CHAPTER 17
VISUAL BINARY STARS

17.1 *Introduction*

Many stars in the sky are seen through a telescope to be two stars apparently close together. By the use of a filar micrometer it is possible to measure the position of one star (the fainter of the two, for example) with respect to the other. The position is usually expressed as the angular distance ρ (in arcseconds) between the stars and the position angle θ of the fainter star with respect to the brighter. (The separation can be determined in kilometres rather than merely in arcseconds if the distance from Earth to the pair is known.) The position angle is measured counterclockwise from the direction to north. See figure XVII.1.

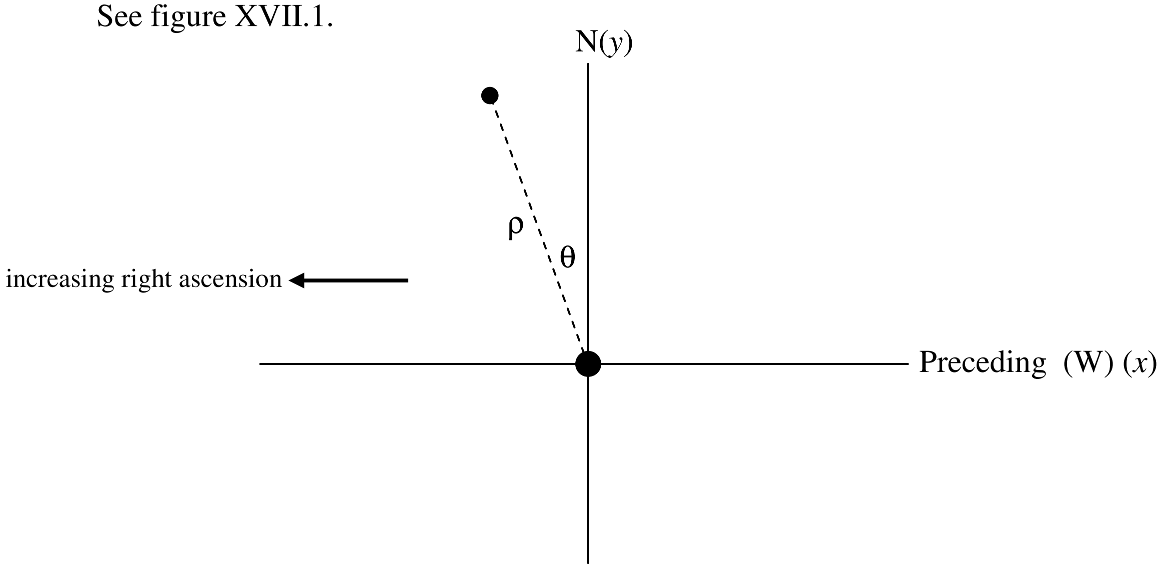


FIGURE XVII.1

These coordinates (ρ , θ) of one star with respect to the other can, of course, easily be converted to (x , y) coordinates. In any case, after the passage of many years (sometimes longer that the lifetime of an astronomer) one ends up with a table of coordinates as a function of time. Because the orbital period is typically of the order of many years, and the available observations are correspondingly spread out over a long period of time, it needs to be pointed out that all position angles, which are measured with respect to the equator of date, need to be adjusted so as to refer to a standard equator, such as that of J2000.0. I don't wish to interrupt the flow of thought here by discussing this point (important though it is) in detail; suffice it to say that

$$\theta_{2000.0} = \theta_t + 20''\times(2000 - t)\sin\alpha\sec\delta, \qquad 17.1.1$$

where t is the epoch of the observation in years, and the position angles are expressed in arcseconds.

If one star appears to move in a straight line with respect to the other, it is probable that the two stars are not physically connected but they just happen to lie almost in the same line of sight. Such a pair is called an *optical pair* or an *optical double.*

However, if one star appears to describe an ellipse relative to the other, then the two stars are physically connected and are moving around their common centre of mass.

The angular separation between the two stars is usually very small, of the order of arcseconds or less, and is not easy to measure. Much more difficult to measure would be the distances of the two stars individually from their mutual centre of mass. Close pairs are usually measured visually with a filar micrometer, and it is then almost invariably the case that what is measured is the position of the secondary with respect to the primary. Wider pairs, however, can be measured from photographs, or, today, from CCD images. In that case, not only are the measurements more precise, but it is possible to measure the position of each component with respect to background calibration stars, and hence to measure the position of each component with respect to the centre of mass of the system. This enables us to determine the mass ratio of the two components. Pairs that are sufficiently wide apart for photographic measurements, however, come with their own set of problems. If their angular separation is large, this could mean either that the real, linear separation in kilometres is large, or else that the stars are not very far from the Sun. In the former case, we may have to wait rather a long time (perhaps more than an average human lifetime) for the two stars to describe a complete orbit. In the latter case, we may have to take account of complications such as proper motion or annual parallax.

The brighter of the two stars is the *primary*, and the fainter is the *secondary*. This will nearly always mean (though not necessarily so) that the primary star is also the more massive of the pair, but this cannot be assumed without further evidence. If the two stars are of equal brightness, it is arbitrary which one is designated the primary. If the two stars are of equal brightness, it can sometimes happen that, when they become very close to each other, they merge and cannot be distinguished until their separation is sufficiently great for them to be resolved again. It may then not be obvious which of the two had been designated the "primary".

