Lecture 2

1 Collisional Excitation

Collisional excitation in O and N is an important source of radiative cooling because their excitation potentials are $\sim kT$. Collisional de-excitation can cause electron to go back to lower level if it doesn't radiate first. Collisional cross section is a strong function of impact energy (and therefore T). Define collision cross section $\sigma_{12}(E)$ in terms of collision strength Ω_{12}

$$\sigma_{12}(E) = (\frac{h^2}{8\pi m_e E})(\frac{\Omega_{12}}{g_1})\mathrm{cm}^2 \tag{1}$$

where g_1 is the statistical weight of the ground state. Compare this with the cross-section for absorption, some similarities in the definition, but the damping constant has been replaced by the collision strength. Note that Ω gets shared for the levels in a term and the 'share' is proportional to the statistical weight ($g = 2J + 1$). E.g. For $^3P_{0,1,2}$ Ω is shared 1:3:5. Also Ω can depend on energy (more so for neutrals than ions) so we're really talking about an average.

1.1 2-Level Atom

Consider a 2-level atom which is excited collisionally by an electron. The cross-section is zero if $E < E_{12}$, i.e. there is a critical energy required for collisional excitation, analagous to the energy required for photo-excitation.

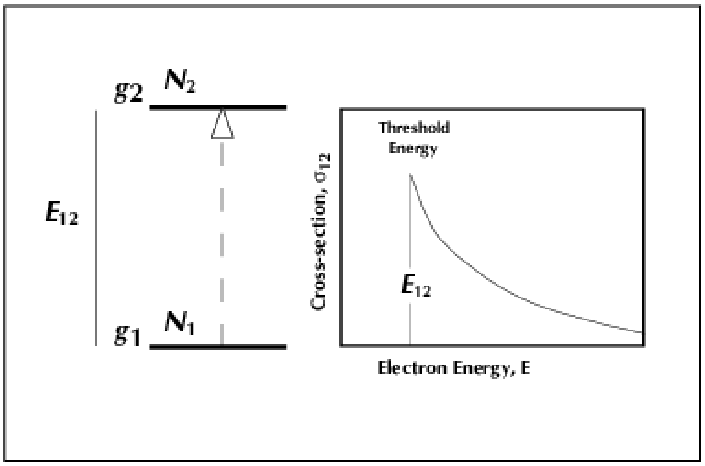

Figure 1: Collisional cross section as a function of electron impact energy.

The rate at which collisions occur is governed by Maxwellian velocity distribution, $f(v)$ which tells us the probability that an electron with have a certain velocity. The number of electrons with velocities between v and $v + dv$ is therefore $N_e f(v)dv$. Since σ is the cross section for interaction the electron will sweep out an 'interaction volume' of σv per second. N_1 is the number of atoms per volume so one elctron can interact $N_1 \sigma v$ times per second. So total number of interactions per second is $(N_1\sigma v).N_e f(v)dv$ integrated over all velocities greater than the threshold value (corresponding to E_{12}). That is the *rate* at which collisional excitations occur (in units of no. cm^{-3} s^{-1}) is

$$N_1 N_e \int_{E_{12}}^{\infty} \sigma_{12} v f(v)dv = N_1 N_e (\frac{2\pi}{m_e kT})^{1/2} \frac{\hbar^2 \Omega_{12}}{m_e g_1} e^{\frac{-E_{12}}{kT}} = N_1 N_e q_{12} \quad (2)$$

where q_{12} is the collision rate coefficient. Combining the constants, we get

$$q_{12} = 8.63 \times 10^{-6} T^{-1/2} \Omega_{12}/g_1 e^{\frac{-E_{12}}{kT}} \quad (3)$$

