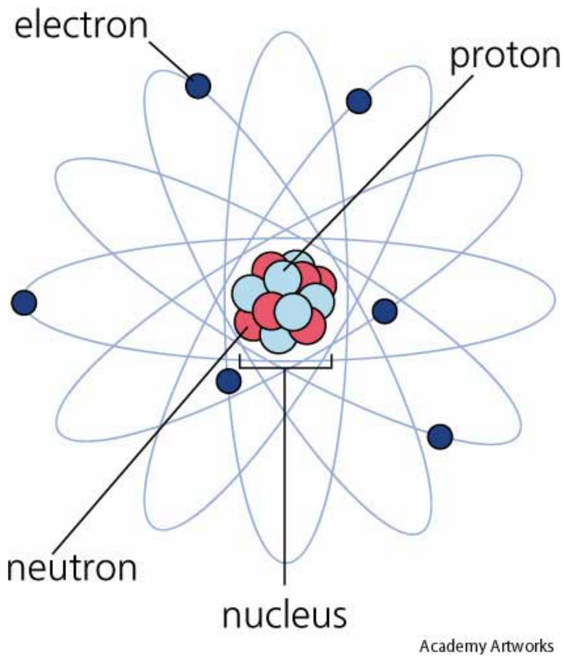
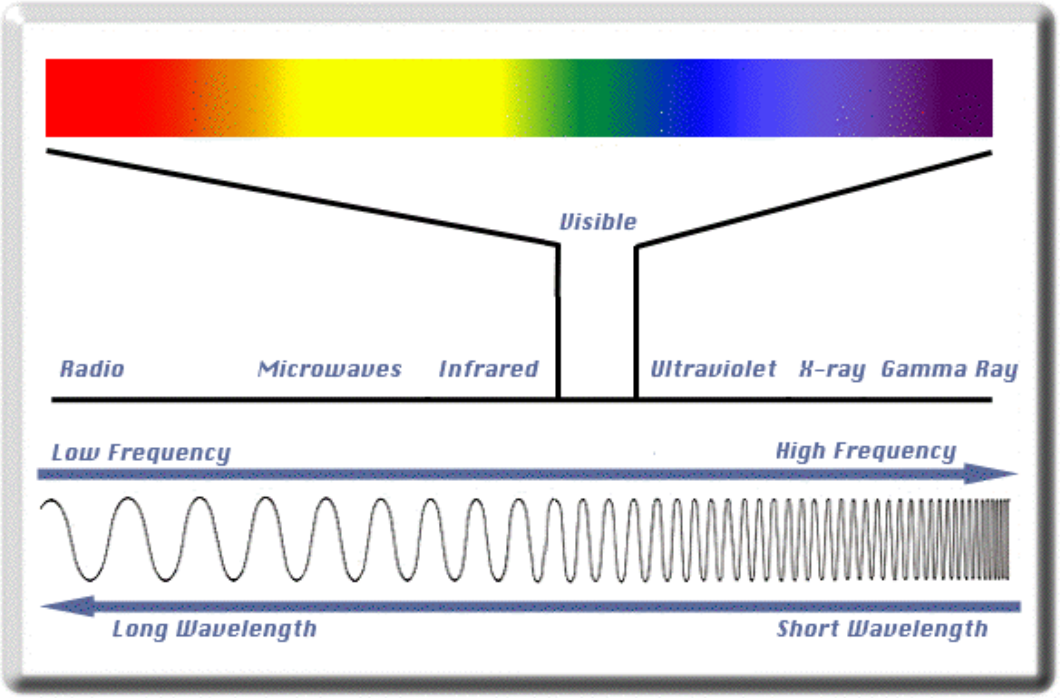
Measuring the properties of stars and galaxies

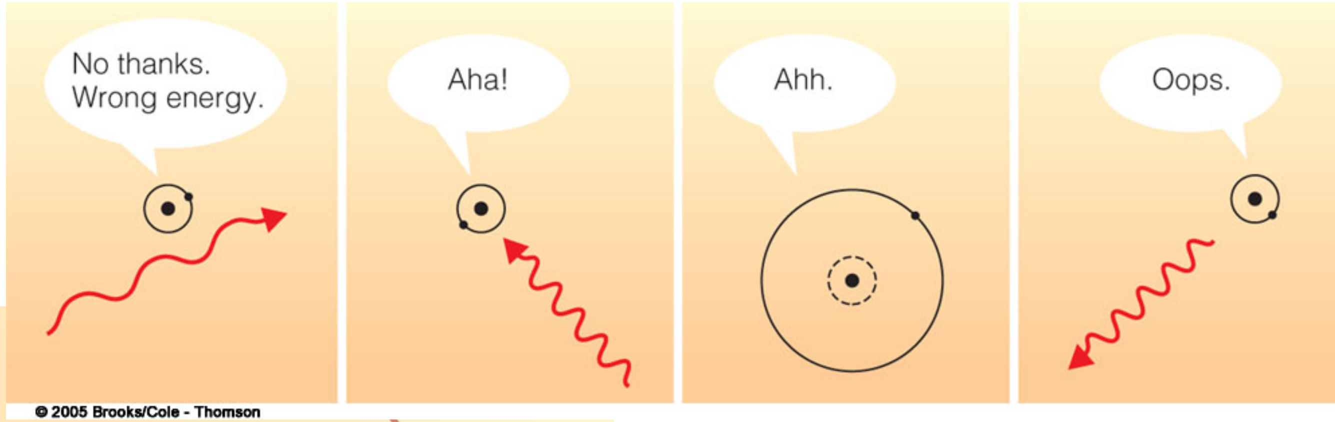
See C&M Chap. 3 (radiation basics), 4 (spectral properties) and 17

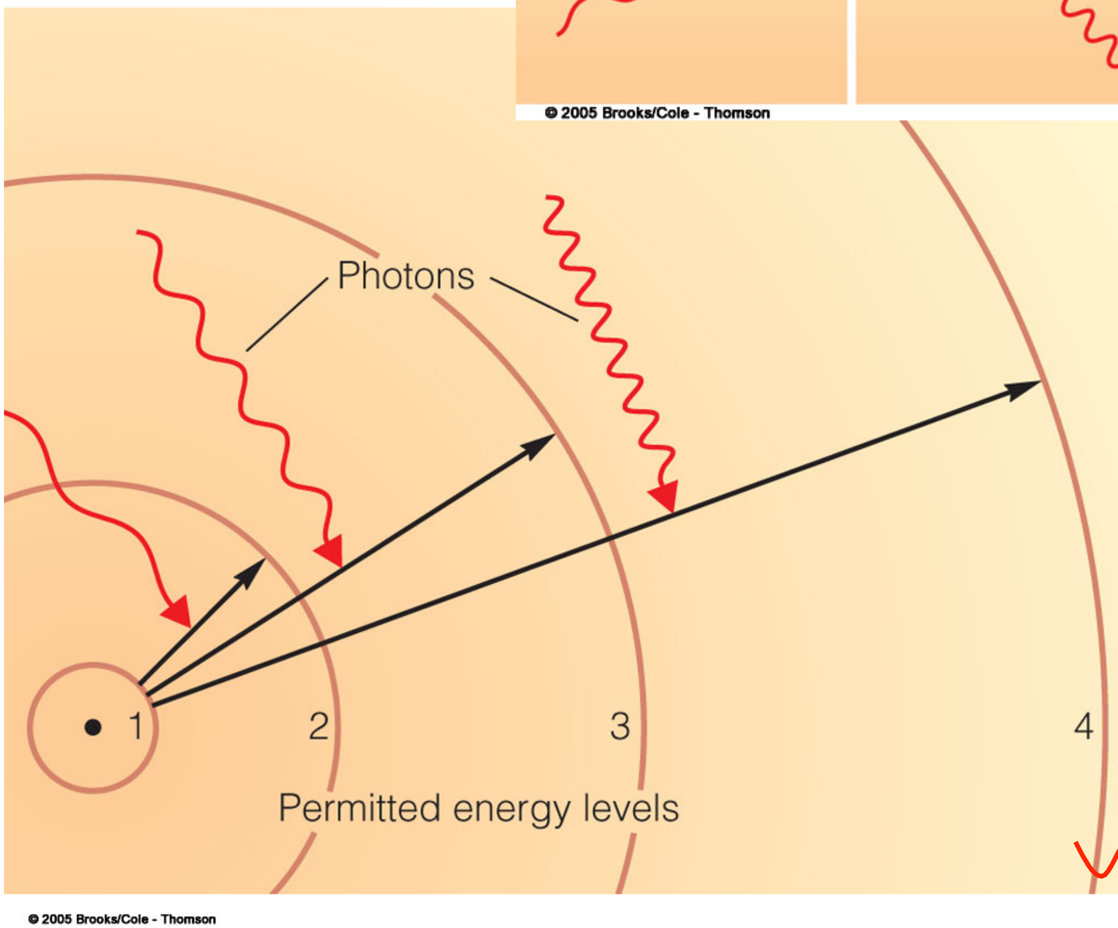
Many properties of stars and galaxies are measured via our understanding of the interaction between light and matter.



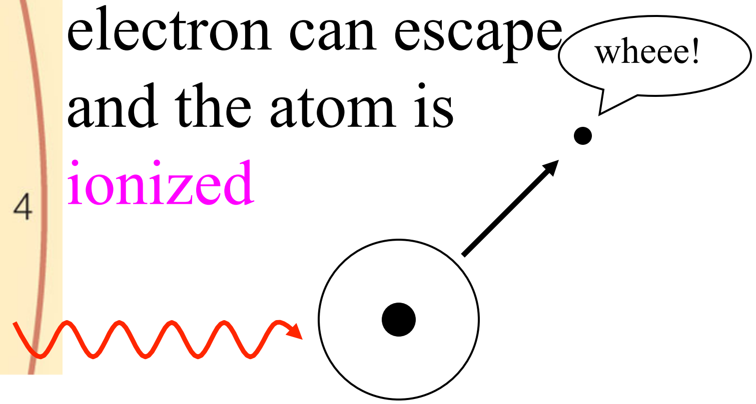


Please review chapter 3 if you need a refresher on what the electromagnetic spectrum is and what we mean by wavelength, frequency etc.