The orbit of the secondary around the primary is, of course, a keplerian ellipse. But what one sees is the *projection* of this orbit on the "plane of the sky". (The "plane of the sky" is the phrase almost universally used by observational astronomers, and there is no substantial objection to it; formally it means the tangent plane to the celestial sphere at the position of the primary component.) The projection of the *true orbit* on the plane of the sky is the *apparent orbit*, and both are ellipses. The centre of the true ellipse maps on to the centre of the apparent ellipse, but the foci of the true ellipse do *not* map on to the foci of the apparent ellipse. The primary star is at a focus of the true ellipse, but it is not at a focus of the apparent ellipse. The radius vector in the true orbit sweeps out equal areas in equal times, according to Kepler's second law. In projection to the plane of the sky, all areas are reduced by the same factor (cos i). Consequently the radius vector in the apparent orbit also sweeps out equal areas in equal times, even though the primary star is not at a focus of the apparent ellipse.

Having secured the necessary observations over a long period of time, the astronomer faces two tasks. First the apparent orbit has to be determined; then the true orbit has to be determined.

17.2 *Determination of the Apparent Orbit*

The apparent orbit may be said to be determined if we can determine the size of the apparent ellipse (i.e. its semi major axis), its shape (i.e. its eccentricity), its orientation (i.e. the position angle of its major axis) and the two coordinates of the centre of the ellipse with respect to the primary star. Thus there are five parameters to determine.

The general equation to a conic section (see Section 2.7 of Chapter 2) is of the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + 1 = 0, \qquad 17.2.1$$

so that we can equally say that the apparent orbit has been determined if we have determined the five coefficients a, h, b, g, f. Sections 2.8 and 2.9 described how to determine these coefficients if the positions of five or more points were given, and section 2.7 dealt with how to determine the semi major axis, the eccentricity, the orientation and the centre given a, h, b, g and f.

We may conclude, therefore, that in order to determine the apparent ellipse all that need be done is to obtain five or more observations of (ρ, θ) or of (x, y), and then just apply the methods of section 2.8 and 2.9 to fit the apparent ellipse. Of course, although five is the minimum number of observations that are essential, in practice we need many, many more (see section 2.9), and in order to get a good ellipse we really need to wait until observations have been obtained to cover a whole period. But merely to fit the best ellipse to a set of (x, y) points is not by any means making the best use of the data. The reason is that an observation consists not only of (ρ, θ) or of (x, y), but also the *time*, t. In fact the separation and position angle are quite difficult to measure and will have quite considerable errors, while the *time* of each observation is known with great precision. We have so far completely ignored the one measurement that we know for certain!

We need to make sure that the apparent ellipse that we obtain *obeys Kepler's second law*. Indeed it is more important to ensure this than blindly to fit a least-squares ellipse to n points.

If I were doing this, I would probably plot two separate graphs – one of ρ (or perhaps ρ^2) against time, and one of θ against time. One thing that this would immediately achieve would be to identify any obviously bad measurements, which we could then reject. I would draw a smooth curve for each graph. Then, for equal time intervals I would determine from the graphs the values of ρ and $d\theta/dt$ and I would then calculate $\rho^2\, d\theta/dt$. According to Kepler's second law, this should be constant and independent of time. I would then adjust my preliminary attempt at the apparent orbit until Kepler's second law was obeyed and $\rho^2\, d\theta/dt$ was constant. A good question now, is, which should be adjusted, ρ or θ ? There may be no hard and fast invariable answer to this, but, generally speaking, the measurement of the separation is more uncertain than the measurement of the position angle, so that it would usually be best to adjust ρ.

If we are eventually satisfied that we have the best apparent ellipse that satisfies as best as possible not only the positions of the points, but also their times, and that the apparent ellipse satisfies Kepler's law of areas, our next task will be to determine the elements of the true ellipse.

17.3 *The Elements of the True Orbit*

Unless we are dealing with photographic measurements in which we have been able to measure the positions of both components with respect to their mutual centre of mass, I shall assume that we are determining the orbit of the secondary component with respect to the primary as origin and focus.