In equilibrium, the number of collisional excitations into level 2 will equal the number of collisional and radiative de-excitations from level 2 to 1:

$$N_e N_1 q_{12} = N_e N_2 q_{21} + N_2 A_{21} \quad (4)$$

In the low density limit, collisional de-excitation unimportant cf radiation ($N_e N_2 q_{21} << N_2 A_{21}$) and we get

$$N_2 = \frac{N_e N_1 q_{12}}{A_{21}} = \frac{N_e N_1 \Omega_{12}}{A_{21} g_1} \frac{8.63 \times 10^{-6}}{T^{1/2}} e^{\frac{-E_{12}}{kT}} \quad (5)$$

The cooling rate (energy lost per second) will be

$$L_c = N_2 A_{21} h\nu_{12} \quad (6)$$

i.e. dominated by exponential al low T and $T^{1/2}$ at high T

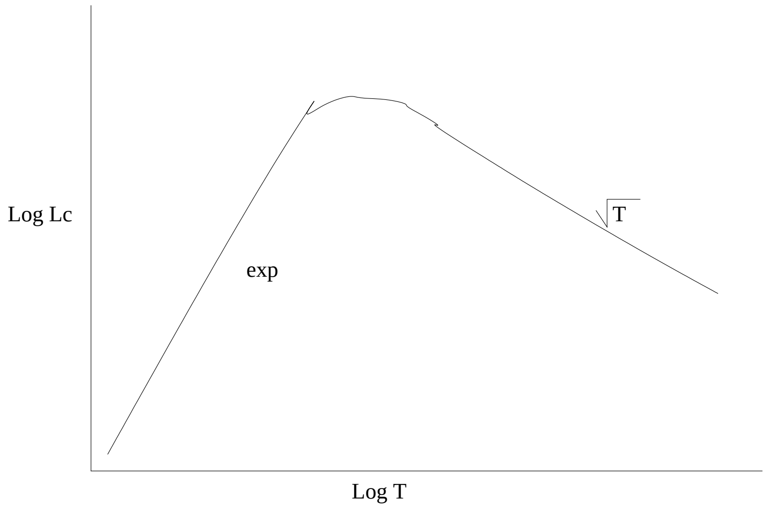

Figure 2: Cooling rate as a function of temperature

In the high density limit ($N_e N_2 q_{21} >> N_2 A_{21}$)

$$\frac{N_2}{N_1} = \frac{q_{12}}{q_{21}} = \frac{\Omega_{12} g_2}{\Omega_{21} g_1} e^{\frac{-E_{12}}{kT}} \quad (7)$$

At high densities, we can consider again the Boltzmann distribution (which is not set up at loe densities). I.e. we recall

$$\frac{N_j}{N_1} = \frac{g_j}{g_1}\, exp(\frac{-\Delta E}{kT}) \tag{8}$$

we thus see that $\Omega_{21} = \Omega_{12}$, i.e. collision strengths are symmetric. The cooling rate in the high density case is

$$L_c = N_2 A_{21} h\nu_{12} = N_1 A_{21} h\nu_{12} g_1/g_2 e^{\frac{-E_{12}}{kT}} \tag{9}$$

1.2 The 3-level Atom

Equations of statistical equilibrium are very similar. We write two equations which consider first the $N = 3$ level as the upper state and then the $N = 2$ as the upper state. The LHS 'feeds' the upper state and the RHS is for reactions which deplete it:

$$N_1 N_e q_{13} + N_2 N_e q_{23} = N_3 N_e q_{31} + N_3 N_e q_{32} + N_3 A_{31} + N_3 A_{32} \tag{10}$$

$$N_1 N_e q_{12} + N_3 N_e q_{32} + N_3 A_{32} = N_2 N_e q_{21} + N_2 N_e q_{23} + N_2 A_{21} \tag{11}$$

