Lecture 3

1 Photoionization

1.1 Definitions

Let I_ν be the specific intensity such that $I_\nu d\nu d\Omega dAdt$ is the energy crossing area dA in time dt and over frequency range dν and within a given solid angle dΩ ($d\Omega = sin\theta d\theta d\phi$, where ϕ is the polar angle and θ is the azimuth).

Also define J_ν, the mean intensity by integrating I_ν over all solid angles. I.e. The total photon energy within $d\nu$ passing through 1 cm^2 per second is (in erg/cm^2/sec)

$$\int_{4\pi} I_\nu d\nu d\Omega = 4\pi J_\nu d\nu \tag{1}$$

Written another way

$$J_\nu = \frac{\int I_\nu d\Omega}{\int d\Omega} = \frac{1}{4\pi}\int I_\nu d\Omega \tag{2}$$

The number of photons per second with frequencies within $d\nu$ is

$$\frac{4\pi J_\nu d\nu}{h\nu} \tag{3}$$

Let a_ν be the ionization cross-section. The number of photoionizations/sec/atom is just the cross-section multiplied by the number of photons:

$$\frac{a_\nu 4\pi J_\nu d\nu}{h\nu} \tag{4}$$

Integrating over all frequencies from the threshold frequency (ν_{min}) to ∞ the total number of photoionizations/sec/cm^3 is

$$N\int_{\nu_{min}}^{\infty} \frac{a_\nu 4\pi J_\nu d\nu}{h\nu} \tag{5}$$

The relation between J_ν and stellar luminosity is easily derived by assuming that the surface is uniform.

Calculate flux at an arbitrary distance from sphere of uniform brightness (intensity), B, i.e. all rays leaving sphere have same brightness. This is clearly isotropic. At P, the specific intensity of the ray is B if the ray intersects the sphere and zero otherwise.

$$F = \int B cos\theta d\Omega = \int B cos\theta sin\theta d\theta \; d\phi \tag{6}$$

$$F = B\int_0^{2\pi} d\phi \int_0^{\theta_c} cos\theta sin\theta d\theta \tag{7}$$

$$F = 2\pi B\int_0^{\theta_c} sin\theta d(sin\theta) = 2\pi B[1/2 sin^2\theta]_0^{\theta_c} \tag{8}$$

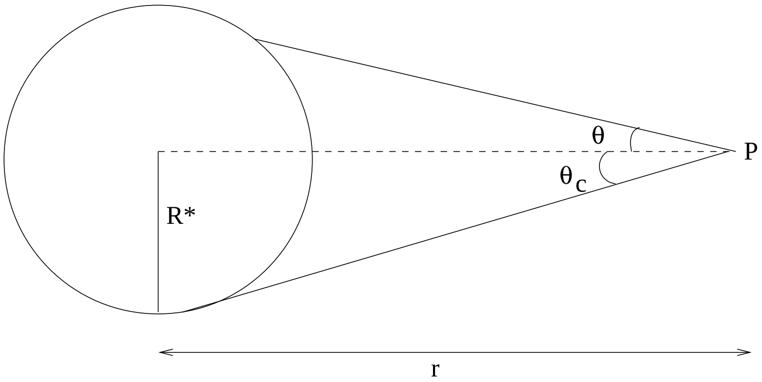

Figure 1: Flux from a uniformly bright sphere

where $sin\theta_c = R_\star/r$. This gives

$$F = \pi B(\frac{R_\star}{r})^2 \tag{9}$$

Setting $r = R$ gives the flux at the surface $F = \pi B$. The brightness is also sometimes referred to as the 'astrophysical flux' $\mathcal{F}$. So we have

$$F = \frac{R^2 \pi \mathcal{F}_\nu}{r^2} \tag{10}$$

Often, we treat ionizing sources as point sources which gives the more familiar

$$F = \frac{L_\nu}{4\pi r^2} \tag{11}$$

1.2 Stromgren Spheres

A Stromgren sphere is the ionized region around a radiation source (e.g. star) which is bounded by a transition zone from the neutral gas outside. Approximate this by assuming all ionizing radiation comes from star. Assume we have a $T = 40,000$ K star surrounded by neutral hydrogen with a density n(HI) = 1 particle cm^{-3}. How long can will an HI atom stay neutral? For ionization of pure H nebula, only radiation with $\nu > \nu_{LL}$ (or $\lambda < 912$ Å) will contribute. The photo-ionization rate at distance r from the star is

$$P = n(HI) \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu \tag{12}$$

$$Q(HI) = \int L_\nu / h\nu \sim 5 \times 10^{48}\ s^{-1} \quad \text{For a T = 40000 K star} \tag{13}$$