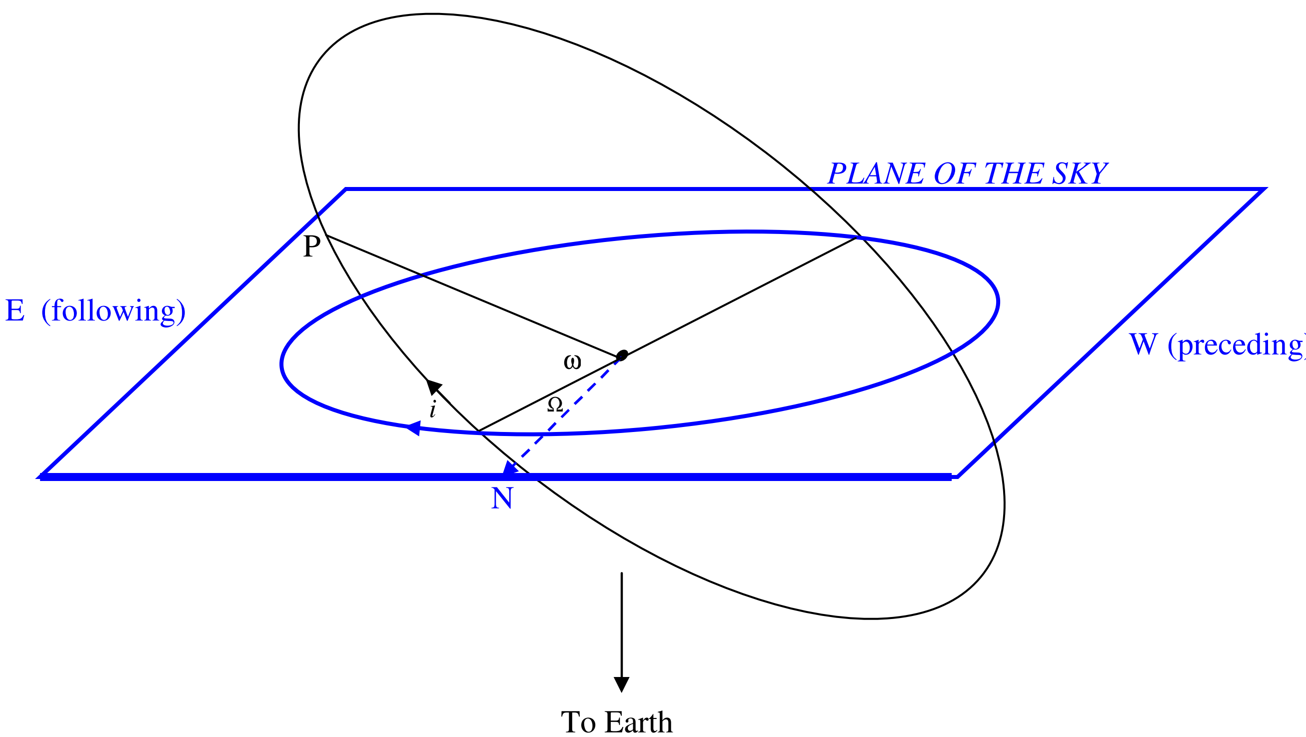


FIGURE XVII.2

In figure XVII.2, which has tested my artistic talents and computer skills to the full, the blue plane is intended to represent the plane of the sky, as seen from "above" – i.e. from outside the celestial sphere. Embedded in the plane of the sky is the apparent orbit of the secondary with respect to the primary as origin and focus. The dashed arrow shows the colure (definition of "colure" – Section 6.4 of Chapter 4) through the primary, and points to the north celestial pole. The primary star is not necessarily at a focus of the apparent ellipse, as discussed in the previous section. As drawn, the position angle of the star is increasing with time – though of course in a real case it is equally likely to be increasing or decreasing with time.

The black ellipse is the true orbit, and of course the primary is at a focus of it. If it does not appear so in figure XVII.2, this is because the true orbit is being seen in projection.

The elements of the true orbit to be determined (if possible) are

a the semi major axis;

e the eccentricity;

i the inclination of the plane of the orbit to the plane of the sky;

Ω the position angle of the ascending node;

ω the argument of periastron;

T the epoch of periastron passage.

All of these will be familiar to those who have read Chapter 10, section 10.2. Some comments are necessary in the context of the orbit of a visual binary star.

Ideally, the semi major axis would be expressed in kilometres or in astronomical units of distance – but this is not possible unless the distance from Earth to the binary star is known. If the distance is not known (as will often be the case), the semi major axis is customarily expressed in arcseconds.

It is sometimes said that, from measurements of separation and position angle alone, and with no further information, and in particular with no spectroscopic measurements of radial velocity, it is not possible to determine the *sign* of the inclination of the true orbit of a visual binary star. This may be a valid view, but, as the late Professor Joad might have said, it all depends on what you mean by "inclination". As with the orbits of planets around the Sun, as described in Chapter 10, Section 10.2, we take the point of view here that the inclination of the orbital plane to the plane of the sky is an angle that lies between 0° and 180° inclusive; that is to say, the inclination is positive, and the question of its sign does not arise. After all an inclination of, say, "-30°" is no different from an inclination of $+150^\circ$. Thus we cannot be ignorant of the "sign" of the inclination. What we do *not* know, however, is which node is the ascending node and which is the descending node.

The Ω that is usually recorded in the analysis of the orbit of a visual binary unsupported by spectroscopic radial velocities is the node for which the position angle is less than 180° – and it is not known whether this is the ascending or descending node.

If the inclination of the orbital plane is less than 90°, the position angle of the secondary will increase with time, and the orbit is described as *direct* or *prograde*. If the position angle decreases with time, the orbit is *retrograde.*

The orbital inclination of a *spectroscopic* binary cannot be determined from spectroscopic observations alone. The inclination of a *visual* binary *can* be determined, although, as discussed above, it is not known which node is ascending and which is descending. If the binary is both a visual binary and a spectroscopic binary, not only can the inclination be determined, but the ambiguity in the nodes is removed. In addition, it may be possible to determine the masses of the stars; this aspect will be dealt with in the chapter on spectroscopic binary stars.

Binary stars that are simultaneously visual and spectroscopic binaries are rare, and they are a copious source of valuable information when they are found. Visual binary stars, unless they are relatively close to Earth, have a large true separation, and consequently their orbital speeds are usually too small to be measured spectroscopically. Spectroscopic binary stars, on the other hand, move fast in their orbits, and this is because they are close together – usually too close to be detected as visual binaries. Binaries that are both visual and spectroscopic are usually necessarily relatively close to Earth.

The element ω, the argument of periastron, is measured from the ascending node (or the first node, if, as is usually the case, the type of node is unknown) from 0° to 360° in the direction of motion of the secondary component.