and

$$N_1 + N_2 + N_3 = N_{ion} \tag{12}$$

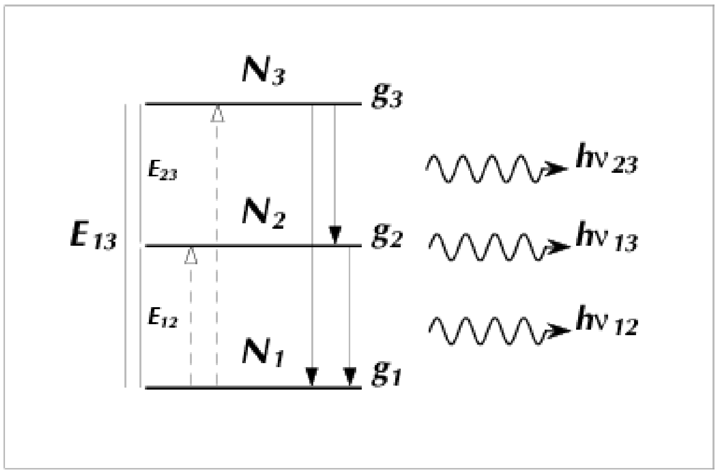

Figure 3: 3 level atom with near equally spaced excited states - good for temperature diagnostic.

Let's consider a simple case where $E_{12} \sim E_{23}$. In the low density limit collisional de-excitation insignificant ($N_3 N_e q_{32}, N_3 N_e q_{31}, N_2 N_e q_{21}$ can be neglected). Also, because of increasing energies $N_3 << N_2 << N_1$, so equations become

$$N_1 N_e q_{13} = N_3 (A_{31} + A_{32}) \tag{13}$$

$$N_1 N_e q_{12} + N_3 A_{32} = N_2 A_{21} \tag{14}$$

Hence

$$N_3 = \frac{N_1 N_e q_{13}}{A_{31} + A_{32}} \qquad\qquad N_2 = \frac{N_1 N_e q_{12}}{A_{21}} \tag{15}$$

The ratio of line intensities is basically a ratio of cooling rates (i.e. how much energy is associated per second with each transition)

$$\frac{I(\nu_{32})}{I(\nu_{21})} = \frac{N_3 A_{32} h\nu_{32}}{N_2 A_{21} h\nu_{21}} = \frac{\Omega_{13} A_{32} \nu_{32}}{\Omega_{12}(A_{31} + A_{32})\nu_{21}} e^{\frac{-E_{23}}{kT}} \tag{16}$$

I.e. line ratio varies as $exp\frac{-E_{23}}{kT}$. This means that the temperature of a gas can be inferred by observing the ratio of lines with different upper energy states. Commonly observed example of collisionally excited lines which are used for this are forbidden [OIII] lines.

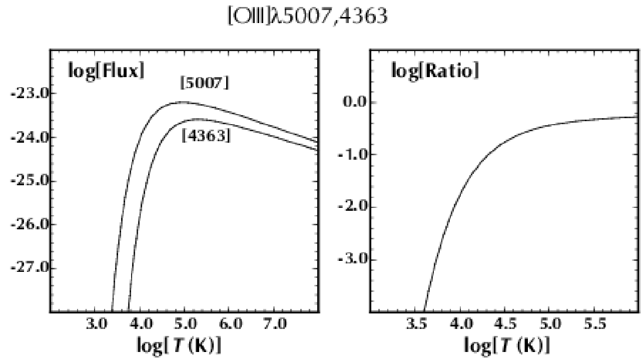

Figure 4: [OIII] flux and line ratios as a function of T.

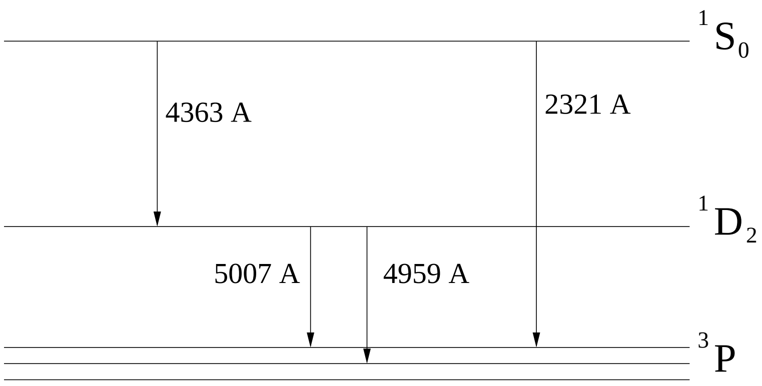

Figure 5: Excitation potentials in oxygen

Once the temperature is known, we can solve for the abundance and figure out [O/H]. Note that (unlike simple absorption measures of metallicity) the strength of a single O and H line alone does not tell us the abundance of O:H.