Here Q(HI) is the number of photons per second. Strictly, we should integrate over frequency, but assuming an average $a_\nu \sim 6 \times 10^{-18}$ cm^2 we find that at 5 pc from the star $P \sim 10^{-8}$ ionizations cm^{-3} s^{-1}. I.e. a neutral H atom can stay neutral for over a year. What fraction of H will be neutral? Let $\xi = n(HI)/n(H)$ The number of recombinations is given by $n_{HII} n_e \alpha_A$, where $\alpha_A \sim 4 \times 10^{-13}$ cm^{-3} s^{-1}. In ionization equilibrium

$$P = n(HI) \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu = n_{HII} n_e \alpha_A \tag{14}$$

In a pure H nebula $n_{HII} = n_e$ and $n_{HII} = n(H).(1-\xi)$, so taking the value of $P = 10^{-8}$ from the example above where we calculated P for n(HI)=1:

$$P = n(HI).10^{-8} \tag{15}$$

$$\xi n(H).10^{-8} = (1-\xi)^2 n(H)^2.4 \times 10^{-13} \tag{16}$$

For typical n(H) ~ 1, $\xi \sim 4 \times 10^{-5}$. How thick is the transition region? Mean free path of an ionizing photon is

$$l = \frac{1}{n(HI)a_\nu} \tag{17}$$

So if the density or cross-section are large, the MFP is small. By definition, in the transition region, half the H is neutral and half ionized.

$$l_{trans} = \frac{1}{0.5n(H)a_\nu} \tag{18}$$

For $a_\nu = 6 \times 10^{-18}$ cm^2 and N(H)=1, this works out to be $\sim$ 0.1 pc - very thin. What is the size of the ionized region? J_ν actually has two components; one from star and one from nebula as it emits photons after electrons re-combine. In particular, recombinations to the ground state emit a photon capable of causing further ionization. This gets complicated, but we can use the 'on-the spot assumption' which says that all photons produced by recombinations to the ground state ($\nu = \nu_0$) are absorbed locally. It's as if recombinations to the ground state never happened! This relies on having a short mean free path (See Osterbrock Chapter 2 for full treatment), i.e. high density or large cross section. The on-the-sot approximation combines the contribution from the diffuse phase with that of the star by correcting for the ground state recombinations:

$$\alpha_B = \alpha_A - \alpha_1 = 2.6 \times 10^{-13} cm^3 s^{-1} \tag{19}$$

α_B is the recombination rate to all except the ground state. Using just the stellar radiation and Eqn 14 with an additional optical depth term to indicate that some photons are absorbed before they reach distance r. Balancing the photo-ionization and recombination rate now gives

$$n(HI) \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu e^{-\tau_\nu} d\nu = n_{HII} n_e \alpha_B \tag{20}$$

Which we'll re-write slightly differently as

$$n(HI) \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} d\nu = 4\pi r^2 n_{HII} n_e \alpha_B \tag{21}$$

Assuming that the density in the nebula is roughly constant and R is the size of the nebula we can integrate both sides over r:

$$\int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} d\nu \int_0^R n(HI) dr = \int_0^R 4\pi r^2 n_{HII} n_e \alpha_B dr \tag{22}$$

since the optical depth as a function of radius depends on the neutral gas number density as a function of radius

$$\tau_\nu(r) = \int_0^r n(HI)(r') a_\nu dr' \longrightarrow \frac{d\tau}{dr} = n(HI) a_\nu \tag{23}$$

Substituting this into eqn 22 and remembering that (by definition) at the edge of the nebula, $\tau = \infty$, the LHS of eqn 22 becomes

$$\int_0^R n(HI) dr \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} \frac{d\tau}{n(HI) a_\nu} = \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} d\nu \int_0^{\infty} e^{-\tau_\nu} d\tau = \int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(HI) \tag{24}$$

Put another way, the photo-ionizing side of the equilibrium equation is just the number of photons produced per second (maybe not surprising, since all of the photons from the star have to be 'used' somehow). Finally, since the ionization fraction inside the nebula is very high so $n(HII) \sim n_e \sim n(H)$ we have

$$\int_{\nu_{LL}}^{\infty} \frac{L_\nu}{h\nu} d\nu = \frac{4}{3}\pi R^3 n(H)^2 \alpha_B = Q(HI) \tag{25}$$

Our 40,000K star will give an ionized region about 50 pc in radius.