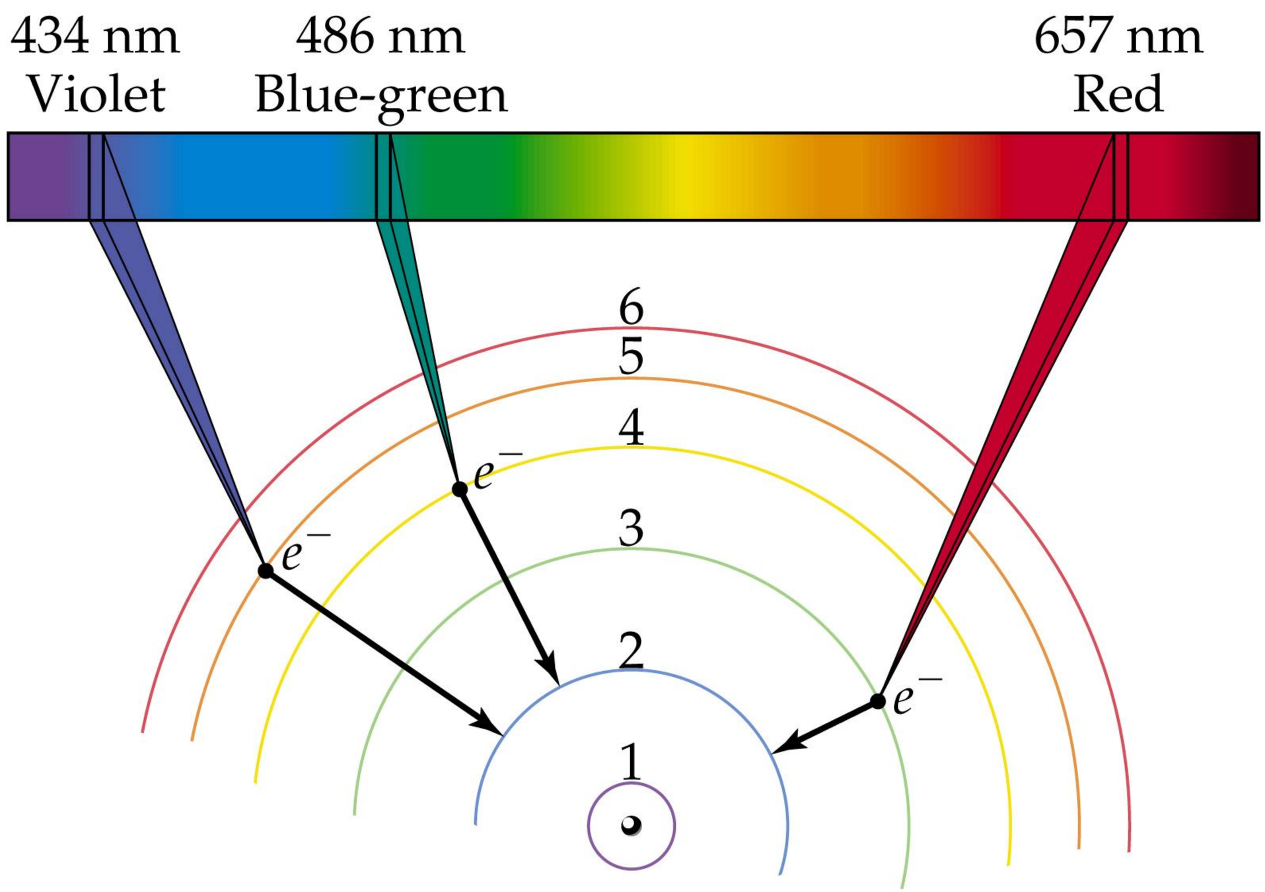


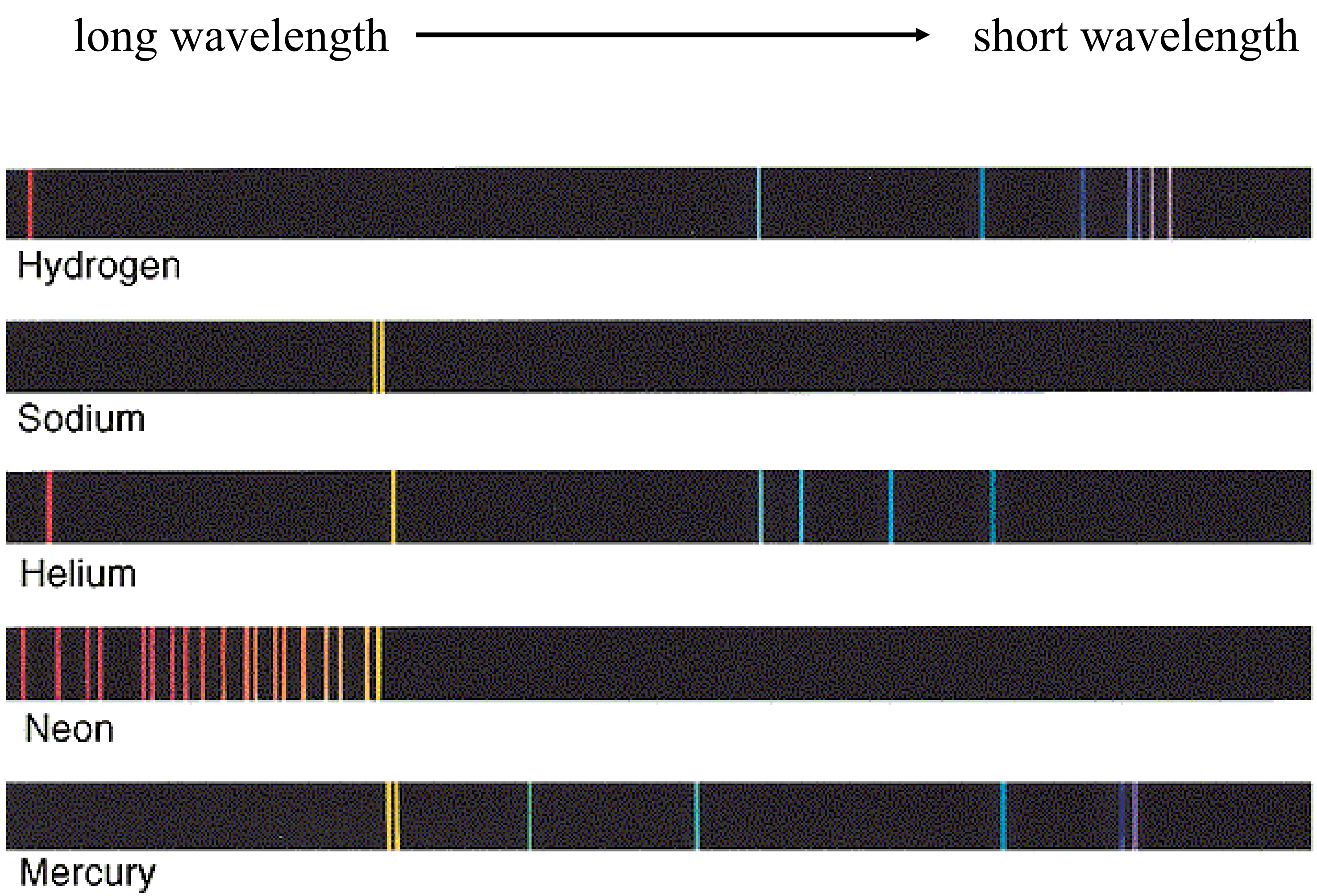


Given a photon of enough energy, the electron can escape and the atom is ionized



A more energetic (blue) photon is needed in order to excite an electron to higher orbits from the ground state (lowest energy level). Less energetic (red) photons are required for smaller jumps.





Each element has a unique configuration of electron orbits, so its emission spectrum is unique.

Estimating stellar temperatures

A continuous spectrum emitted by an ideal non-reflecting surface is called blackbody radiation and has a very distinctive shape with total flux (energy per unit area per unit time, or power per area) given by the Stefan-Boltzmann law:

$F = \sigma T^4$ (σ is the Stefan Boltzmann constant = 5.67e-8 W/m^2K^4)

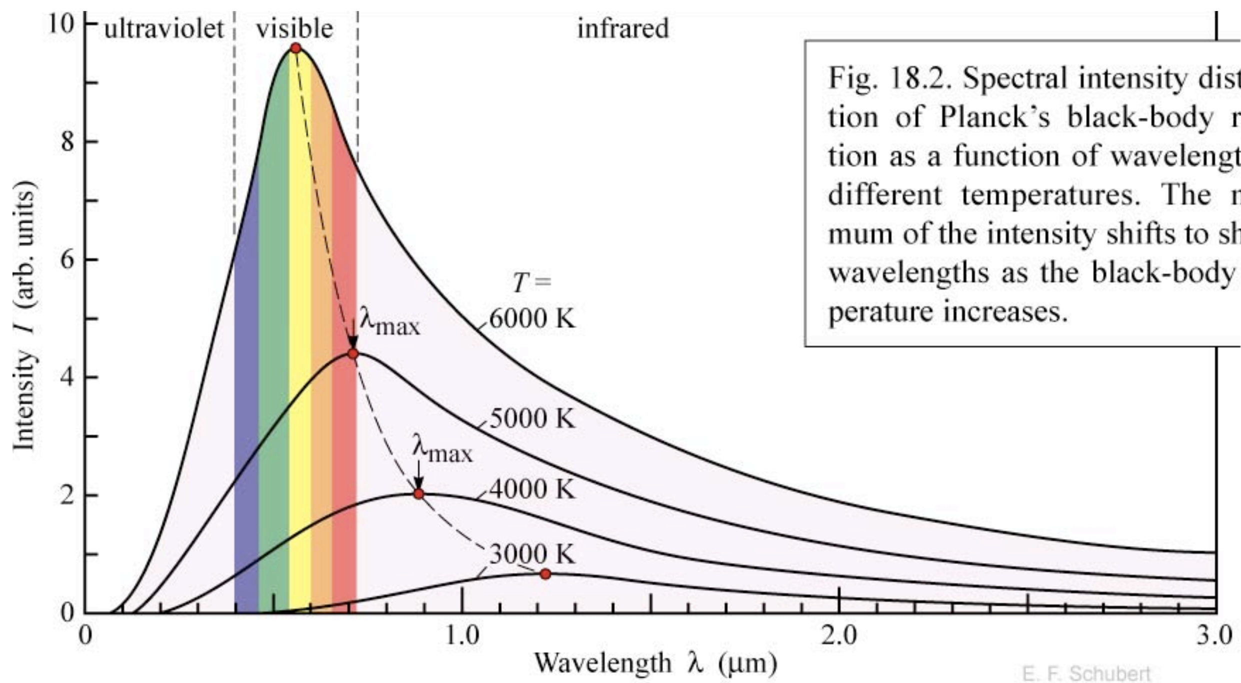
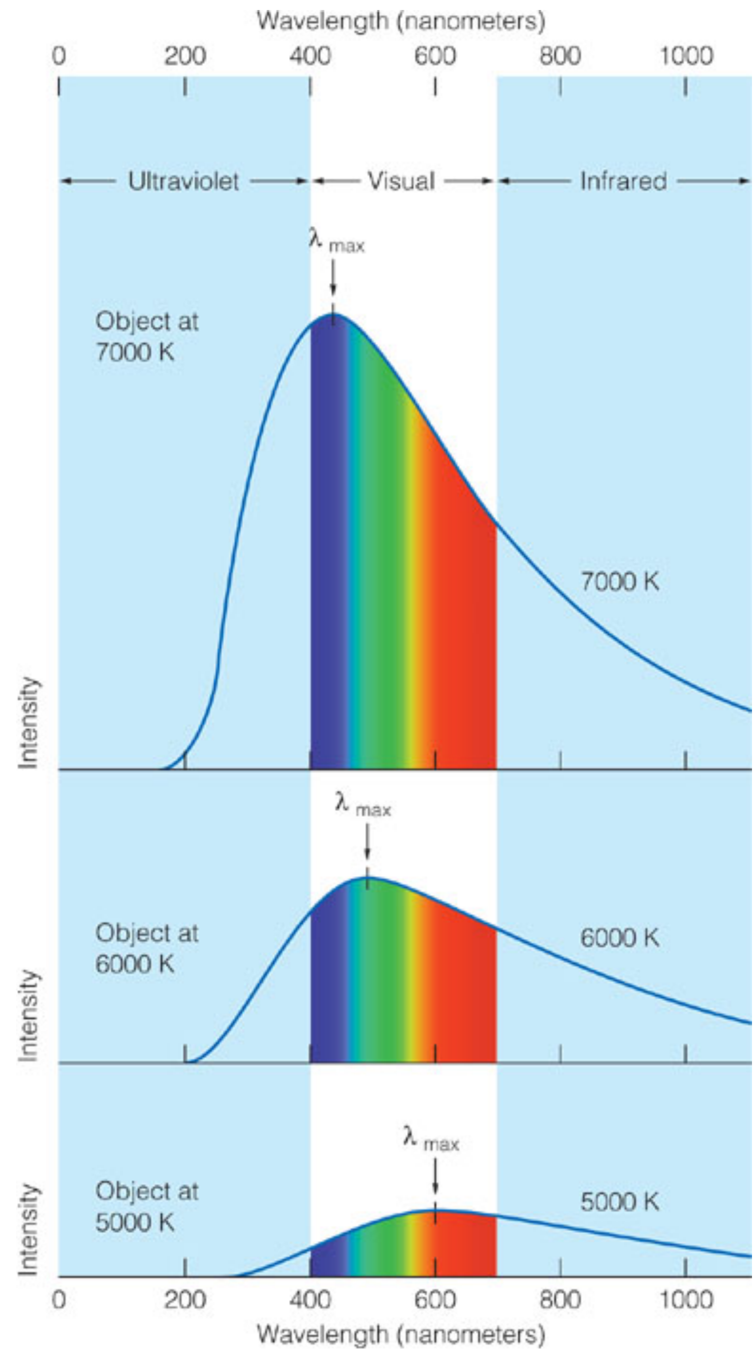


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

The value of λ_{max} and temperature are connected by Wien's displacement law:

$$\lambda_{max}\,(\text{nm}) = \frac{2.9 \times 10^6}{\text{Temperature (K)}}$$

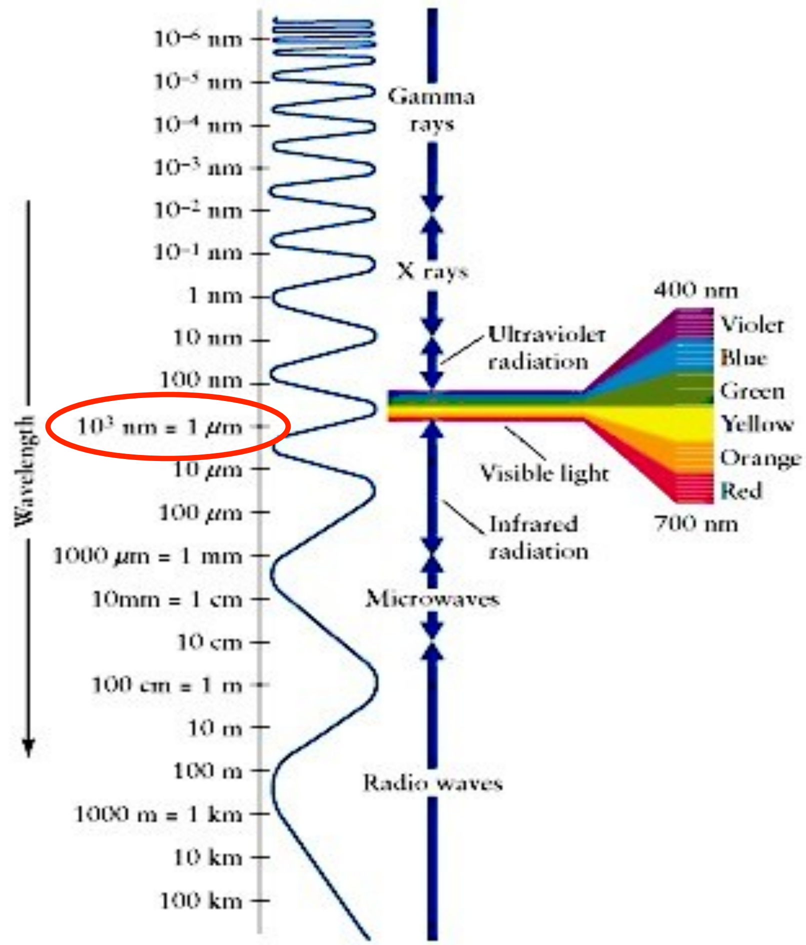


Example: Your body temperature is about 37 degrees centigrade, what is this in kelvin? At what wavelength do you emit most of your radiation? What part of the electromagnetic spectrum is this?