In practice the 4363 Å line is only seen at relatively low metallicities and is abundances can't be directly determined using the above method when $Z > 1/10 Z_{\odot}$. In order to get abundances from

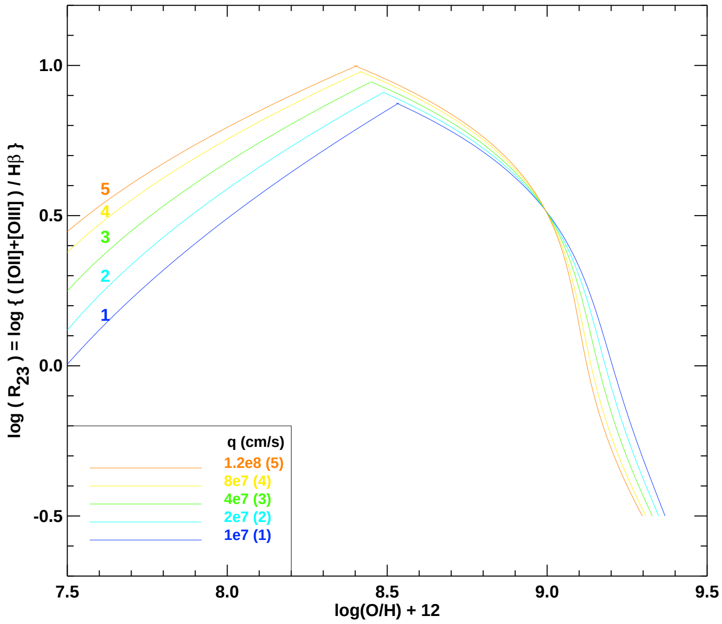

Figure 6: R_{23} calibrations taken from Kobulnicky & Kewley (2004).

just the strong lines, many empirical calibrations have been developped. R_{23} is the most famous of these, developped by Bernard Pagel. Many different calibrations, some empirical, some theoretical.

$$R_{23} = \frac{I([OII]\lambda 3727) + I([OIII]\lambda\lambda 4959 + 5007)}{I(H\beta)} \tag{17}$$

Note that the 4959 line is often too weak to be observed, so we infer its strength because the doublet occurs in the ratio 1:3. The advantage is that only the strongest lines need to be observed. Disadvantage is double-valued nature because at low metallicities cooling is no longer efficient. Some *apriori* knowledge of abundance required. Also, abundances are poorly determined in turnover region and there is a dependence on ionization parameter, q.

$$q = \frac{S_{\mathrm{H}^0}}{n} \tag{18}$$

where S_{H^0} is the ionizing photon flux passing through a unit area, and n is the local number density of hydrogen atoms. The ionization parameter q, can be physically interpreted as the maximum velocity of an ionization front that can be driven by the local radiation field. This ionization parameter is divided by the speed of light to give the more commonly used ionization parameter $\mathcal{U}$;

$$\mathcal{U} \equiv q/c = \frac{U}{h\nu n} \tag{19}$$

where U is the local photon energy density. Thus, $\mathcal{U}$ is simply the local ratio of the density of photons to the density of atoms. To break the R_{23} degeneracy, another line is required, e.g. [NII]. Other strong line ratios appear monotonic with metallicity, but are only easily applied at low redshift. E.g. [NII]/Hα

However, different calibrations can give quite different results.