1.3 Ionization Fronts - The Time Dependent Case

Stromgren spheres are not created instantaneously. During the early stages of ionization only a few photons are required to maintain equilibrium and most go to expanding the front. Later, an equilibrium is set-up. We therefore should consider a time dependent case of ionizing fronts. Recall from a previous lecture that the ionization parameter gives a measure of the velocity of the ionization front

$$q = \frac{S}{n(HI)} = \frac{dr}{dt} \tag{26}$$

The number of new ionizations per second in an expanding shell (with volume $4\pi r^2 dr$) plus the number of recombinations (which equals the number of ionizations if we're in equilibrium) equals the ionizing flux of the star:

$$Q(HI) = \frac{4}{3}\pi \alpha n(HII)^2 r^3 + 4\pi r^2 n(HII) \frac{dr}{dt} \tag{27}$$

Note that the 1st term on the RHS is from the Stromgren sphere equation, but I've assumed $n(HII) \sim n_e \sim n(H)$. Re-arranging and integrating:

$$\int dt = \int \frac{4\pi r^2 n(HII) dr}{(Q(HI) - (4\pi/3) r^3 n(HII)^2 \alpha)} \tag{28}$$

$$-tn(HII)\alpha = ln(\frac{Q(HI) - (4\pi/3)r^3 n(HII)^2\alpha}{Q(HI)}) \quad (29)$$

As $t \to \infty$ the LHS of the previous equation $\to 0$ (once exponetial of both sides is taken), so

$$Q(HI) \to \frac{4\pi}{3} r^3 n(HII)^2 \alpha \quad (30)$$

Here $r = R_s$, the Stromgren radius which we have already derived, i.e. the maximum radius which can be supported against recombination by the stellar flux. Substituting $R_s = (3Q(HI)/4\pi n(HII)^2\alpha)^{1/3}$ into equation 29 we get

$$r = R_s(1 - exp[-\alpha n(HII)t])^{1/3} \quad (31)$$

The characteristic timescale at which the Stromgren radius is approached is

$$\tau_s = \frac{1}{\alpha n(HII)} \sim 10^5/n(HII)\text{years} \quad (32)$$

This is short in comparison with the lifetime of a star. The boundary of the ionization front has velocity

$$v \sim R_s/\tau_s \sim 300(Q_{48} n(HII))^{1/3} \quad \text{km/s} \quad (33)$$

The front is supersonic compared with the unionized medium (which has a sound speed ~ 1 km/sec. Two types

1. R-type (rapid/rarefied) which is supersonic wrt both ionized and unionized gas. This is usually the case in a young nebula.

2. D-type (dense) fronts occur as the velocity slows to the sound speed of the medium. The strong pressure gradient across the front can cause a shock.

1.4 Cosmological Stromgren Spheres

The theory of Stromgren spheres can be used to infer the epoch of re-ionization. Current limits come from

1. Presence of high redshift ($z \sim 6$) Lyα emitters. To be seen, these must be embedded in an ionized medium.

2. Gunn-Peterson trough in $z \sim 6.5$ QSOs shows low transmission, so higher neutral fraction. Rapid evolution $z = 6 - 6.5$? Are we seeing the end of re-ionization?

