigator may prefer to keep P (hence n) as a fixed, known constant during the calculation. Or again, if the period is short, the investigator may wish (perhaps on the basis of additional knowledge) to suppose that the two stars are close together and that the orbits of the components are circular, and hence fix $e = 0$ throughout the calculation. I am always a little uneasy about making an assumption that some element has some desired value; it seems to me that, once one starts this, one might as well assume values for *all* of the elements. This would have the advantage that one need not make any observations or do any calculations and can just assume all the results according to personal taste. Whether an assumption that P or e can be held as fixed and known, or whether one should let the computer do the entire calculation without any intervention, is something that requires the experience of someone who has been calculating orbits for years.

18.6 Finding the Period

The first five sections of this chapter have dealt with calculating the relations between the orbital elements and the radial velocity curve, and that really completes what is necessary in a book whose primary focus is on celestial mechanics. In practice, the celestial mechanics part is the least of the difficulties. The equations may look forbidding at first sight, but at least the equations are unambiguous and clear cut. There are lots of problems of one sort or another that in practice occupy much more of the investigator's time than merely the computation of the orbit, which nowadays is done in the blink of an eye. I mention a few of these only briefly in the remaining sections, partly because they are not particularly concerned with celestial mechanics, and partly because my personal practical experience with them is limited.

If you were able to measure the radial velocity every five minutes throughout a complete period, there would be no difficulty in obtaining a nice velocity curve. In practice, however, you measure a radial velocity "every so often" – with perhaps many orbital periods between consecutive observations. Finding the period, then, is obviously a bit of a problem. (That there is an initial difficulty in finding the period is ultimately compensated for in that, once a preliminary value for the period is found, it can often be calculated to great precision, if the star has been observed over many decades.)

If you have a large number of observations spread out over a long time, it may be possible to identify several observations in which the radial velocity is a maximum, and you might then assume that the least time between consecutive maxima is an integral number of orbital periods. Of course you don't know what this integral number is, but you might be able to do a little better. For example, you might find that there are 100 days between two consecutive maxima, so that there are an integral number of periods in 100 days. You might also find that two other maxima are separated by 110 days. You now know that there are an integral number of periods in 10 days – which is a great improvement.

A difficulty arises if you observe the star at regular and equal intervals. While there is an obvious answer to this – *i.e.* don't do it – it may not in practice be so easy to avoid. For example: if you always observe the star when it is highest in the sky, on the meridian, then you are always observing it at an integral number of sidereal days. You then get a stroboscopic effect. Thus, if you have a piece of machinery that is cycling many times per second, you can illuminate it stroboscopically with a light that flashes periodically, and you can then see the machinery moving apparently much more slowly than it really is. The same thing happens if you observe a spectroscopic binary star at precisely regular intervals – it will appear to have a much longer period than is really the case.

It is easier to understand the effect if we work in terms of frequency (reciprocal of the period) rather than period. Thus let ν ($= 1/P$) be the orbital frequency of the star and let n' ($= 1/T'$) be the frequency of observation (the frequency of the stroboscope flash, to recall the analogy). Then the apparent orbital frequency ν' of the star is given by

$$|v' - v| = mn, \quad 18.6.1$$

where m is an integer. Returning to periods, this means that you can be deceived into deducing a spurious period P' given by

$$\frac{1}{P'} = \frac{1}{P} \pm \frac{1}{mn}. \quad 18.6.2$$

You don't have to make an observation every single sidereal day to experience this stroboscopic effect. If your stroboscope is defective and it misses a few flashes, the machinery will still appear to slow down. Likewise, if you miss a few observations, you may still get a spurious period.

Once you have overcome these difficulties and have determined the period, in order to construct a radial velocity curve you will have to subtract an integral number of periods from the time of each observation in order to bring all observations on to a single velocity curve covering just one period.

18.7 *Measuring the Radial Velocity*

In a text primarily concerned with celestial mechanics, I shan't attempt to do justice to the practical details of measuring a spectrum, but one or two points are worth mentioning, if only to draw the reader's attention to them.

To measure the radial velocity, you obtain a spectrum of the star and you measure the wavelength of a number of spectrum lines (i.e. you measure their positions along the length of the spectrum) and you compare the wavelengths with the wavelengths of a comparison laboratory spectrum, such as an arc or a discharge tube, adjacent to the stellar spectrum. If the spectra are obtained on a photographic plate, the measurement is done with a measuring microscope. If they are obtained on a CCD, there is really no "measurement" in the traditional sense to be done – a computer will read the pixels on which the lines fall. If the stellar lines are displaced by $\Delta\lambda$ from their laboratory values λ , then the radial velocity v is given simply by

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda}. \quad 18.7.1$$

Note that this formula, in which c is the speed of light, is valid only if $v \ll c$. This is certainly the case in the present context, though it is not correct for measuring the radial velocities of distant galaxies. (The z in the galaxy context is the measured $\Delta\lambda/\lambda$, and knowledge of both relativity and cosmology is necessary to translate that correctly into radial velocity.)

The accurate measurement of wavelengths in stellar spectra has its own set of difficulties. For example, the spectrum lines of early type stars are broad and diffuse as a result of the

high temperatures and quadratic Stark broadening of the lines, as well as the rapid rotation of early type stars. The lines of late-type stars are numerous, closely crowded together and blended. Thus there are difficulties at both ends of the spectral sequence.

One very nice technique for measuring radial velocities involves making use of the entire spectrum rather than the laborious process of measuring the wavelengths of individual lines. Suppose that you are, for example observing a G-type star. You will prepare an opaque mask on which are inscribed, in their correct positions, transparent lines corresponding to the lines expected of a G-type star. During observation, the spectrum of the star is allowed to fall on this mask. Some light gets through the transparent inscribed lines on the mask, and this light is detected by a photoelectric cell behind the mask. The mask is moved parallel to the spectrum until the dark absorption lines in the stellar spectrum fall on the transparent inscribed lines on the mask, and at this moment the amount of light passing through the mask and reaching the photoelectric cell reaches a sharp minimum. Not only does this technique make use of the whole spectrum, but the radial velocity is obtained immediately, *in situ*, at the telescope.

I end by briefly mentioning two little problems that are well known to observers, known as the *rotation effect* and the *blending effect*.

If the orbital inclination is close to 90° , the system, as well as being a spectroscopic binary, might also be an eclipsing binary. In this case, we can in principle get a great deal of information about the system – but there is a danger that the information might not be correct. For example, suppose that the system is a single-lined binary, and that the bright star (the one whose spectrum can be seen) is a rapid rotator and is being partially eclipsed by the secondary. In that case we can see only part of the surface of the primary star – perhaps that part of the star that is (by rotation) moving towards us. This will give us a wrong measurement of the radial velocity.

Or again, suppose that we have a double-lined binary. For much of the orbital period, the lines from one star may be well separated from those of the other. However, there comes a time when the two sets of lines approach each other and become partially blended. I show in figure XVIII.6 two partially blended gaussian profiles. You will see that the minima of the blended profile, shown as a dashed curve, occur closer together than the true minima of the individual lines. If you measure the minima of the blended profile, this will obviously give the wrong radial velocity and will result in a distortion of the velocity curve and corresponding errors in the orbital elements. Many years ago I made some calculations on the amount of the blending effect for gaussian and lorentzian profiles for various separations and relative intensities. These calculations were published in *Monthly Notices of the Royal Astronomical Society*, **141**, 43 (1968).

FIGURE XVIII.6