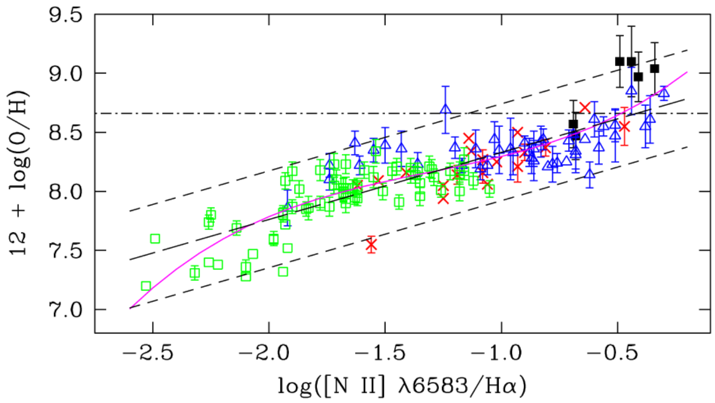

Figure 7: [NII]/Hα calibration taken from Pettini & Pagel (2004).

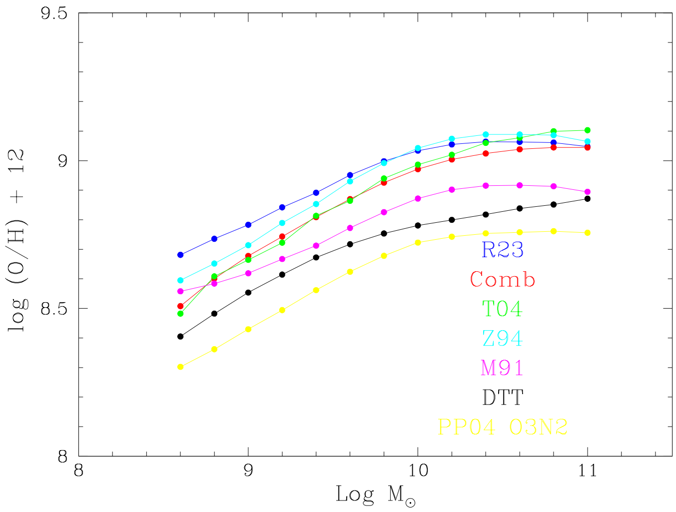

Figure 8: Mass-metallicity relation for various different calibrations.

For any of these calibrations, it is assumed that stars are the major source of ionizing photons. If AGN are present they no longer work. Line ratios indicate whether there is AGN contamination. Other pitfalls of these empirical calibrations are 1) need to account for dust (see lecture notes) and 2) need to account for underlying Balmer absorption.

2 Line Transfer Effects

Consider a few of the effects of line transfer that lead to the emission of radiation.

2.1 Resonance Fluorescence

Photons emitted by one ion are re-absorbed by a different ion and de-excite into two or more photons of lower energy. Some specific examples are