$$T_{kelvin} = T_{centigrade} + 273 = 37 + 273 = 310 \text{ K}$$

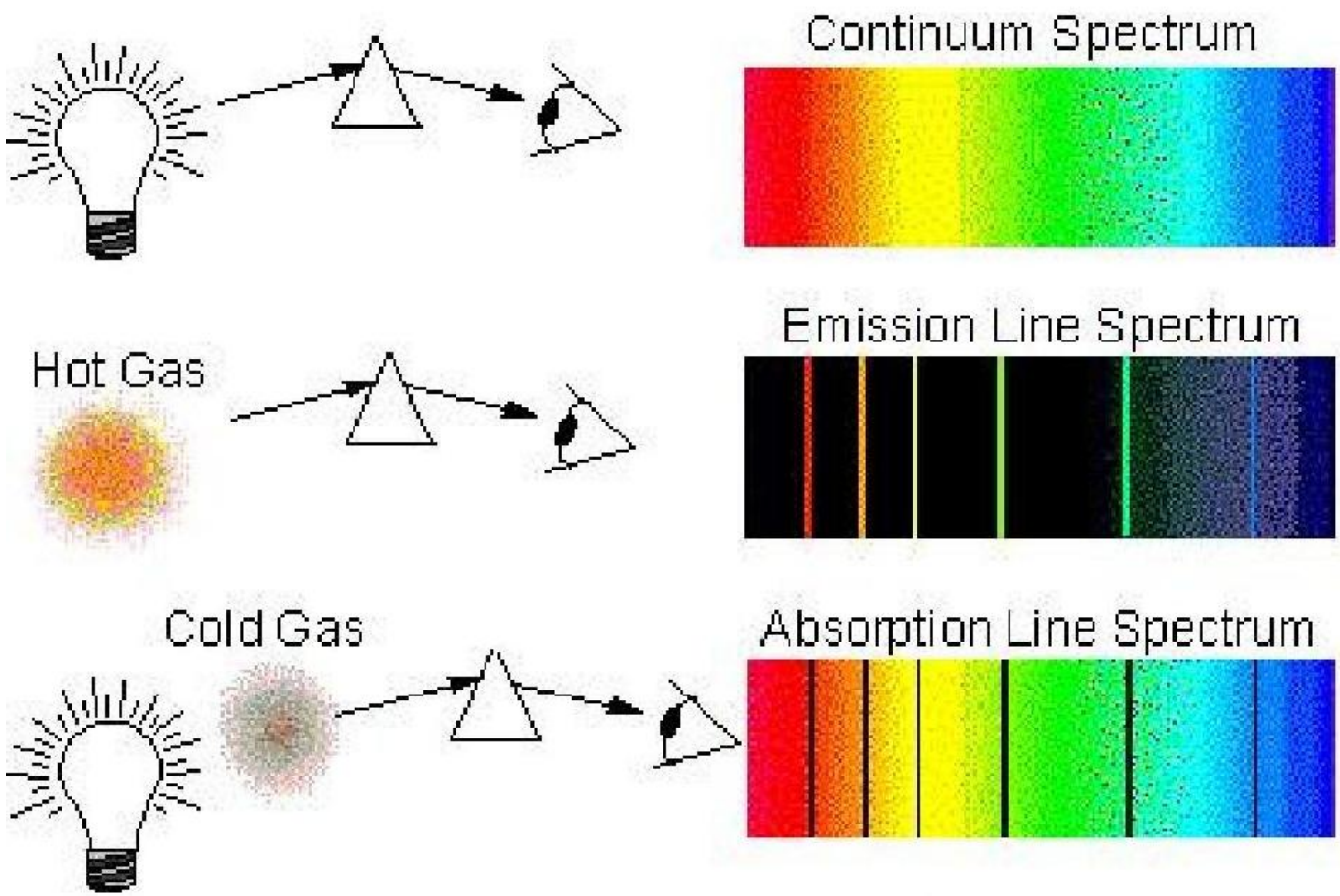
$$\lambda_{max}\,(\text{nm}) = \frac{2.9 \times 10^6}{\text{Temperature (K)}}$$

$$= \frac{2.9 \times 10^6}{310} = 9355 \text{ nm}$$

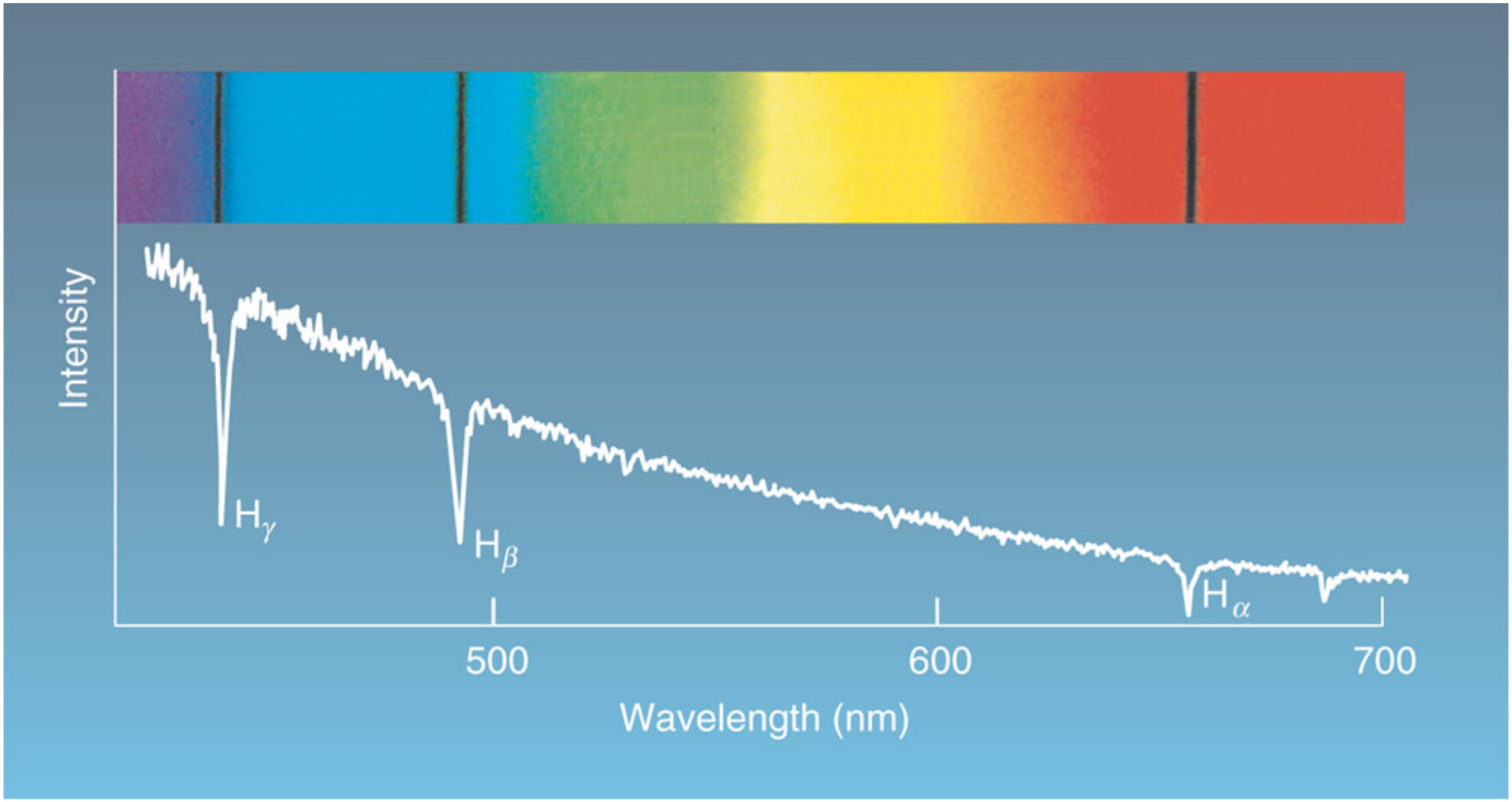


1000 nm is the same a 1μm (1 micron), so 9355nm = 9.4 μm , which is in the infra-red.

Continuous, emission and absorption spectra:

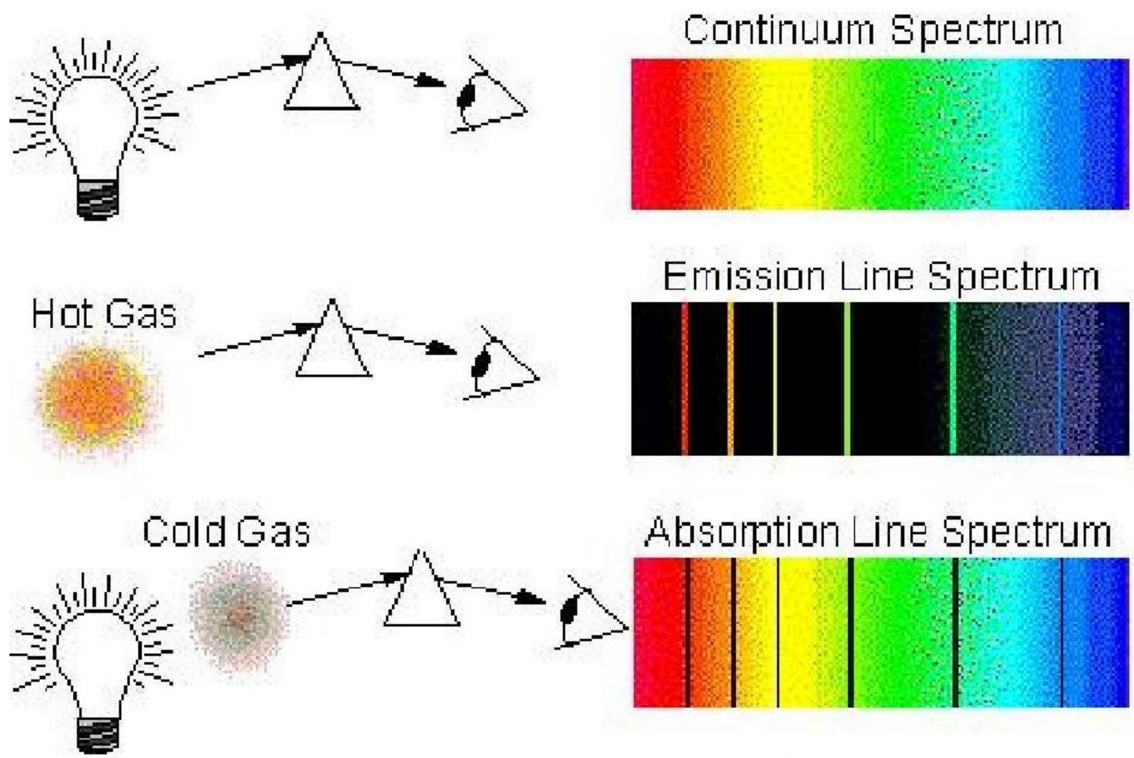


Typical stellar spectrum:

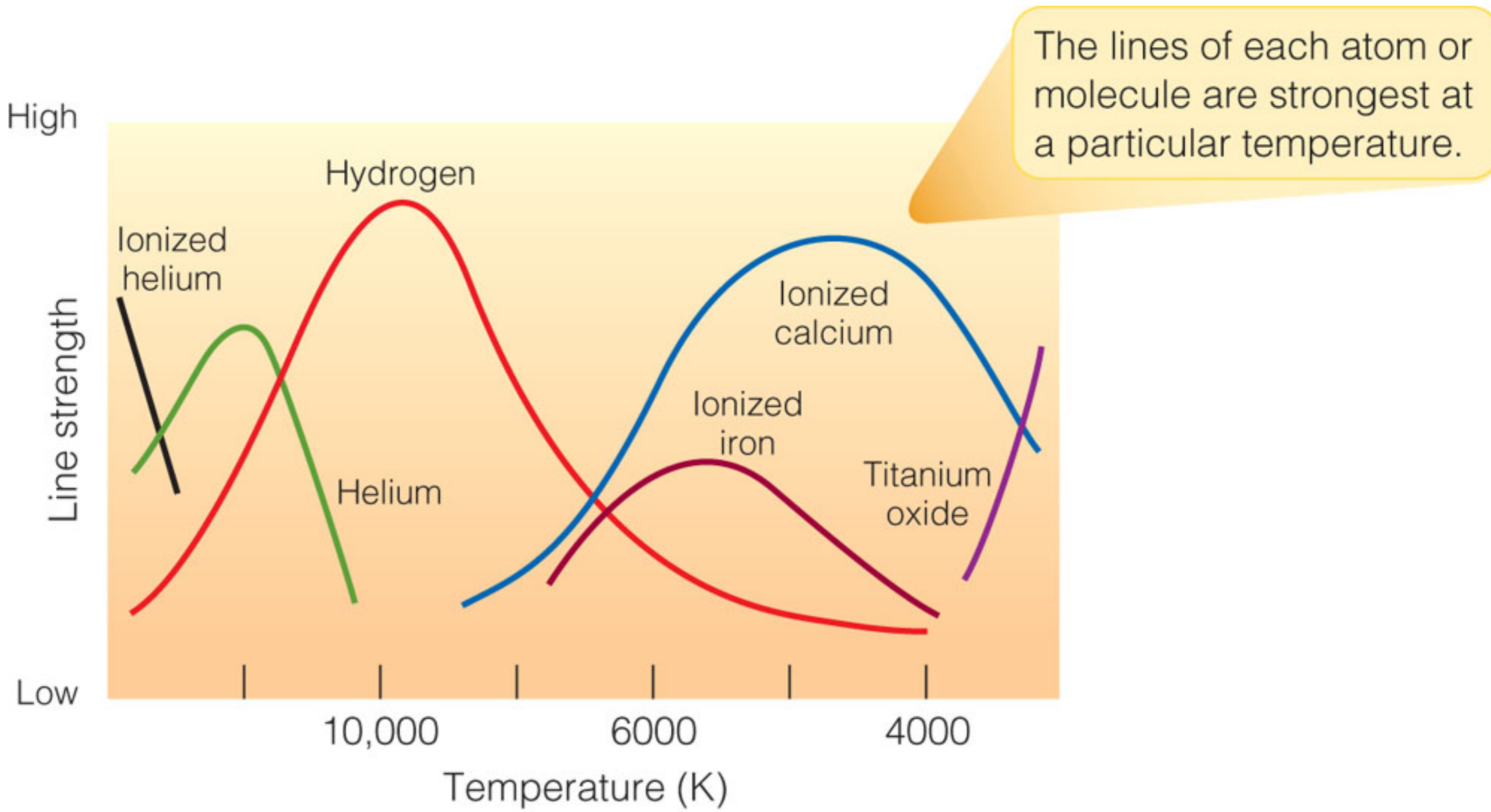


These 3 types of radiation are summarised by Kirchoff's laws

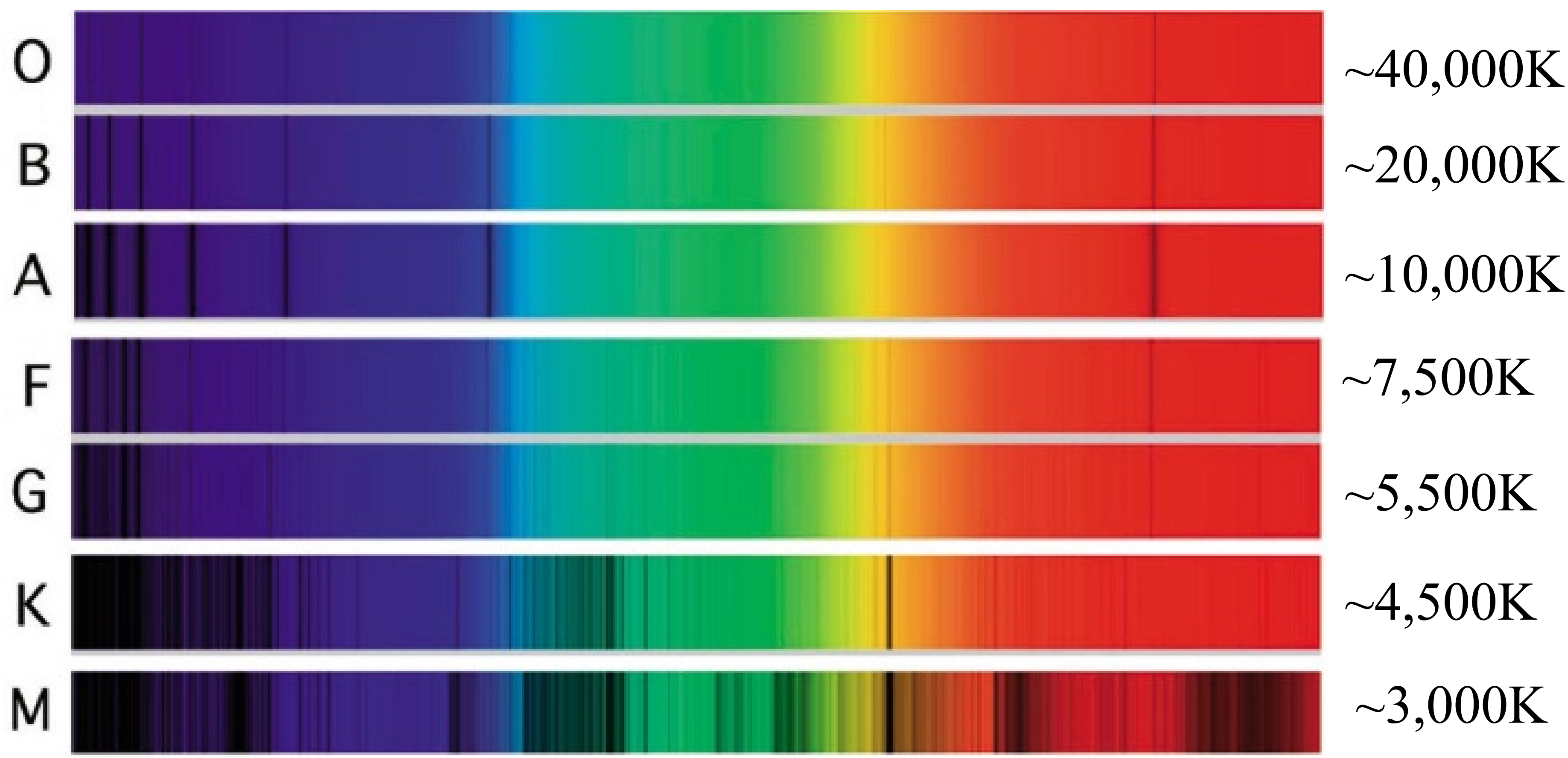
1) A solid or dense gas will produce a continuous spectrum
2) A hot low density gas will produce an emission spectrum with lines at fixed wavelengths
3) A cold gas in front of a continuum source will produce an absorption spectrum



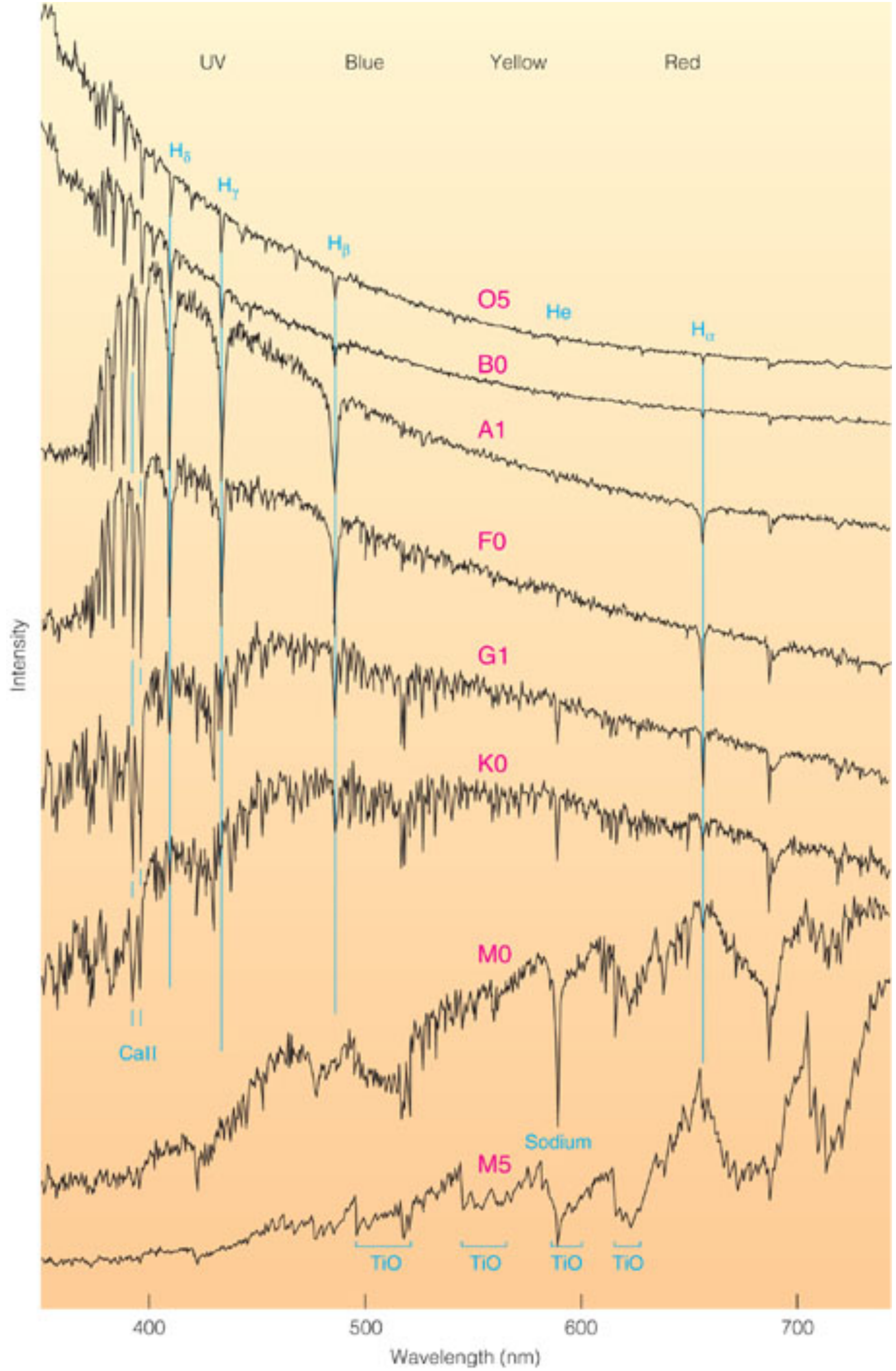
Spectroscopy provides an accurate thermometer of surface temperature, based on what spectral lines are seen.



For convenience, we refer to stellar types (classified by temperature) by a letter



Oh Be A Fine Girl/guy Kiss Me !!



UV
Blue
Yellow
Red
Hδ
Hγ
Hβ
O5
He
Hα
B0
A1
F0
G1
K0
M0
CaII
Sodium
M5
TiO
TiO
TiO
TiO
Intensity
400
500
600
700
Wavelength (nm)

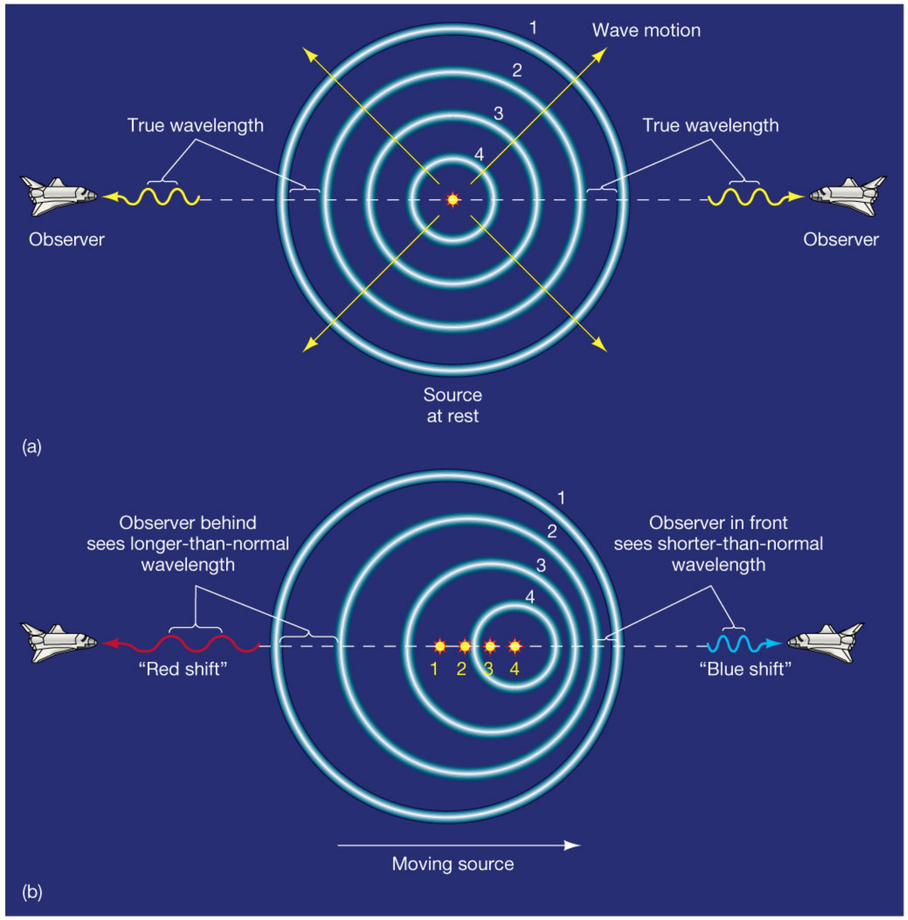
Absorption lines also tell us about abundances:

TABLE 7-2
The Most Abundant Elements in the Sun

Element	Percentage by Number of Atoms	Percentage by Mass
Hydrogen	91.0	70.9
Helium	8.9	27.4
Carbon	0.03	0.3
Nitrogen	0.008	0.1
Oxygen	0.07	0.8
Neon	0.01	0.2
Magnesium	0.003	0.06
Silicon	0.003	0.07
Sulfur	0.002	0.04
Iron	0.003	0.1

Cecelia Payne Gaposchkin

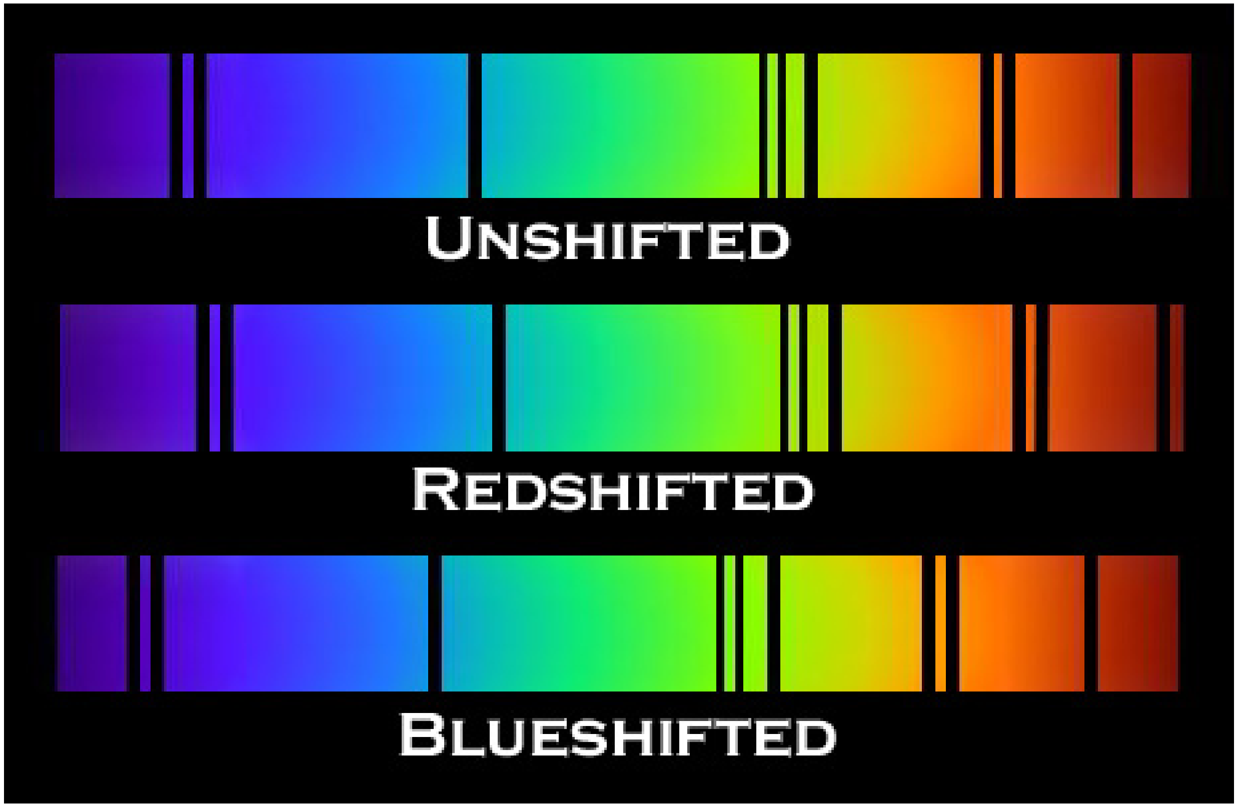
Stellar motions in space



In addition to its temperature and chemical composition, a spectrum can also tell us the radial velocity of an object.