3. WMAP (1st year) optical depth measurements indicate early re-ionization $z \sim 17$

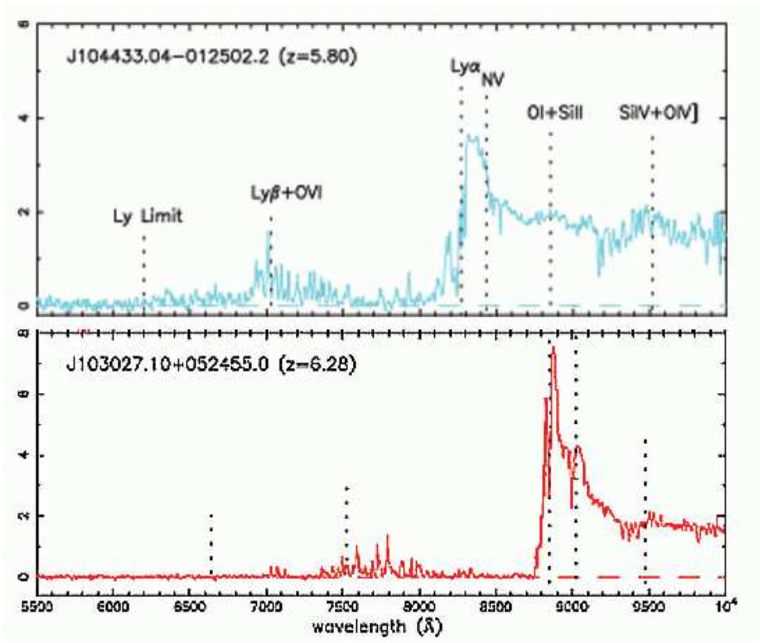

Figure 2: Gunn-Peterson trough in high redshift QSOs

All of these techniques have caveats. Lyα emitters could be sitting in a local stromgren sphere, the Gunn Peterson trough becomes black by $\xi \sim 10^{-3}$ and optical depths from WMAP have a very broad distribution. Nonetheless, there seems to be a discrepancy with GP measurements which have evoked some exotic multi-reionization scenarios. We would like to map the optical depth to higher precision. One possibility may be with QSOs, since they will ionize a Stromgren sphere in a neutral medium, the size of which will depend on the neutral fraction of the medium. The optical depth between an observer at $z = 0$ and a source at $z = z_s$ is

$$\tau(\lambda_{obs}) = \int_0^{z_s} dzc\frac{dt}{dz}n_H(z)\xi_H(z)\sigma(\frac{\lambda_{obs}}{1+z}) \tag{34}$$

where $c.dt/dz$ is the line element in a given cosmology and σ is the absorption cross section. However, since QSOs sit inside Stromgren spheres, the total optical depth is a sum of the contributions from inside and out: $\tau = \tau_i + \tau_o$.

$$\tau_i(\lambda_{obs}) = \int_{z_{HII}}^{z_s} dzc\frac{dt}{dz}n_H(z)\xi_H(z)\sigma(\frac{\lambda_{obs}}{1+z}) \tag{35}$$

$$\tau_o(\lambda_{obs}) = \int_{z_{end}}^{z_{HII}} dzc\frac{dt}{dz}n_H(z)\xi_H(z)\sigma(\frac{\lambda_{obs}}{1+z}) \tag{36}$$

z_{HII} is the redshift corresponding to the edge of the Stromgren sphere and z_{end} corresponds to the redshift at which the HI absorption becomes insignificant.

1.5 Balmer Radiation/Decrement

It is easily shown that the Balmer radiation (photons down to $n = 2$) in an HII region is directly related to the ionizing flux. Virtually all recombinations will reach the ground state before another