17.4 *Determination of the Elements of the True Orbit*

I am assuming at this stage that we have used all the observations plus Kepler's second law and have determined the apparent orbit well, and can write it in the form

$$\mathit{a}x^2 + 2hxy + \mathit{b}y^2 + 2gx + 2fy + c = 0. \qquad 17.4.1$$

[The coefficients a and b here, and e in equation 17.4.3, do not, of course, mean the semi major axis a, the semi minor axis b and eeccentricity e of the true ellipse. It is thought that the reader will be unlikely confused by this, but I have nevertheless used slightly different fonts for them.]

The origin of coordinates here is the primary star, which, although it is at the focus of the true ellipse, is not at the focus of the apparent ellipse. The x-axis points west (to the right) and the y-axis points north (upwards), and position angle θ (measured counterclockwise from north) is given by $\tan \theta = -x/y$. Our task is now to find the elements of the true orbit.

During the analysis we are going to be obliged, on more than one occasion, to determine the coordinates of the points where a straight line $y = mx + d$ intersects the ellipse, so it will be worth while to prepare for that now and write a quick program for doing it instantly. The x-coordinates of these points are given by solution of

$$(a + 2hm + bm^2)x^2 + (2hd + 2bmd + 2g + 2fm)x + bd^2 + 2fd + c = 0, \quad 17.4.2$$

and the y-coordinates are given by solution of the equation

$$(b + 2hn + an^2)y^2 + (2he + 2ane + 2f + 2gn)y + ae^2 + 2ge + c = 0, \quad 17.4.3$$

where $n = 1/m$ and $e = -d/m$. If m is positive the larger solution for y corresponds to the larger solution for x; If m is negative the larger solution for y corresponds to the smaller solution for x.

If the line passes through F, so that $d = 0$, these equations reduce to

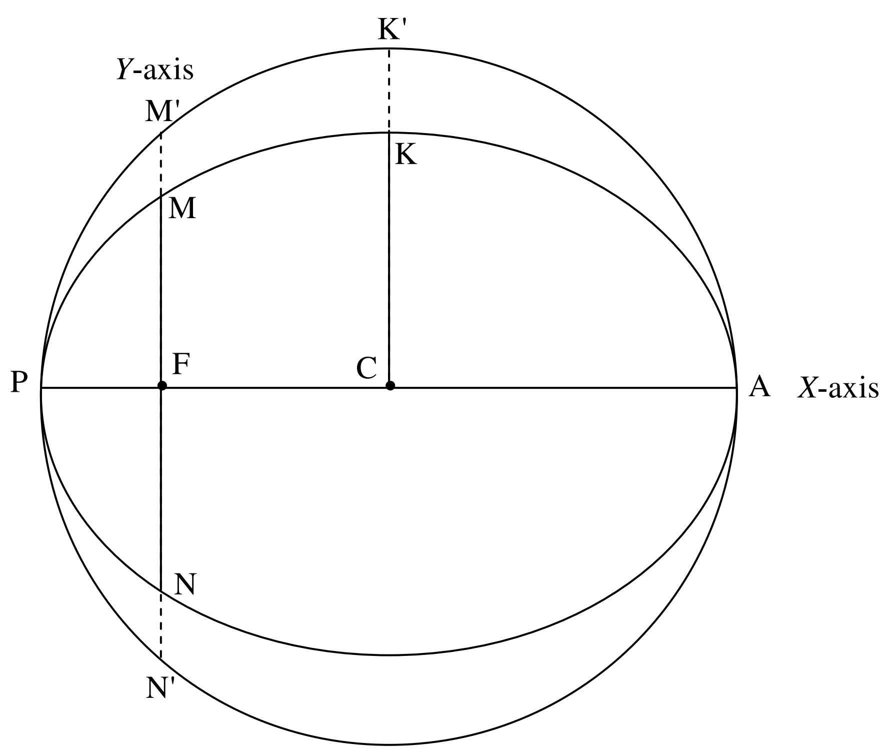
$$(a + 2hm + bm^2)x^2 + (2g + 2fm)x + c = 0, \quad 17.4.4$$

and

$$(b + 2hn + an^2)y^2 + (2f + 2gn)y + c = 0. \quad 17.4.5$$

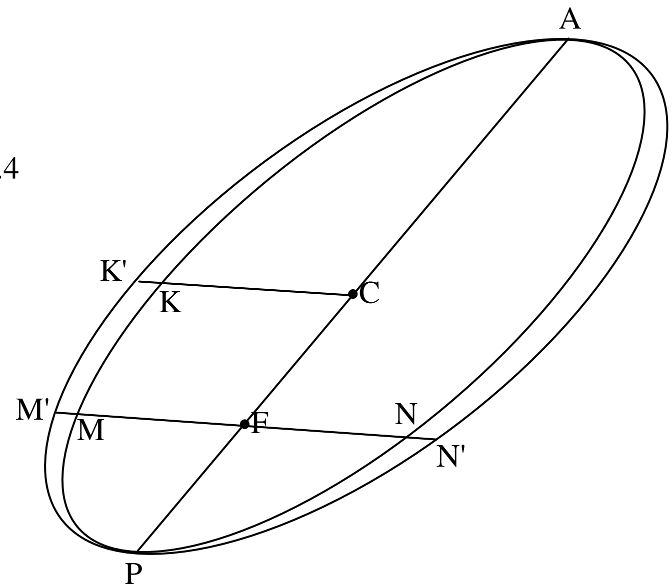
In figure XVII.3 I draw the *true ellipse* in the plane of the orbit. F is the primary star at a focus of the true ellipse. C is the centre of the ellipse. I have drawn also the auxiliary circle, the major axis (with periastron P at one end and apastron A at the other end), the latus rectum MN through F and the semi minor axis CK. The ratio FC/PC is the eccentricity e of the true ellipse, and the ratio of minor axis to major axis is $\sqrt{1-e^2}$. This is also the ratio of any ordinate on the auxiliary circle to the corresponding ordinate on the ellipse. Thus I have extended the latus rectum and the semi minor axis by the reciprocal of this factor to meet the auxiliary circle in M' , N' and K'.