If a light source is approaching us, we see its spectral lines shifted towards the blue end of the spectrum: blueshift. A blueshift results in spectral lines having a shorter observed wavelength

If a light is receding from us, we see its spectral lines shifted towards the red end of the spectrum: redshift. A redshift results in spectral lines having a longer observed wavelength



We calculate the radial velocity with this simple formula:

$$\frac{\text{velocity}}{\text{speed of light}} = \frac{\text{change in wavelength}}{\text{original (rest) wavelength}}$$

A shorter way of writing this is: $\frac{v}{c} = \frac{\Delta\lambda}{\lambda_{rest}}$

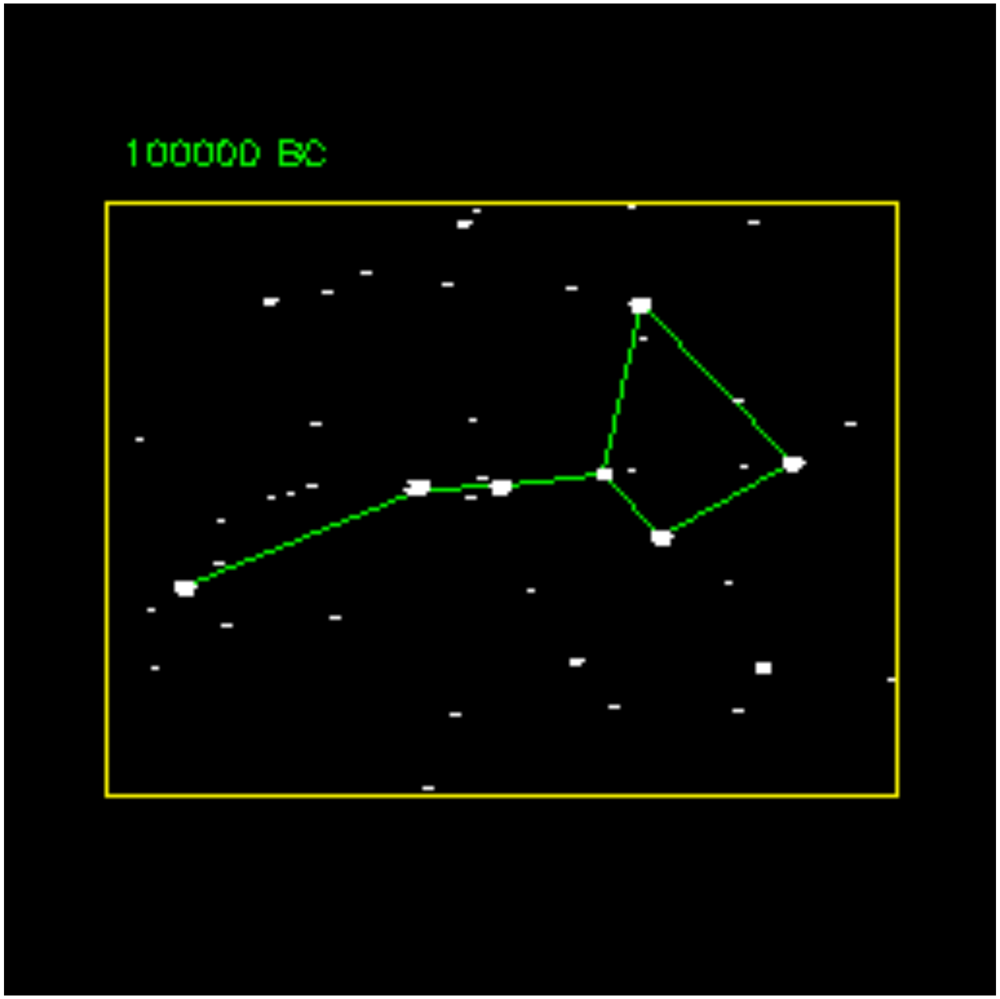
Example: You take a spectrum of a star and notice that the Lyman alpha line is shifted from its rest wavelength of 121.6 nm to 121.8nm. What is the velocity of the star?

In this example, λ=121.6 nm and we know that c=300,000 km/s.

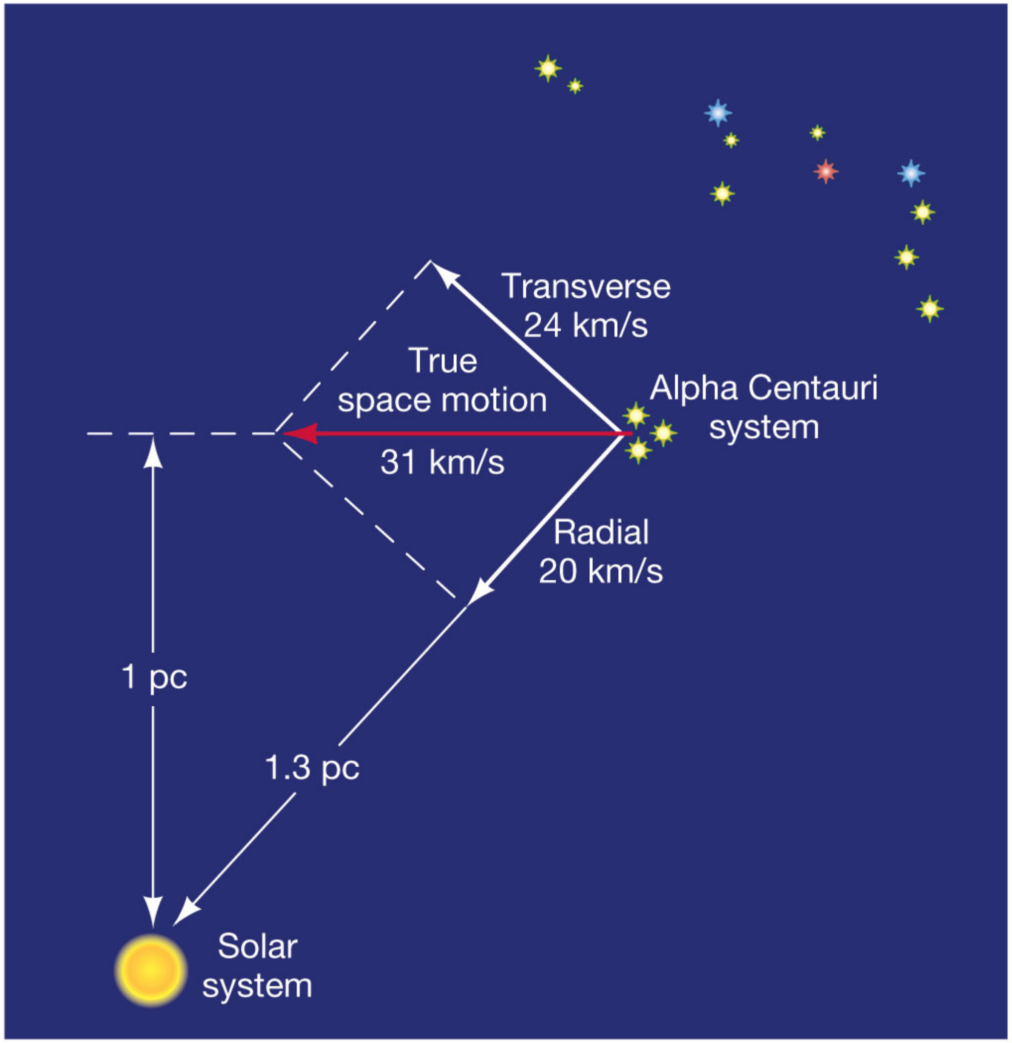
v = (121.8-121.6)/121.6 x 300,000

= 493 km/s because the observed wavelength is larger than the rest wavelength, this is a REDSHIFT.

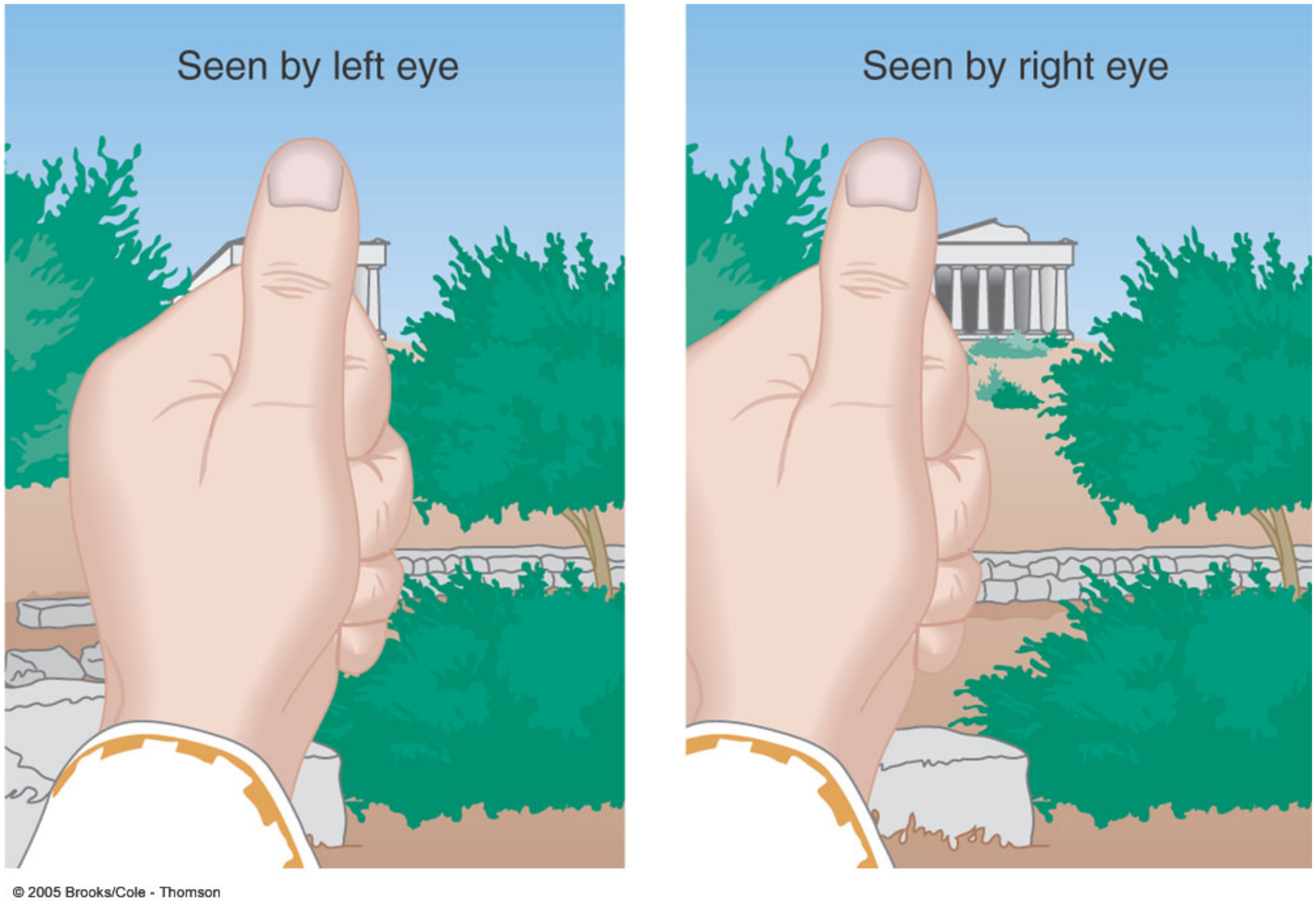
Radial motions can only be detected spectroscopically, but 3 dimensional motions show up as proper motions (measured in arcsec/year).



The true space motion is a combination of transverse (I.e. proper) and radial velocities.