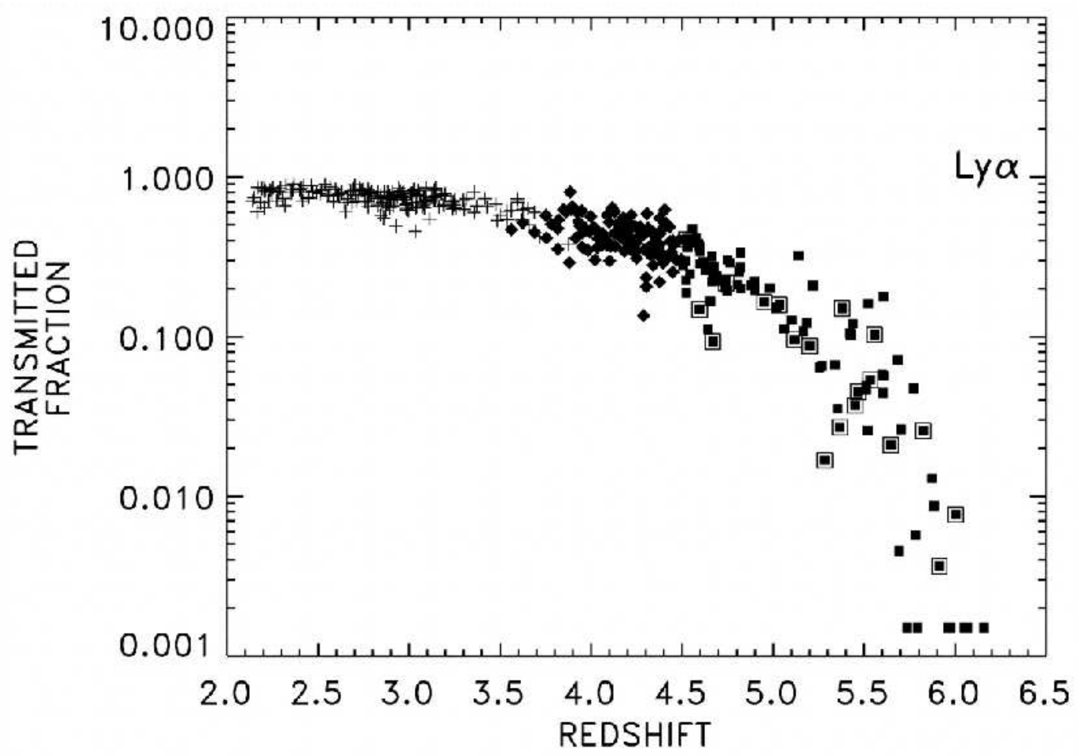

Figure 3: The Lyα optical depth is a very sensitive probe of neutral fraction. Taken from Songaila, 2004, AJ, 127, 2598.

ionizing photon is absorbed. Moreover, the only way to avoid creating a Balmer photon is to decay directly from $n > 2$ to $n = 1$, i.e. creating a Lyman photon. However, Lyman photons undergo heavy scattering, providing a second opportunity to cascade down. Balmer photons all generally escape the nebula so the number of photons leaving the HII region per second must equal Q(HI). I.e.

Number of ionizing photons emitted by star/sec = number of Balmer photons produced by nebula/sec

Moreover, the ratios of the Balmer lines depend on their emission coefficients. For example, relative to Hβ ($= 4 \rightarrow 2$)

$$\frac{F(n,2)}{F(4,2)} = \frac{F(Hn)}{F(H\beta)} = \frac{N_n A_{n,2} \nu_{n,2}}{N_4 A_{4,2} \nu_{H\beta}} \tag{37}$$

The different emission coefficients of the Balmer series only differ very slightly with temperature, so line ratios governed mostly by constants and an assumption about whether the nebula is optically thin or thick.

- Case A: The nebula is optically thin and every photon produced through a photon to the ground state is free to leave the nebula.

- Case B: The nebula is optically thick (more realistic). Each recombination to the ground state contributes to photo-ionizing background. The state of ionization is therefore higher