FIGURE XVII.3



Now, in figure XVII.4, we are going to look at the same thing as seen projected on the plane of the sky.

FIGURE XVII.4



The *true ellipse* has become the *apparent ellipse*, and the *auxiliary circle* has become the *auxiliary ellipse.* At the start of the analysis, we know only the apparent ellipse, which is given by equation 17.4.1, and the position of the focus F, which is at the origin of coordinates, (0 , 0). F is not at a focus of the apparent ellipse, but C *is* at the centre of the apparent ellipse.

From section 2.7, we can find the coordinates $(\bar{x}\ ,\ \bar{y})$ of the centre C. These are $(\bar{g}/\bar{c}\ ,\ \bar{f}/\bar{c})$, where the bar denotes the cofactor in the determinant of coefficients. Thus the slope of the line FC, which is a portion of the true major axis, is $\bar{f}/\bar{g}$. We can now write the equation of the true major axis in the form $y = mx$ hence, by use of equations 17.4.4 and 5, we can determine the coordinates of periastron P and apastron A. We can now find the distances FC and PC; and the ratio FC/PC, which has not changed in projection, is the eccentricity e of the true ellipse.

<u>*Thus e has been determined.*</u>

Our next step is going to be to find the slope of the projected latus rectum MN and the projected semi minor axis CK, which is, of course, parallel to the latus rectum. If the equation to the projected latus rectum is $y = mx$, we can find the x-coordinates of M and N by use of equation 17.4.4. But if MN is a latus rectum, it is of course bisected by the major axis and therefore the length FM and FN are equal. That is to say that the two solutions of equation 17.4.4 are equal in magnitude and opposite in sign, which in turn implies that the coefficient of x is zero. Thus the slope of the latus rectum (and of the minor axis) is $-g/f$.

(It is remarked in passing that the projected major and minor axes are *conjugate diameters* of the apparent ellipse, with slopes $\bar{f}/\bar{g}$ and $-g/f$ respectively.)

Now that we have determined the slope of the projected latus rectum, we can easily calculate the coordinates of M and N by solution of equations 17.4.4 and 17.4.5. Further, CK has the same slope and passes through C, whose coordinates we know, so it is easy to write the equation to the projected minor axis in the form $y = mx + d$ (d is $\bar{y} - m\bar{x}$), and then solve equations 17.4.2 and 17.4.3 to find the coordinates of K.

Now we want to extend FM, FN, CK to M' , N' and K'. For M' and N' this is done simply by replacing x and y by kx and ky, where k is the factor $1/\sqrt{1-e^2}$. For K' , it is done by replacing x and y by $\bar{x} + k(x - \bar{x})$ and $\bar{y} + k(y - \bar{y})$ respectively.

We now have five points, P, A, M' , N' and K', whose coordinates are known and which are on the auxiliary ellipse. This is enough for us to determine the equation to the auxiliary ellipse in the form of equation 17.4.1. A quick method of doing this is described in section 2.8 of Chapter 2.

The slopes of the major and minor axis *of the auxiliary ellipse* (written in the form of equation 17.4.1) are given by

$$\tan 2\theta = \frac{2h}{a-b}. \qquad 17.4.6$$

This equation has two solutions for θ, differing by 90°, the tangents of these being the slopes of the major and minor axes of the auxiliary ellipse. Now that we know these slopes, we can write the equation to these axes in the form $y = mx + d$ (d is $\bar{y} - m\bar{x}$) and so we can determine where the axes cut the auxiliary ellipse and hence we can determine the lengths of the both axes of the auxiliary ellipse.

This has been hard work so far, but we are just about to make real progress. The major axis of the auxiliary ellipse is the only diameter of the auxiliary circle that has not been foreshortened by projection, and therefore it is equal to the diameter of the auxiliary circle, and hence the major axis of the auxiliary ellipse is also equal to the major axis of the true ellipse.