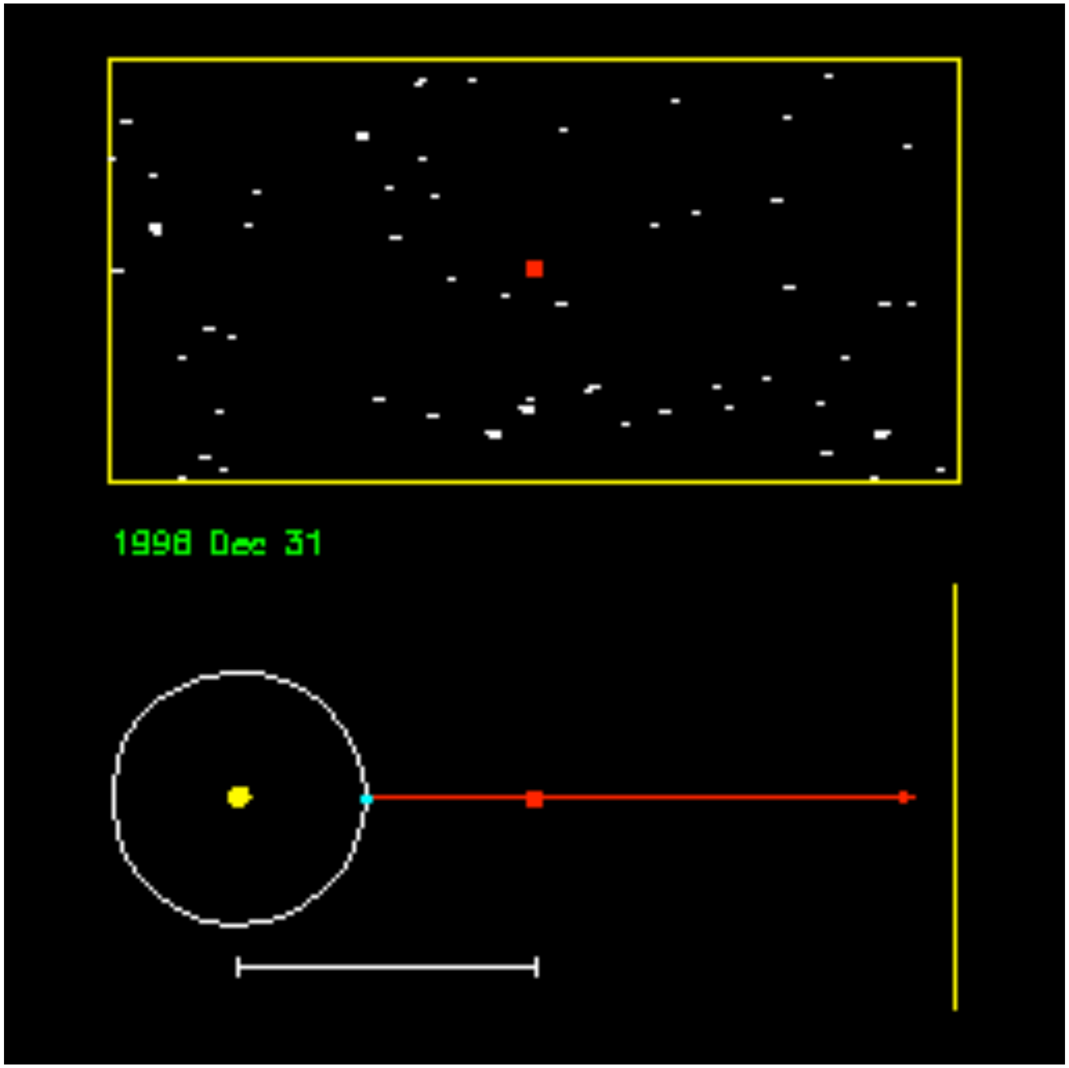


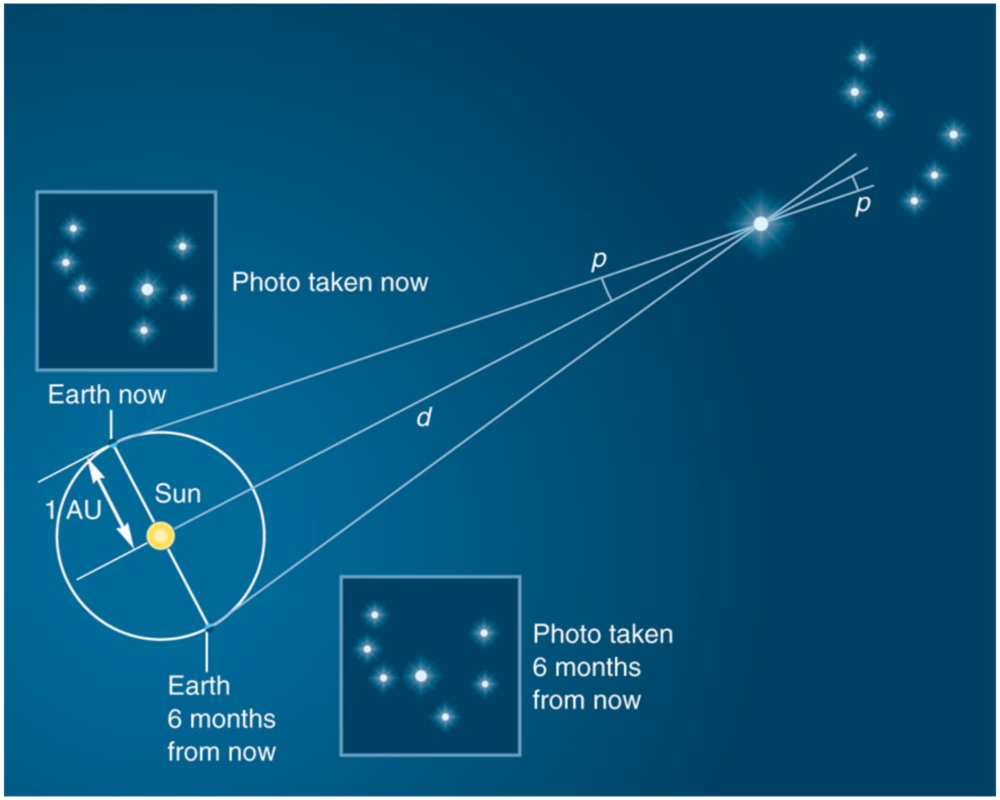
Measuring distances to the (nearest) stars



Distances to nearby stars can be measured by using parallax, whereby an object's position appears slightly different when viewed from different vantage points (see also previous lecture notes).

Parallax is an "apparent" motion, rather than a "true" motion.





The convention for stellar parallax is that the angle of parallax, p, is half the angle that a star appears to move in a 6 month period.

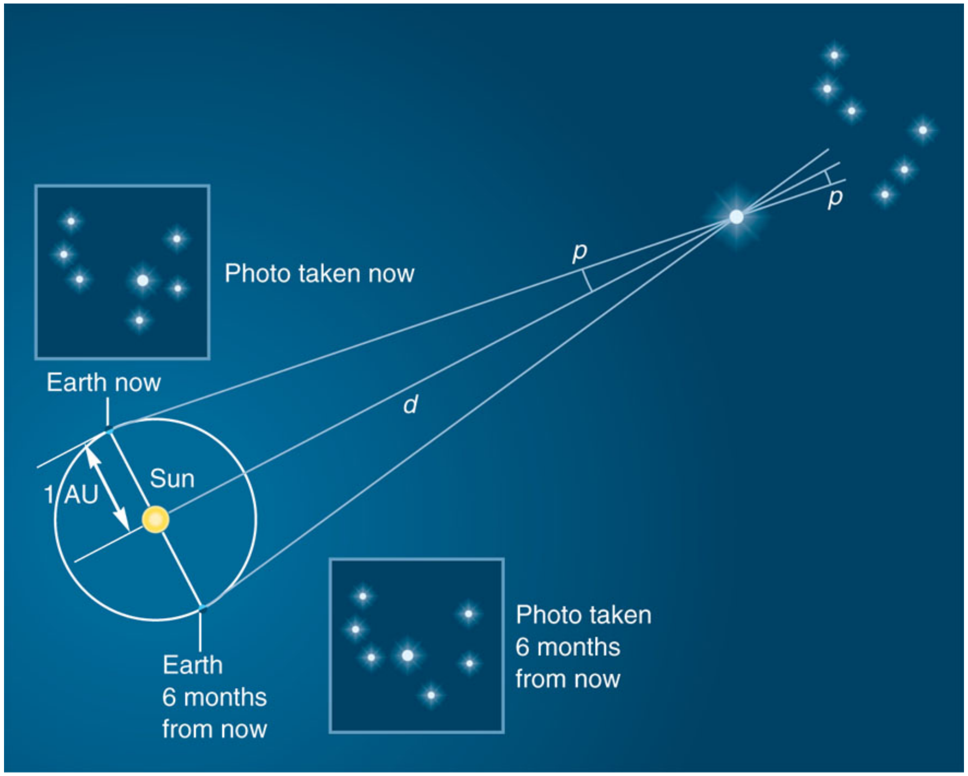
tan p = a/d

Where a is the semi-major axis and d is the distance to the star.

For small p (in radians)

p = a/d

For "simplicity" define new unit: the parsec



1 parsec is the distance to a star whose angle of parallax is 1 second of arc. A larger parallax angle therefore means the star is closer.

Parsecs make it easy to determine the distance to a star if you know the angle of parallax, p:

d (pc) = a (AU) / p (arcsecs) = 1/p (as seen from Earth)

Example: If the parallax of a given star is 0.5” as viewed from mercury, what will its parallax be as seen from Jupiter?

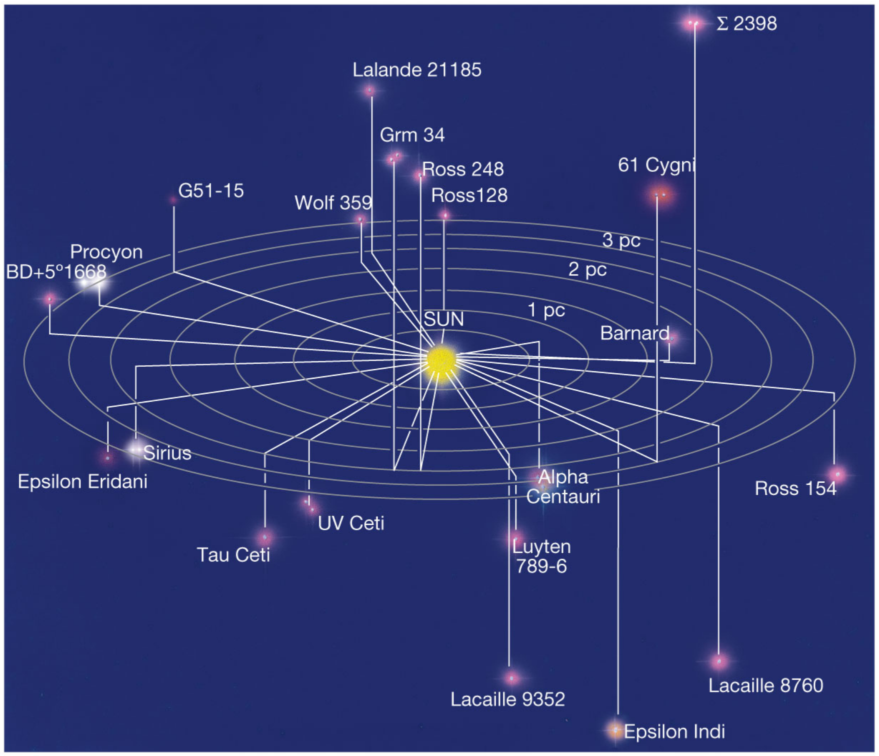
d (pc) = a (AU) / p (arcsecs)

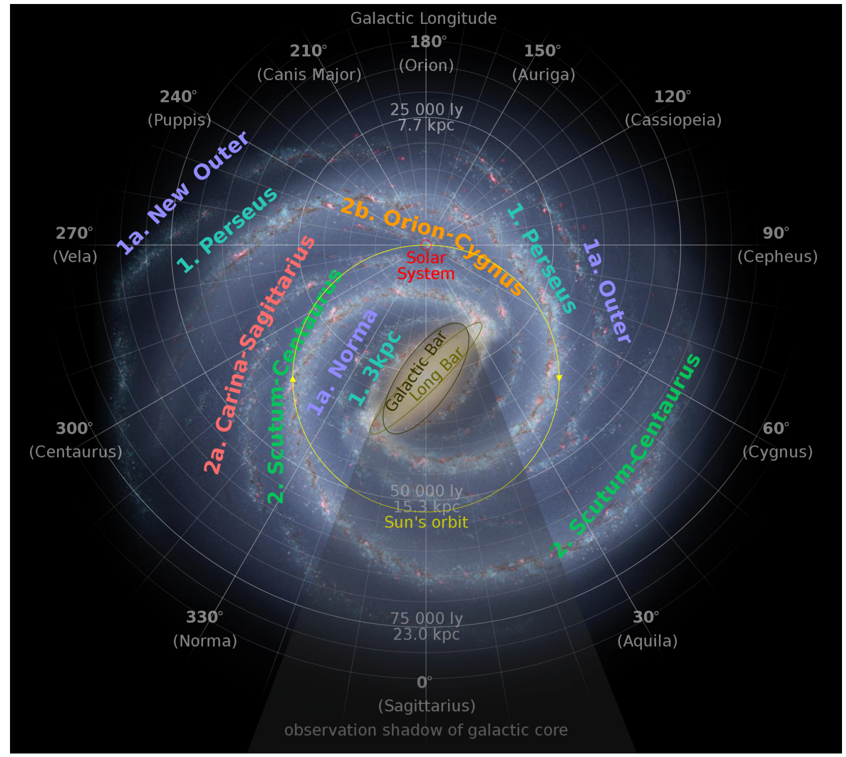
Answer: Mercury’s semi-major axis is 0.39 AU, so distance to the star = 0.39 AU / 0.5” = 0.78 pc.

The semi-major axis of Jupiter is 5.2 AU, so
p = 5.2 AU/ 0.78 pc = 6.67 “

Note that parallax increases as your baseline is bigger.

1 parsec (pc) is equal to 206, 265 AU or 3.26 lightyears, or 3.09 x 10^{13} km. We also use kilo parsecs (kpc) for larger distances, where 1 kpc = 1000 pc.





Professional astronomers tend to use:

- AU within the solar system
- pc and kpc for distances within galaxies
- kpc and Mpc between galaxies

Why would we want to use satellites for the measurement of parallax?

ESA's Gaia satellite, launched in Dec. 2013 will map the proper motions of 1 billion stars in the Milky Way.

Example: A star at 12 pc has a proper motion of 1"/year and a radial velocity of 50 km/s. What is its real space (3D) velocity?

Use the small angle formula to determine physical size of 1" at 12 pc distance:

$$\boxed{\frac{\text{angular diameter (arcsecs)}}{206{,}265} = \frac{\text{linear diameter}}{\text{distance}}}$$

$\frac{1 \text{ x } 12}{206265}$ = 5.8×10^{-5} pc (transverse distance travelled by star in 1 year)

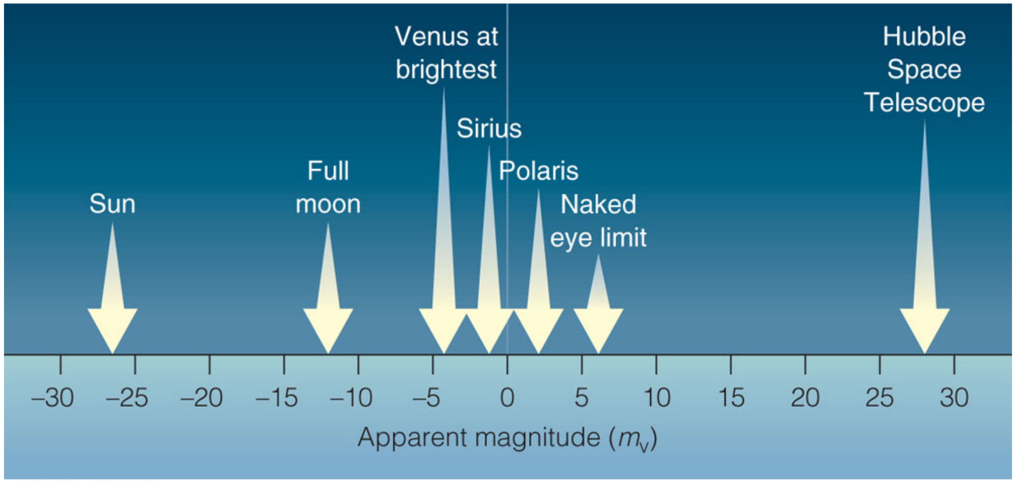
$= 1.8 \times 10^{9}$ km

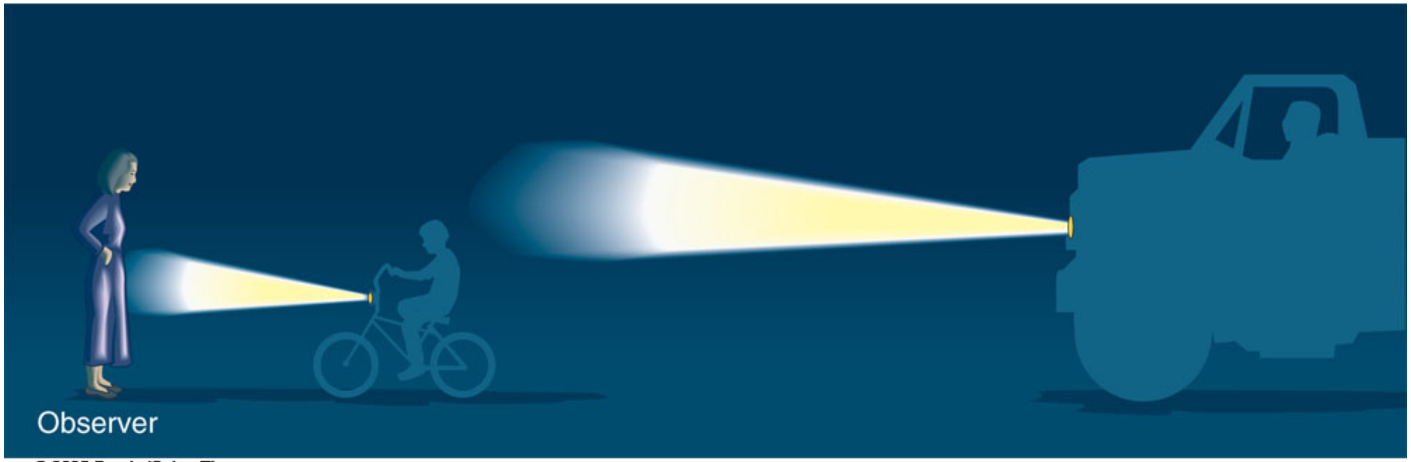
1 year = 3.154×10^{7} secs, so $v_t = 1.8 \times 10^{9} / 3.154 \times 10^{7} = 57$ km/s