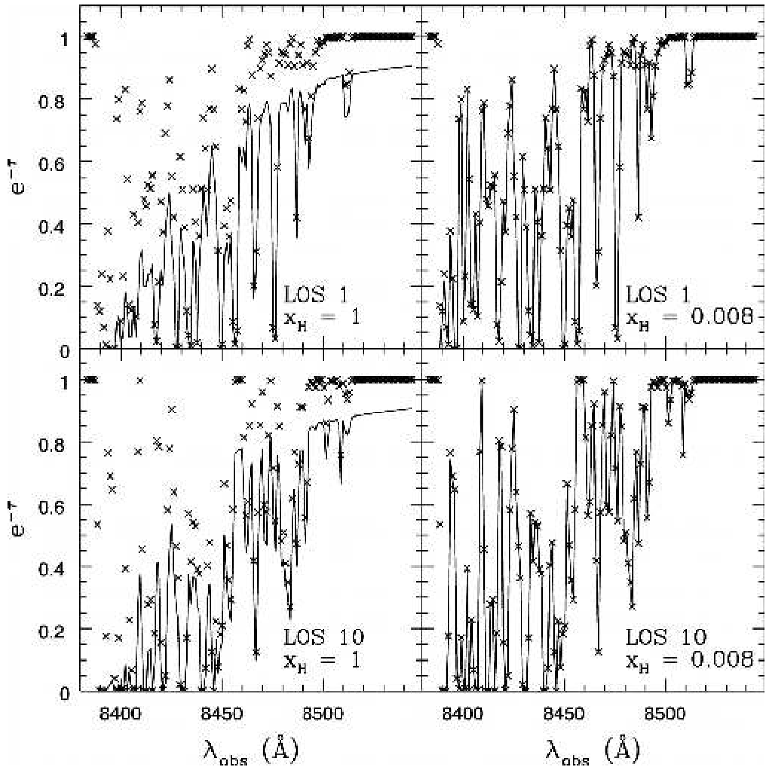

Figure 4: Simulated spectra with different ionization fractions with R_s=43 Mpc. Crosses show the optical depth contribution just from the inside of the sphere, and the solid line shows the optical depth from the medium in which it is embedded. On the right, most of the gas is ionized so the dominant contribution is τ_i. On the left the medium is completely neutral. Taken from Mesinger, Haiman, Cen 2004, ApJ, 613, 23.

than expected from just the stellar contribution (see the on-the-spot approx.)

Compare the theoretical line ratios of Hα and Hγ to Hβ:

	Case A		Case B		Mean
	10^4 K	2×10^4 K	10^4 K	2×10^4 K	observed
Hα	1.91	1.99	2.71	2.79	2.77
Hγ	0.59	0.57	0.51	0.50	0.50

Deviations from the case B ratios can be caused by dust which preferentially extinguishes bluer lines causing a 'Balmer decrement'. We usually assume a Balmer decrement of 2.85 between Hα and Hβ.

1.6 Hydrogen and Helium Nebulae

Ionization potentials are: IP(HeI) = 24.6 eV, IP(HeII) = 54.4 eV. So the radiation field can be considered in three parts:

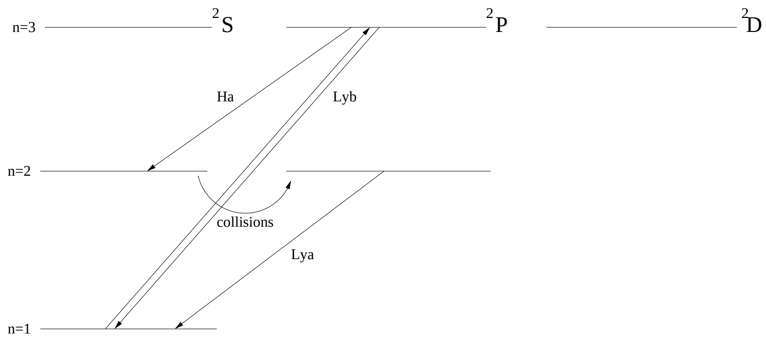

Figure 5: Scattering in HI produces Balmer photons unless it goes directly from $n > 2$ to $n = 1$ thus creating a Ly photon.

Band A: H-ionizing	$1.0 < \nu/\nu_H < 1.8$
Band B: H-ionizing, HeI-ionizing	$1.8 < \nu/\nu_H < 4.0$
Band C: H-ionizing, HeI, HeII-ionizing	$4.0 < \nu/\nu_H$

Note that $IP(HI) = 13.6$ eV is also known as 1 Rydberg. These bands cause 'edges' in the spectrum i.e. boundaries at which photons with higher energies are largely used up (opacity is high). In the HI zone, the first species with IP above 1 Ryd will dominate. In the He I (H II) zone, the first species with IP > 1.8 Ryd will dominate and in the He II (H II) zone, the first species with IP > 4.0 Ryd will dominate.