<u>*Thus a has been determined.*</u>

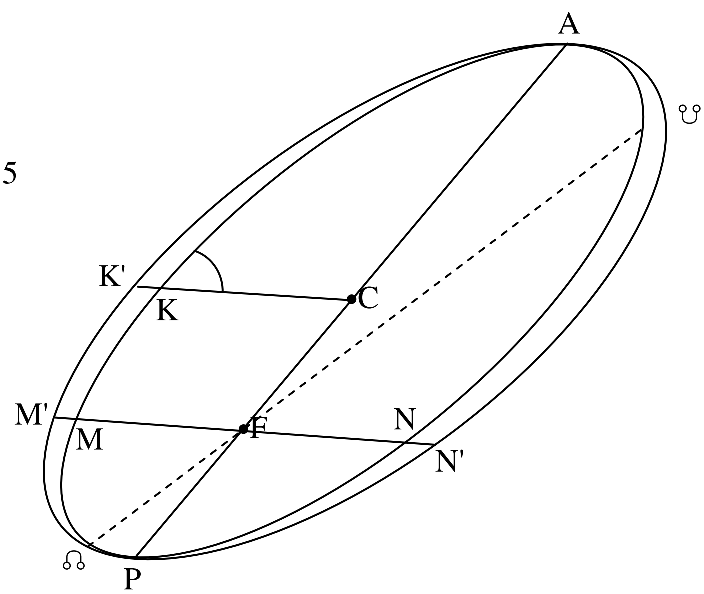
The ratio of the lengths of the minor to major axes of the auxiliary ellipse is equal to the amount by which the auxiliary circle has been flattened by projection. That is, the ratio of the lengths of the axes is equal to $|\cos i|$. Since the lengths of the axes are essentially positive, we obtain only $|\cos i|$, not cos i itself. However, by our definition of i, it lies between 0° and 180° and is less than or greater than 90° according to whether the position angle of the secondary component is increasing or decreasing with time. For example, if $|\cos i| = \frac{1}{2}$, i is 60° or 120°, to be distinguished by the sense of motion of the secondary component.

The *line of nodes* passes through F and *is parallel to the major axis of the auxiliary ellipse.* This indeed is the reason why the major axis of the auxiliary ellipse was unchanged from its original diameter of the auxiliary circle. We therefore already know the slope of the line of nodes and hence we know the position angle of the first node.

Thus Ω has been determined.

In figure XVII.5 I have added the line of nodes, parallel to the (not drawn) major axis of the auxiliary ellipse. I have used the symbols ☊ and ☋ for the first and second nodes, but we do not know (and cannot know without further information) which of these is ascending and which is descending.

FIGURE XVII.5



We can also determine the position angle of P but this is not yet ω, the argument of periastron. Rather, it is a plane-of-sky *longitude* of periastron. Let's call the angle ☊FP λ and have a look at figure XVII.6, in which the symbol ☊ refers, of course, to the nodal point, not the angle Ω.

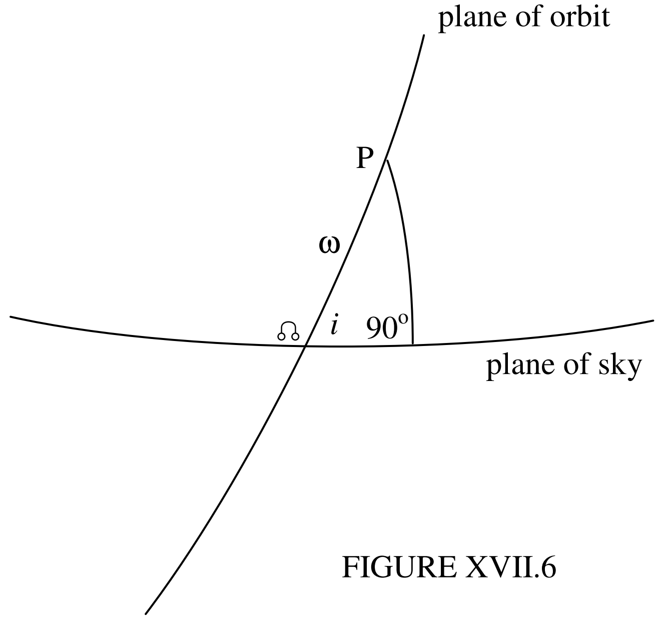


FIGURE XVII.6

Solution of the spherical triangle gives us

$$\tan\omega = \tan\lambda \sec i. \qquad 17.4.7$$

<u>*Thus ω has been determined.*</u>

We still have to determine the period P and the time T of periastron passage, but we have completed the purely geometric part, and a numerical example might be in order.

Let us suppose, for example, that the equation to the apparent ellipse is

$$14x^2 - 23xy + 18y^2 - 3x - 31y - 100 = 0.$$

Figures XVII.4, 5 and 6 were drawn for this ellipse.

I give here results for various intermediate stages of the calculation to a limited nmber of significant figures. The calculation was done by computer in double precision, and you may not get exactly all the numbers given unless you, too, retain all significant figures throughout all stages of the calculation.

Centre of apparent ellipse:	(+1.71399 , +1.95616)
Slope of true major axis:	1.14123
Coordinates of P:	(−1.73121 , −1.97582)
Coordinates of A:	(+5.15919 , +5.88814)

Length of FC:	2.60083
Length of PC:	5.22780
True eccentricity:	0.49750
Slope of latus rectum and minor axis:	−0.09677
Coordinates of M:	(−2.46975 , +0.23901)
Coordinates of N:	(+2.46975 , −0.23901)
Coordinates of K:	(−1.13310 , +2.23168)
Lengthening factor k:	1.15279
Coordinates of M':	(−2.84709 , +0.27552)
Coordinates of N':	(+2.84709 , −0.27552)
Coordinates of K':	(−1.56810 , +2.27378)

Equation to auxiliary ellipse:

$$10.5518x^2 - 16.9575xy + 15.3528y^2 - 3.0000x - 31.000y - 100 = 0$$

Slope of its major axis:	0.75619
Lengths of semi axes:	5.66541 , 2.47102
True semi major axis:	5.66541
Inclination:	64° 08' or 115° 52'
Longitude of the node:	127° 06'
λ:	11° 41'
Argument of periastron:	25° 21' or 154° 39'

That completes the purely geometrical part. It remains to determine the period P and the time of periastron passage T.