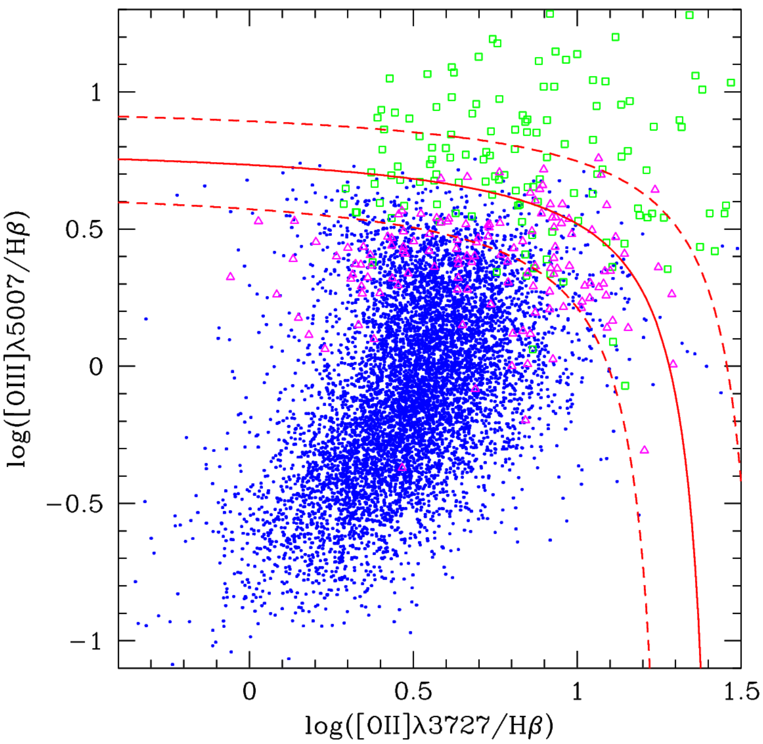

Figure 9: Separation of starbursting galaxies and AGN from Lamareille et al (2004)

1. Bowen Fluorescence. Near coincidence of HeII Lyα ($\lambda = 303.78$ Å) and the $2p^2\ ^3P_2 - 3d\ ^3P_2$ in [OIII] ($\lambda = 303.80$Å) allows conversion of HeII Lyα photons to [OIII] photons.

2. Fluorescence of H_2 with hydrogen Lyα. In warm collisionally excited molecular gas that surrounds a source of Lyα photons, a number of H_2 transitions can absorb Lyα and convert this radiation to several emission lines in the 1100 – 1700 Å range. This has been seen e.g. in HH objects.

3. Resonance scattering of Lyman lines. Because H is so abundant, the Lyman series is particularly thick to scattering. This means that any flavour photon (e.g. Lyα) can't survive many interactions before being absorbed again. Consider a Lyβ photon which excites an electron from n=1 to n=2. The electron's decay probability back through Lyβ is 0.882 but there is a 0.118 probability that the electron will de-excite to Hα. The survival probability after n scatterings is therefore 0.882^n i.e. only 1 in a million after 100 scatterings. In this way, an $n > 1$ member of the Lyman series can get converted into Lyα (lowest energy transition of Lyman series) plus members of another series.

2.2 Charge Exchange

An atom and ion can interact and exchange one or more electrons without being accompanied by the absorption or emission of radiation. E.g.

OI + HII $\rightleftharpoons$ OII + HI + 0.02 eV
NI + HII $\rightleftharpoons$ NII + HI + 0.94 eV

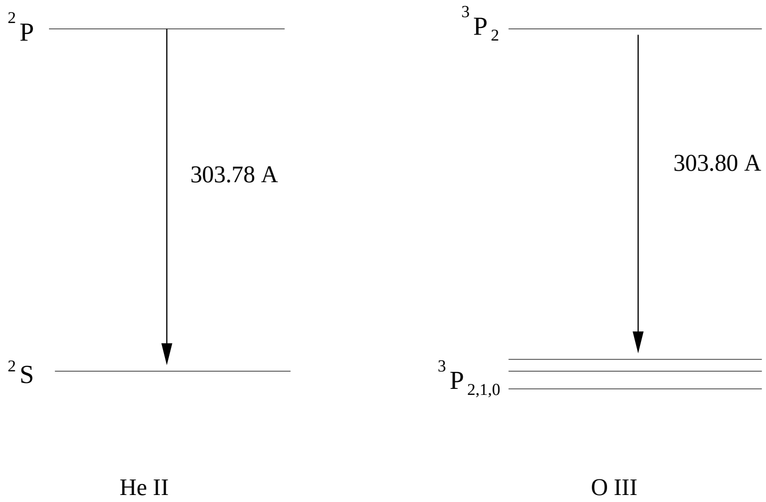

Figure 10: Bowen fluorescence.

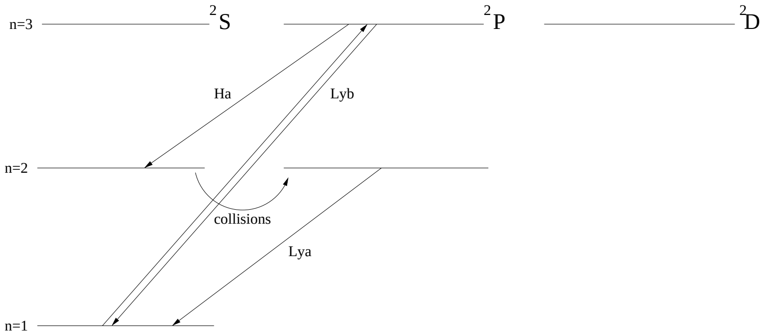

Figure 11: Scattering through the Lyman series.