$V^2 = v_t^2 + v_r^2 = 57^2 + 50^2$

V = 76 km/s

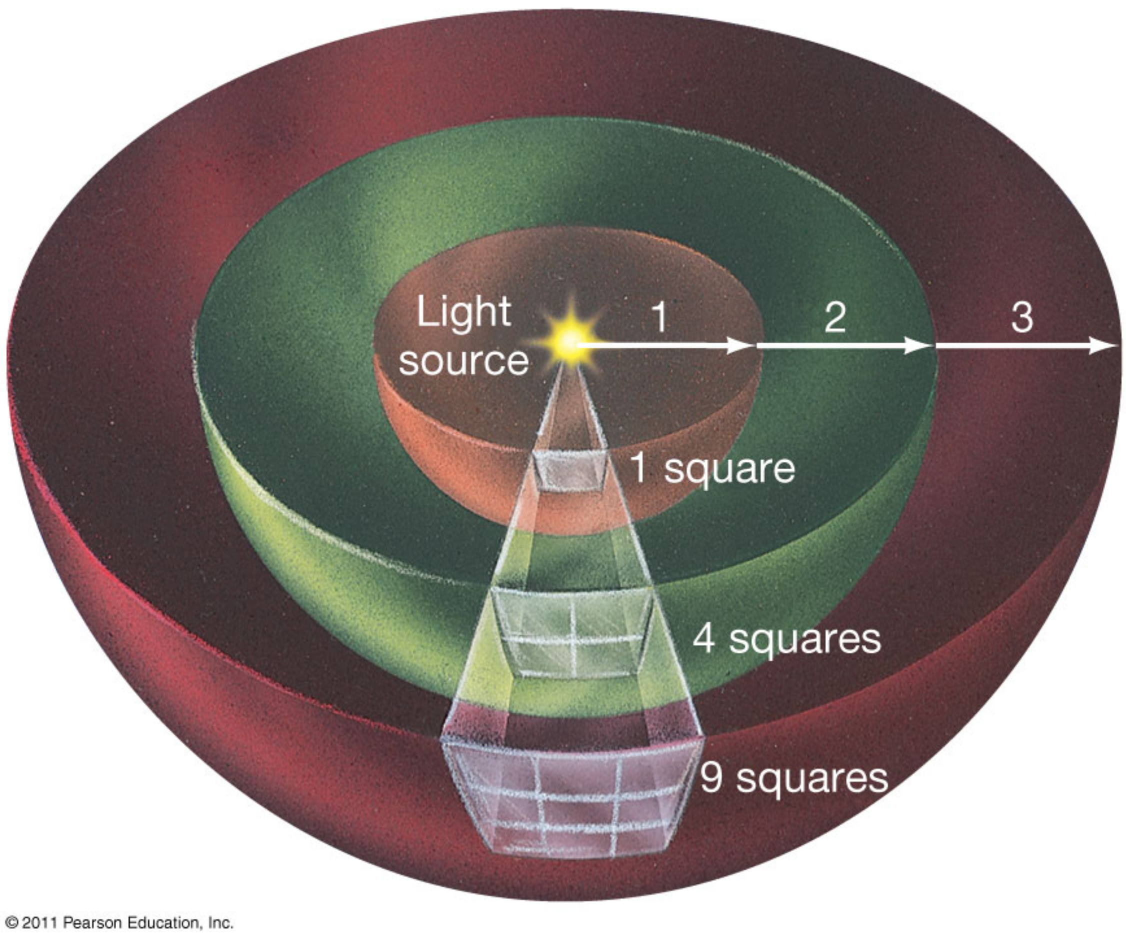
Magnitudes (again) and luminosity





Knowing the distance to an object is critical in determining it's absolute magnitude, or more fundamentally, its luminosity.

Like gravity, light also follows an inverse square law:



$$\text{Apparent brightness} \propto \frac{\text{luminosity}}{\text{distance}^2}$$

Absolute magnitude (M): the apparent magnitude a star would have it were 10 pc away

In this way, all stars can be put on an equal footing. The actual equation for absolute magnitude is

$$m - M = -5 + 5 \log d(pc).$$

m-M is also known as the distance modulus.

TABLE 9-1
Distance Moduli

$m_V - M_V$	d(pc)
0	10
1	16
2	25
3	40
4	63
5	100
6	160
7	250
8	400
9	630
10	1000
⋮	⋮
15	10,000
⋮	⋮
20	100,000
⋮	⋮

$m - M = -5 + 5 \log d(pc)$.

Example 1: What is the distance modulus for a star that is 100 pc away?

Answer: Distance modulus is just m-M, which we read off the table, or determine from the eqn: 5 magnitudes.

TABLE 9-1
Distance Moduli

$m_v - M_v$	d(pc)
0	10
1	16
2	25
3	40
4	63
5	100
6	160
7	250
8	400
9	630
10	1000
⋮	⋮
15	10,000
⋮	⋮
20	100,000
⋮	⋮

Example 2: If a star has an apparent magnitude of 4.3 and is 40 pc away, what is its absolute magnitude (M)?

Answer: from the table we see that for a star 40 pc away, m-M = 3. If the apparent magnitude (m) is 4.3 then the absolute magnitude, M=m-3 = 4.3 - 3 = 1.3.

Example 3: Which star is intrinsically brighter, A or B?

	Star A	Star B
Apparent magnitude (m)	3.3	2.1
Distance (pc)	63	16

TABLE 9-1
Distance Moduli

$m_V - M_V$	d(pc)
0	10
1	16
2	25
3	40
4	63
5	100
6	160
7	250
8	400
9	630
10	1000
⋮	⋮
15	10,000
⋮	⋮
20	100,000
⋮	⋮

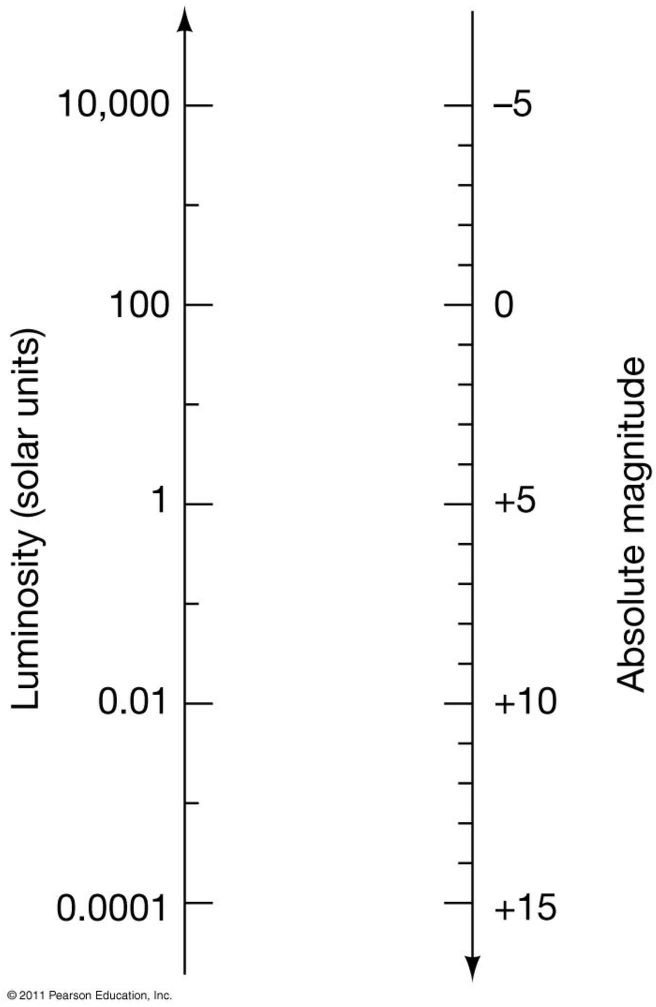
Answer: Calculate the absolute magnitude for both stars using the table of distance moduli.

Star A: For d=63 pc, m-M=4. Therefore M=m-4 = -0.7

Star B: For d=16pc, m-M=1. Therefore M=m-1=1.1

Star A has a smaller absolute magnitude (-0.7 is less than 1.1) so it is intrinsically brighter, even though its apparent magnitude is fainter.

It is often convenient to compare to the Sun's luminosity. Recall that a difference of 5 mags is a flux ratio ~ 100.

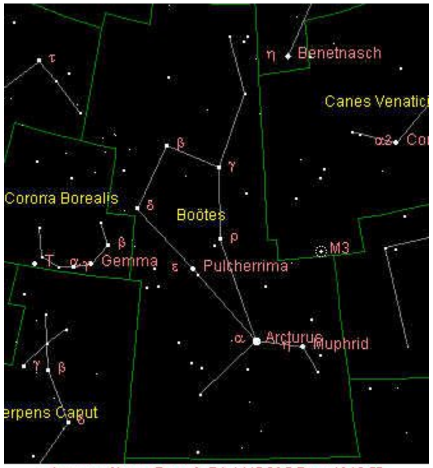


Sun's absolute magnitude in the V band is $M_V = 4.83$.

Absolute bolometric magnitude is $M = 4.74$.

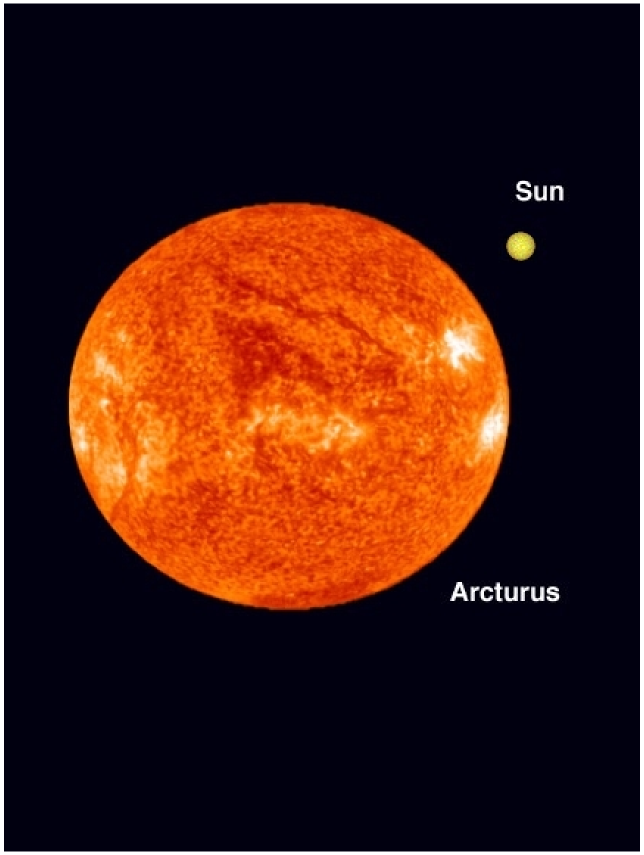
The Sun has a small bolometric correction. Why?

Example: The star Arcturus has an absolute bolometric magnitude of -0.3. How many times more luminous is Arcturus than the sun (whose absolute bolometric magnitude is 4.7)?



Arcturus (Alpha Bootis), RA 14 15 39.7 Dec +19 10 57

Answer: The sun's bolometric magnitude is 5 magnitudes fainter than Arcturus. Each magnitude is a factor of 2.5 in brightness, so 5 magnitudes gives 2.5x2.5x2.5x2.5x2.5 = 100. 100 times brighter means 100 times more luminous, so Arcturus is 100 times more luminous than the sun.



Example: An astronomer looking through HST can see a star with solar luminosity out to a distance of 100,000 pc. The brightest Cepheid stars have luminosities 30,000 times greater than the sun. Taking the sun's absolute magnitude to be 5, calculate the absolute magnitudes of these brightest Cepheids.

$F_1/F_2 = 10^{-0.4\,(M1\,-\,M2)}$
$30{,}000 = 10^{-0.4\,(M-5)}$
$M = -6.2$

Example: How far away could HST see such a star?