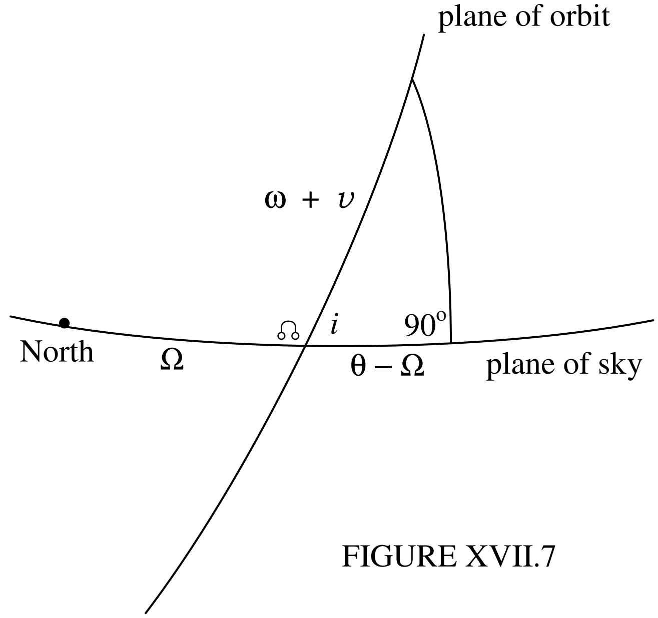


FIGURE XVII.7

Figure XVII.7 shows the secondary component somewhat past periastron, when its true anomaly is υ, so that its argument of latitude is $\omega + \upsilon$, and its position angle is θ. By solution of the spherical triangle we have (exactly as for equation 17.4.7)

$$\tan(\omega + \upsilon) = \frac{\sin(\theta - \Omega)}{\cos(\theta - \Omega)} \sec i, \qquad 17.4.8$$

so that we can determine the true anomaly υ for a given position angle θ.

[In an earlier version of these notes, equation 17.4.8 was written as $\tan(\omega + \upsilon) = \tan(\theta - \Omega)\sec i$. I am indebted to Rod Letchford of Wagga Wagga, NSW, Australia, for pointing out that this can lead to quadrant ambiguity. To avoid quadrant ambiguity it is necessary to preserve the signs of $\sin(\theta - \Omega)$ and $\cos(\theta - \Omega)$ separately. This is facilitated on many computers or calculators by means of an ATAN2 function. This is a useful reminder in orbit computation always to be alert for quadrant ambiguities!]

From the true anomaly we can now calculate the eccentric and mean anomalies in the usual manner from equations 2.3.16 or 17 and 9.6.5. So, for a given time t, we know the mean anomaly $\boldsymbol{M}$. Equation 9.6.4 is

$$\boldsymbol{M} = \frac{2\pi}{P}(t - T). \qquad 9.6.4$$

With $\boldsymbol{M}$ known for two instants t, we can solve two equations of the type 9.6.4 to obtain P and T. Better, of course, is to obtain $\boldsymbol{M}$ for *many* (perhaps hundreds) values of t and hence obtain best (least squares) solutions for P and T. To do this, a table, or graph, will be prepared, of $\boldsymbol{M}$ versus t. If, during the time covered by the observations, the stars go through periastron, it is then important to remember not to subtract 360ºfrom $\boldsymbol{M}$, but to allow $\boldsymbol{M}$ to continue to increase, so that the graph of $\boldsymbol{M}$ versus t continues as a straight line (equation 9.6.4), from which P and T can be obtained.

Recall that we used *all* of the observations (plus Kepler's second law) to obtain the best apparent ellipse. Once this has been done, the auxiliary ellipse is unique and it can be determined by just five points on it. To obtain P and T, we again have to use all the observations to obtain optimum values.

Since the above was written, Esmat Bekir has devised a method is which he calculates the elements of the auxiliary ellipse explicitly in terms of the elements of the apparent ellipse. His interesting method can be found, clearly explained, in
http://dergipark.gov.tr/uploads/articlefiles/73f0/7f53/f49e/5bdc55b9b621c.pdf

17.5 *Construction of an Ephemeris*

An ephemeris is a table giving the predicted separation and position angle as a function of time. The position angle will be given with respect to a standard equator, such as that of J2000.0, whereas observations are necessarily made with respect to the equator of date.

In the plane of the orbit it is easy (for those who have mastered Chapter 9) to calculate the true anomaly υ and the separation r as a function of time, and we can calculate the rectangular coordinates (X , Y) (figure XVII.3) from $X = r\cos\upsilon$ and $Y = r\sin\upsilon$. What we would like to do would be to calculate the plane-of-sky coordinates (x , y) (figure XVII.1). This can be done from

$$x = X\cos(x,X) + Y\cos(x,Y) \qquad 17.5.1$$

and

$$y = X\cos(y,X) + Y\cos(y,Y), \qquad 17.5.2$$

where the direction cosines can be found either (by those who have mastered Section 3.7) by Eulerian rotation of axes or (by those who have mastered Section 3.5) by solution of appropriate spherical triangles. (I'm sorry, rather a lot of mastery seems to be called for!) I make it

$$\cos(x,X) = -\cos i\sin\Omega\sin\omega + \cos\Omega\cos\omega, \qquad 17.5.3$$

$$\cos(x,Y) = -\cos i\sin\Omega\cos\omega - \cos\Omega\sin\omega, \qquad 17.5.4$$

$$\cos(y,X) = +\cos i\cos\Omega\sin\omega + \sin\Omega\cos\omega \qquad 17.5.5$$

and

$$\cos(y,Y) = +\cos i\cos\Omega\cos\omega - \sin\Omega\sin\omega. \qquad 17.5.6$$

The (x , y) and (X , Y) coordinate systems are shown in figure XVII.8 as well as in figures XVII.1 and 3.