At temperatures higher than the difference in IP charge exchange reactions tend to set up an equilibrium. Define

$$\delta = \int_0^\infty \sigma_{ce} v f(v) dv \tag{20}$$

Where δ is charge exchange rate cofficient, similar to q in collisional excitations. So, analogously to collisional excitation equations we have the reaction rates

- N(OI)N(HII)δ is the rate/cm^3/sec for the forward reaction
- N(OII)N(HI)δ' is the rate/cm^3/sec for the reverse reaction

Taking the ratios of these reactions and including the values of δ

$$\frac{N(OI)}{N(OII)} = \frac{9N(HI)}{8N(HII)} \quad \frac{N(NI)}{N(NII)} = \frac{2N(HI)}{9N(HII)} \tag{21}$$

The coefficients (basically δ/δ') come from solving the Milne equation for transition cross sections (see Osterbrock if interested in the details). In brief, the Milne relation is a formula which relates the cross-section for photo-ionization to the cross-section for recombination in LTE.

This brings us to an important effect, which we'll re-visit later. Normally, the dominant species in an HI cloud is the first species to have an IP $>$ 13.6 eV, because higher energies are 'used' to ionize HI. E.g. MgII and SiII are dominant. If the cloud is partially ionized, this may not be the case, but charge exchange reactions maintain OI and NI as the dominant states even when the cloud is partially ionized.

Complete Ionization Potentials for the First 10 Elements (eV)											
Z	**element**	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**	**10th**
1	**H**	13.6									
2	**He**	24.6	54.4								
3	**Li**	5.4	75.6	122							
4	**Be**	9.3	18.2	154	218						
5	**B**	8.3	25.2	37.9	259	340					
6	**C**	11.3	24.4	47.9	64.5	392	490				
7	**N**	14.5	29.6	47.5	77.5	97.9	552	667			
8	**O**	13.6	35.1	54.9	77.4	114	138	739	871		
9	**F**	17.4	35.0	62.7	87.1	114	157	185	953	1103	
10	**Ne**	21.6	41.0	63.5	97.1	126	158	207	239	1196	1362

First 5 Ionization Potentials (eV) only, for other "A" group elements													
Z	**element**	**1st**	**2nd**	**3rd**	**4th**	**5th**	**Z**	**element**	**1st**	**2nd**	**3rd**	**4th**	**5th**
11	**Na**	5.1	47.3	71.6	98.9	138	38	**Sr**	5.7	11.0	43.6	57	71.6
12	**Mg**	7.6	15.0	80.1	109	141	49	**In**	5.8	18.9	28.0	54	?
13	**Al**	6.0	18.8	28.4	120	154	50	**Sn**	7.3	14.6	30.5	40.7	72.3
14	**Si**	8.2	16.3	33.5	45.1	167	51	**Sb**	8.6	16.5	25.3	44.2	56
15	**P**	10.5	19.7	30.2	51.4	65.0	52	**Te**	9.0	18.6	28.0	37.4	58.8
16	**S**	10.4	23.3	34.8	47.3	72.7	53	**I**	10.5	19.1	33	?	?
17	**Cl**	13.0	23.8	39.6	53.5	67.8	54	**Xe**	12.1	21.2	32.1	?	?
18	**Ar**	15.8	27.6	40.7	59.8	75.0	55	**Cs**	3.9	25.1	?	?	?
19	**K**	4.3	31.6	45.7	60.9	82.7	56	**Ba**	5.2	10.0	?	?	?
20	**Ca**	6.1	11.9	50.9	67.1	84.4	81	**Tl**	6.1	20.4	29.8	?	?
31	**Ga**	6.0	20.5	30.7	64	?	82	**Pb**	7.4	15.0	31.9	42.3	68.8
32	**Ge**	7.9	15.9	34.2	45.7	93.5	83	**Bi**	7.3	16.7	25.6	45.3	56.0
33	**As**	9.8	18.6	28.4	50.1	62.6	84	**Po**	8.4	?	?	?	?
34	**Se**	9.8	21.2	30.8	42.9	68.3	85	**At**	9.5	?	?	?	?
35	**Br**	11.8	21.8	36	47.3	59.7	86	**Rn**	10.7	?	?	?	?
36	**Kr**	14.0	24.4	37.0	52.5	64.7	87	**Fr**	4	?	?	?	?
37	**Rb**	4.2	27.3	40	52.6	71.0	88	**Ra**	5.3	10.1	?	?	?