Answer 1: Using inverse square law.
sqrt(30,000) x 100,000 = 1.7 e 7 pc = 17 Mpc

Answer 2: By calculating apparent magnitude limit of HST.
$m= -5 + 5\log d + M$
$= -5 + 5 \log 100{,}000 + 5$
$= 25$

Distance at which HST can see M=-6.2 => $(m-M+5)/5 = \log d$
$d = 10^{(25\,-\,(-6.2)\,+5\,)/5} = 1.7e7$ pc = 17 Mpc

Estimating stellar sizes (radii)

Recall Stefan-Boltzmann law: $F = \sigma T^4$

Stefan-Boltzmann law integrated over a sphere:

$L = 4\pi R^2 . \sigma . T^4$

or re-writing in solar units:

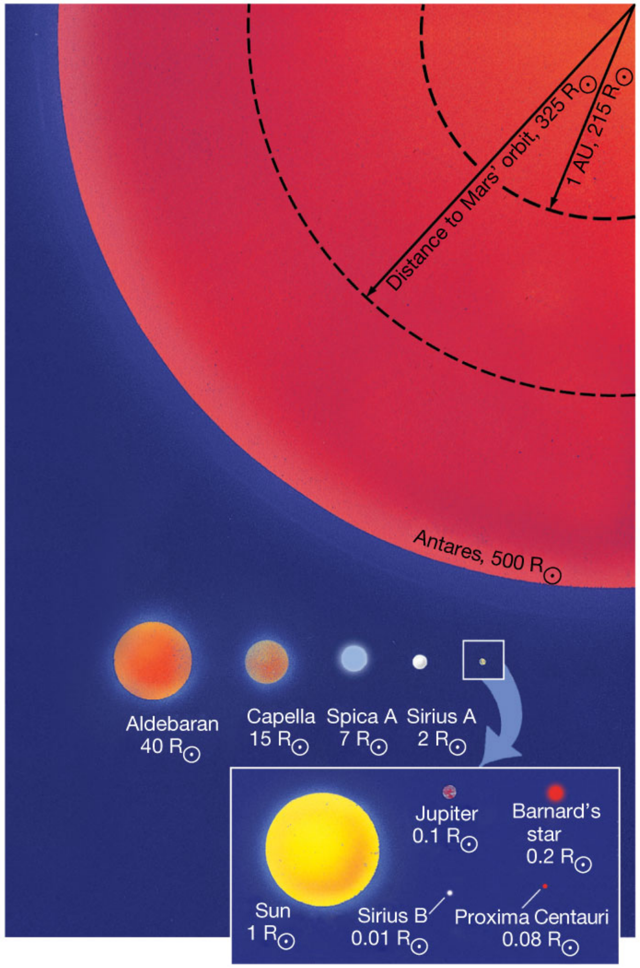
$$L/L_{sun} = (R/R_{sun})^2 \times (T/T_{sun})^4$$

Qualitatively notice that:

Higher surface area means brighter star.

Luminosity increases dramatically with T.

Stellar sizes vary greatly.



Example 1: A star has the same temperature as the sun, but twice the radius, what is its luminosity?

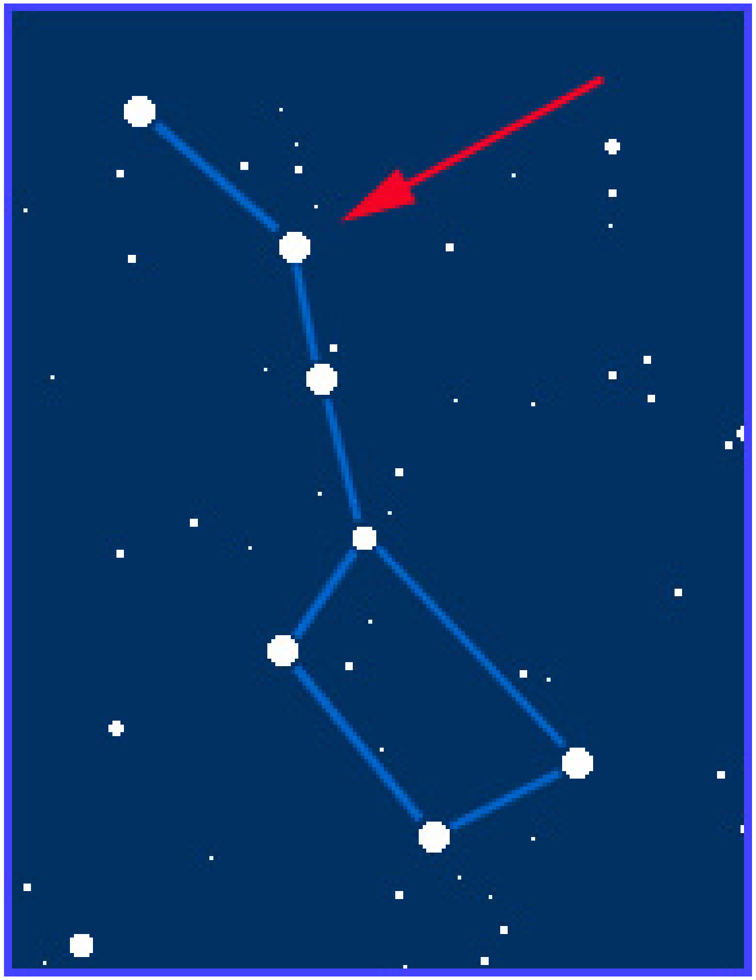
Answer: $L/L_{sun} = (R/R_{sun})^2 \times (T/T_{sun})^4$
The temperature of the star is the same us the sun, so $(T/T_{sun})^4 = 1$ and $(R/R_{sun})^2 = 2^2 = 4$.
So $L = 1x4{=}4\ L_{sun}$

Example 2: A star has half the sun's temperature, but four times the radius, what is its luminosity?

Answer: $L/L_{sun} = (R/R_{sun})^2 \times (T/T_{sun})^4$
$(T/T_{sun})^4 = 0.5^4 = 0.0625$ and $(R/R_{sun})^2 = 4^2 = 16$.
So $L = 0.0625 \times 16 = 1\ L_{sun}$

Determining stellar masses

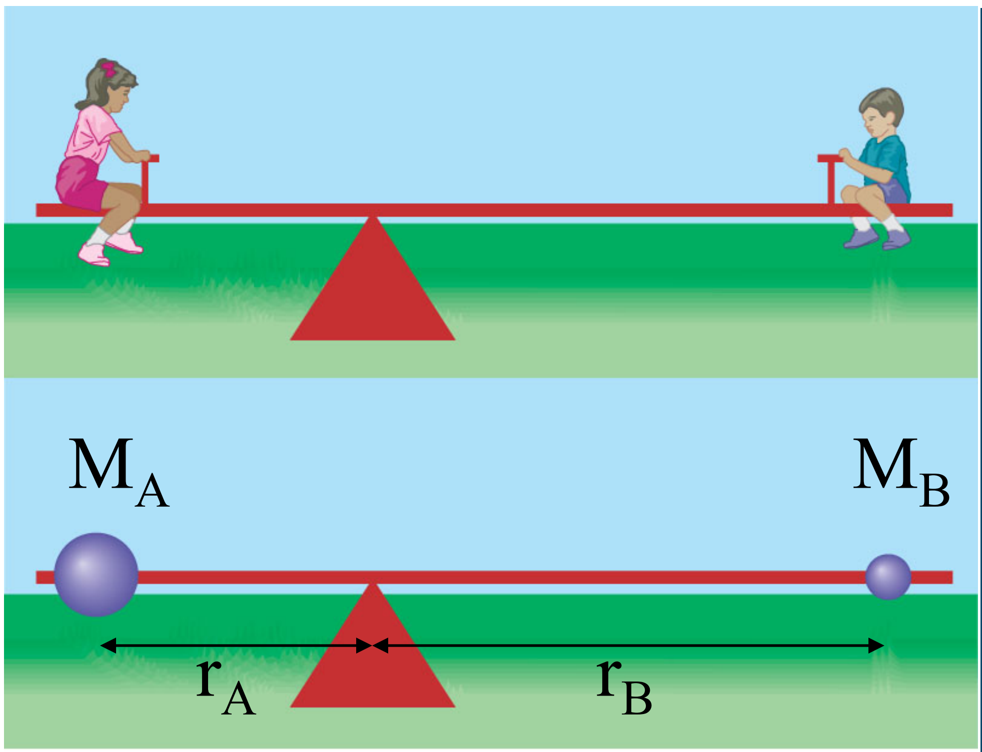
In order to measure the mass of a star, we need to find binary stars.

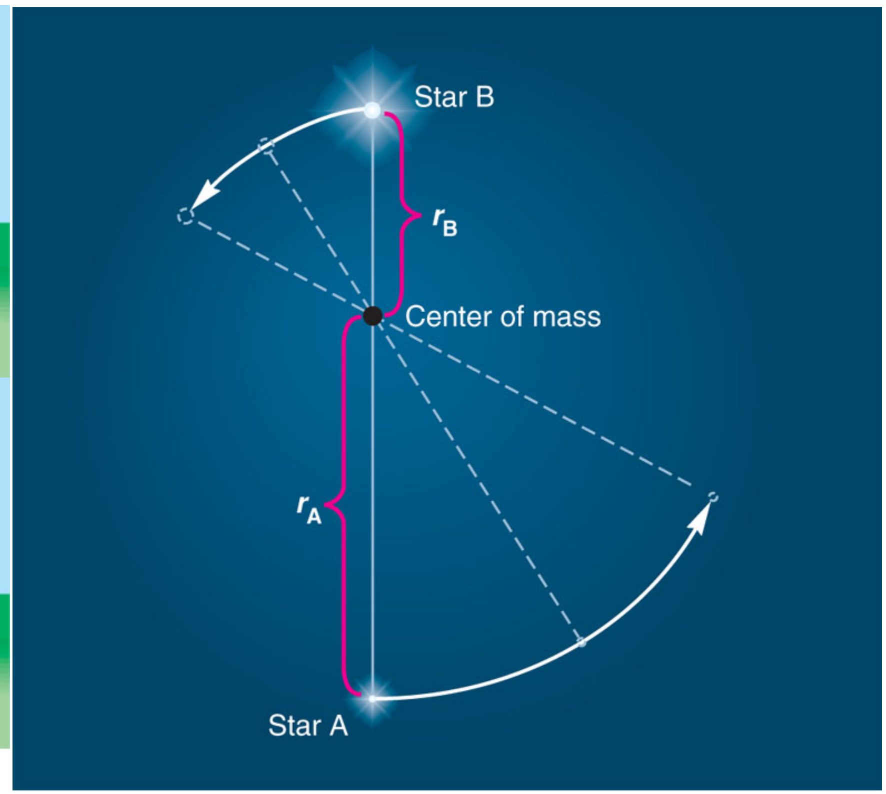


Mizar is a double star in the Big Dipper

The key to using binary stars is that they will orbit around a common centre of mass. Remember, the centre of mass will be closer to the higher mass object, like on a seesaw. The ratio of masses is related to the ratio of radii like this:

$$\frac{M_A}{M_B} = \frac{r_B}{r_A}$$





The formula $M_A/M_B = r_B/r_A$ only tells us about relative masses. In order to determine individual masses, we need to combine this formula with Kepler's 3rd law:

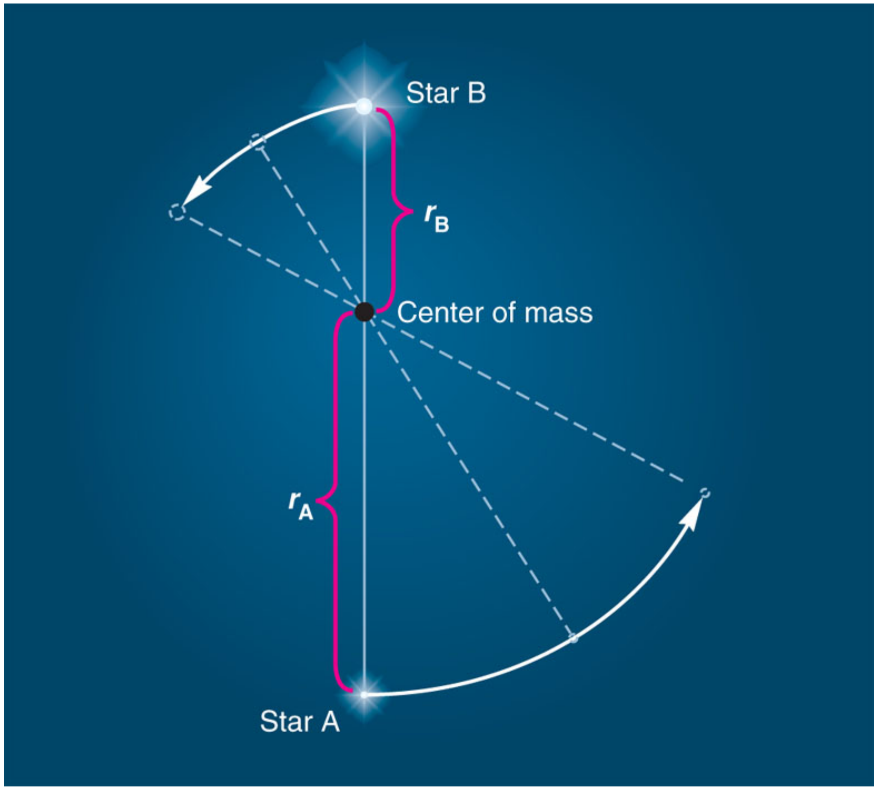
$$P^2 = \frac{a^3}{M}$$

In the case of a binary star, the M is the total of both stars masses: $M_A + M_B = a^3 / P^2$

Example: A binary system has a period of 32 years and an average separation of 16 AU, what is the total mass of the system?

Answer: $M_A + M_B = a^3 / P^2 = 16^3/32^2 = 4$ solar masses

Example 2: If the 2 stars in the previous example have distances of 12AU and 4AU from the centre of mass, what are the individual masses?



Answer: We're told that r_B=4AU and r_A= 12AU.

Now, using $M_B/M_A = r_A/r_B$ we know that $r_A/r_B = M_B/M_A = 3$. We know (from the previous example) that the total mass is 4 solar masses and now we also know that M_B must be 3 times larger than M_A. This gives M_B = 3 solar masses and M_A= 1 solar mass.

Review:

- Temperatures determined from Wien's law and/or spectral features.
- Radial velocities measured from Doppler shifts. Transverse velocities measured from proper motions.
- We can measure the distances to nearby stars by parallax.
- Once we know the distance, we can work out the absolute magnitude from the apparent magnitude and distance modulus.
- We can compare the absolute bolometric magnitude of any star with the sun and hence determine its luminosity, L
- Combining T with L we know the star's size from Stefan-Boltzmann law: $L/L_{sun} = (R/R_{sun})^2 \times (T/T_{sun})^4$
- We can determine the mass of a star if it is in a binary system