The separation and predicted position angle are then found from

$$\rho^2 = x^2 + y^2, \qquad 17.5.7$$

$$\cos\theta = y/\rho, \qquad 17.5.8$$

and

$$\sin\theta = -x/\rho. \qquad 17.5.9$$

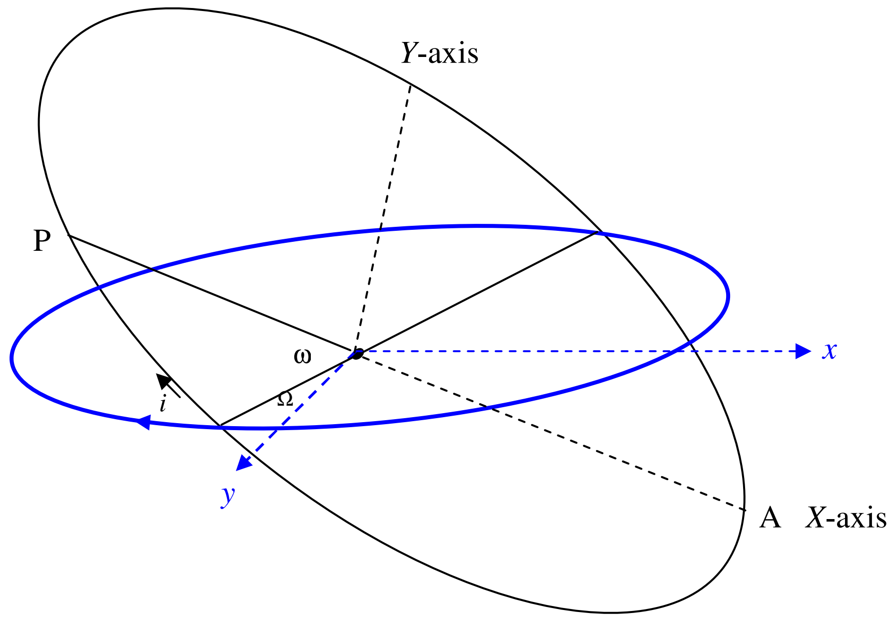


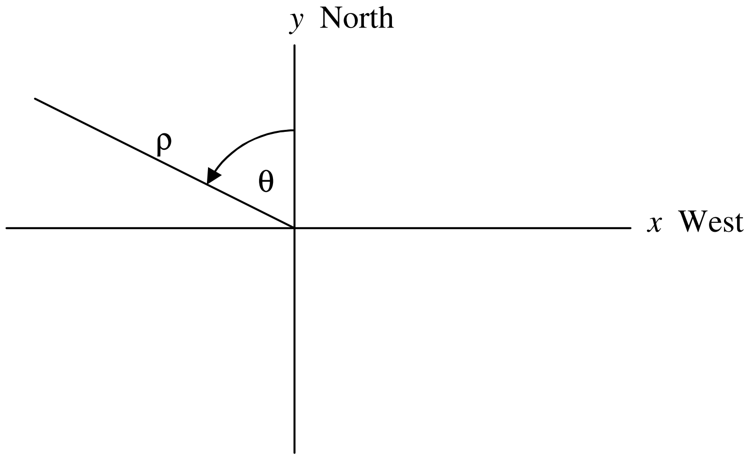
FIGURE XVII.8

17.6 *Sign Conventions*

It will occur to the reader that there are several ways in which the various angles θ, Ω, ω, i might be defined. That is, where is the starting point for their measurement, and in which direction (clockwise or counterclockwise) should they be expressed?

The following are recommended, in accordance with current practice. They have all been mentioned in the main text, but it is thought to be useful to gather them all together here, since they can be a source of difficulty.

Position angle.



East is towards increasing right ascension. West (x) is towards decreasing right ascension. Measure p.a. counterclockwise from North.

Your measurements will be made referring to the equinox and equator of date, and, if you publish your *original measurements* of position angle, this should be stated explicitly and unambiguously. Before computing an orbit, however, these should be referred to a standard equinox and equator (at present taken to be J2000.0), and this should be stated in publishing the final orbital elements. See equation 17.1.1.

In the following , it is assumed that no radial velocity data are available. Consequently the sign of the inclination and which node is ascending are unknown.

Inclination.

i is a positive number between 0 and 180º.

If the secondary is moving counterclockwise, i is between 0 and 90º.
If the secondary is moving clockwise, i is between 90 and 180º.

Ω

The term to be used in describing this angle is The Position Angle of the First Node. I.e. the node whose position angle is less than 180º.

ω

This is a positive number that goes from 0º to 360º. It is measured from the First Node, in the direction of motion of the secondary.

Further, it is **very strongly mentioned**, that whenever you write on this subject, whether in a paper, an article and a book, you always, every time, state explicitly what you mean by these angles, and do not assume that your reader will automatically use the same convention that you do.