Figure 12: Selected ionization potentials

3 Collisional Ionization

This is the process by which an electron strikes an ion and liberates an electron

$$A^{i+} + e^- \rightarrow A^{(i+1)+} + 2e^- - \Delta E_{IP} \tag{22}$$

where ΔE_{IP} is the ionization potential of ion A^{i+} and is the amount of energy removed from the incoming electron. This process therefore cools the gas. The equations for collisional ionization are analogous to those for collisional excitation, except that instead of excitation to a single level it occurs to the continuum of levels above the IP. The number of collisional ionizations per second produced by the electron gas is

$$N_e N_i \int_I^{\infty} \sigma_{coll} v f(v) dv = N_e N_i r_{coll} \tag{23}$$

where r_{coll} is the collisional ionization rate (analogous to q_{12} for collisional excitation) and I is the ionization potential. Similarly, σ_{coll} is the cross-section for interaction. The cross-section for any *individual* electron shell can be expressed in the form

$$u I_n^2 \sigma_n = A_n(1 - 1/u) + B_n(1 - 1/u)^2 + C_n ln(u) + \frac{D_n}{u} ln(u) \tag{24}$$

I_n is the ionization potential and u is dimensionless ratio of the energy of an incident electron and the IP, i.e. $E = uI_n$. Coefficients are tabulated in, e.g., Arnaud & Rothenflug (1985, A&A, 60, 425). The total collision cross section is weighted by the energies associated with the cross-section for each level within the ion

$$\sigma_{coll} = \Sigma_n \frac{1}{u I_n^2} \sigma_n \tag{25}$$

3.1 Auto-Ionization

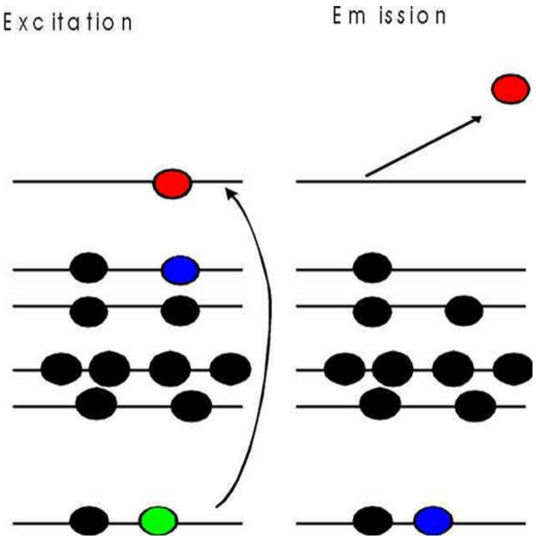

Figure 13: Autoionization

At sufficiently high energies more than one electron may be excited, leaving the ion in an unstable state. This is stabilised by the ejection of an electron, possibly followed by a radiative decay of the ion back to its ground state.
$A^{i+} + e^- \rightarrow A_*^{i+} + e^- - E_1$
$A_*^{i+} \rightarrow A_*^{(i+1)+} + e^- + E_2$
$A_*^{(i+1)+} \rightarrow A^{(i+1)+} + h\nu$