Due to its high photo-ionization cross-section HeI competes well with H for band B photons. However, there is some 'payback' because HeII recombinations can produce photons energetic enough to ionize HI. Approximately 25% of HeII recombinations go to singlets and 75% to triplets. Effectively all the triplets finally decay from 2^3P to 1^1S (inter-combination line) with $E > 1$ Ryd. 2/3 of the singlets end up in the 2^1P state which decays to 1^1S, again with $E > 1$ Ryd. The 1/3 which end up in the 2^1S state undergo two-photon emission (via a virtual p-state) and there is a 56% probability that their energies will be > 1 Ryd. Total probability of a He recombination producing an HI-ionizing photon is

$$P = 0.75 + 0.25[0.66 + 0.33(0.56)] = 0.96 \tag{38}$$

We can therefore largely de-couple the treatment of H and He.

$$Q(HeI) = \frac{4\pi R_{HeII}^3 n_{HeII} N_e \alpha_r(HeI)}{3} \tag{39}$$

$$Q(HI) = \frac{4\pi R_{HII}^3 n_{HII} N_e \alpha_r(HI)}{3} \tag{40}$$

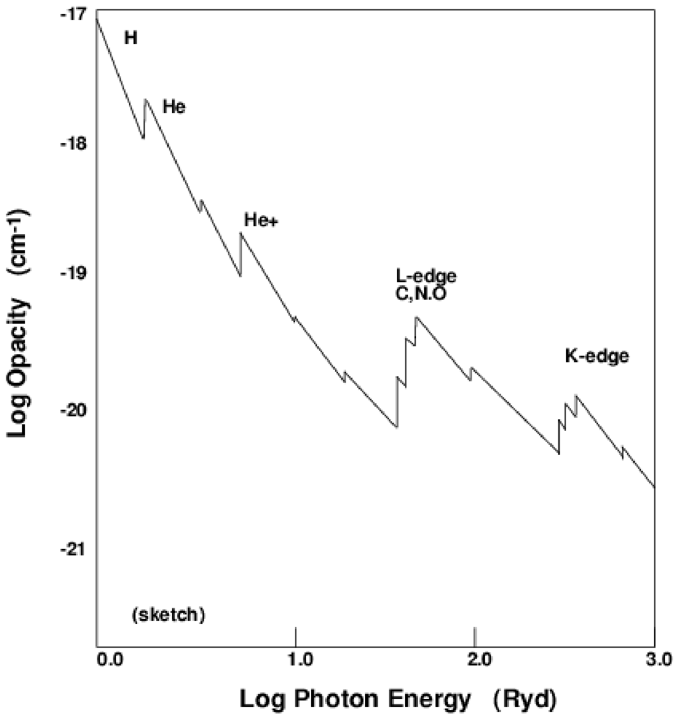

Figure 6: A stellar spectrum is suffers absorption in different bands according to frequency due to H and He.

2 Recombination

2.1 Radiative Recombination

Process by which an electron is captured to an excited state (high n high L captures more probable) followed by cascade through lower states.

$$A^{+n} + e^{-} \rightarrow A^{*+(n-1)} \rightarrow A^{+(n-1)} + h\nu_1 + h\nu_2 + ... \tag{41}$$

The recombination rate is the number of recombinations per unit time per unit volume

$$r_{rec} = N_e N_{ion} \int_0^\infty \sigma_{rec}(v,n) v f(v) dv = N_e N_{ion} \alpha_n \tag{42}$$

This is analogous to equations for collisional excitation. σ is once again a cross-section (this time for recombination) and α is a rate coefficient (similar to q in collisional excitation). As you might expect, the cross-sections for photo-ionization and recombination are related

$$\sigma_{rec}(v,n) = \frac{n^2 (h\nu)^2 a_\nu}{(m_e c)^2 v^2} \tag{43}$$

Whilst typical recombination cross sections are $\sigma_{rec}(v,n) \sim 10^{-20}$ cm^{-2}, the cross section for elastic collisions (scattering) is $\sim 10^{-13}$ cm^{-2}. So many interactions occur before an electron is captured. As a consequence the velocity distributions are closely Maxwellian.

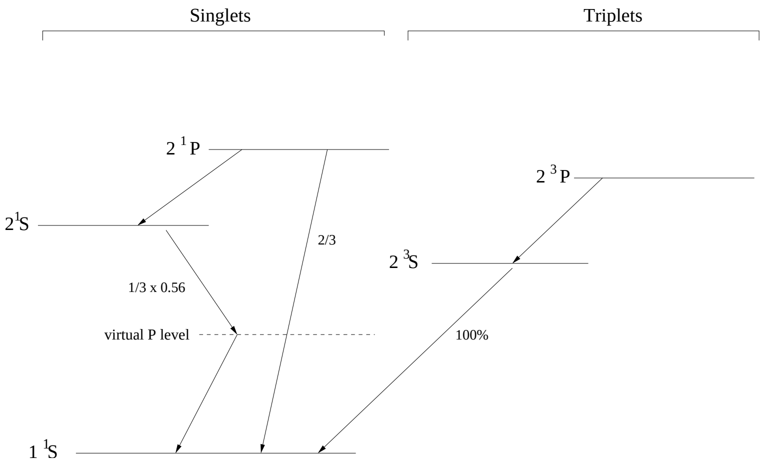

Figure 7: He recombination - effectively all recombinations produce a H-ionizing photon.

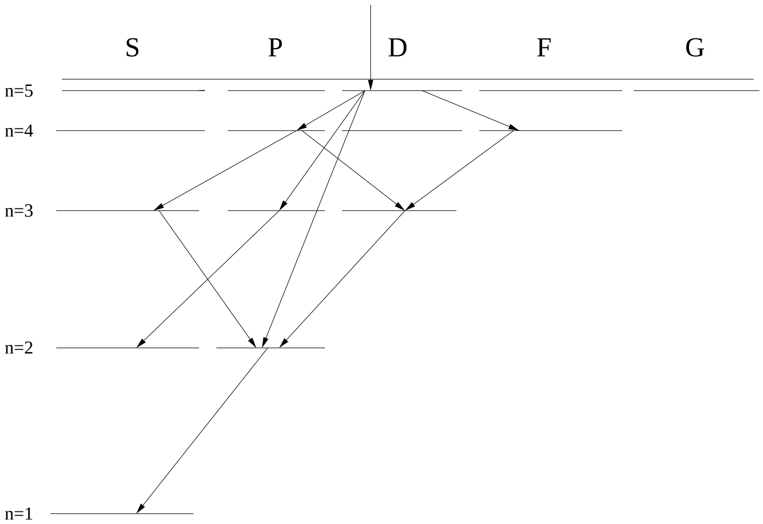

Figure 8: Recombination cascade.

2.2 Dielectronic Recombination

The main pathway for this reaction is through excitation of a core electron by capture of a passing electron. One electron is promoted to an excited state $(n2, l2)$ and the other is in an auto-ionizing state $(n1, l1)$. First the ion stabilizes itself when one of the valence electrons radiates back to the ground state. Then a radiative cascade follows:

$A^{i+} + e^- \rightarrow A_*^{(i-1)+}(n1l1; n2l2)$
$A_*^{(i-1)+}(n1l1; n2l2) \rightarrow A_*^{(i-1)+}(n3l3; n2l2)$
$A_*^{(i-1)+}(n3l3; n2l2) \rightarrow A^{(i-1)+}(n3l3; n4l4) + h\nu_1 + h\nu_2 + ...$

Because of the generally large energy required to excite the bound electron, dielectronic recombination is only important at high temperatures. The total recombination cross-section is therefore

$$\alpha_{tot} = \alpha_r + \alpha_d \tag{44}$$

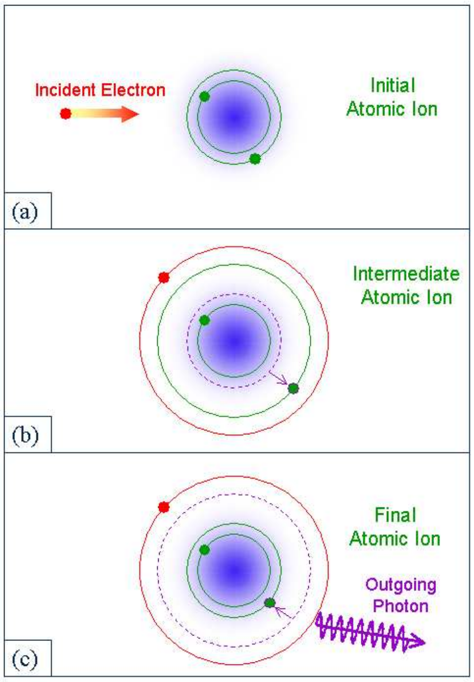

Figure 9: Dielectronic recombination.