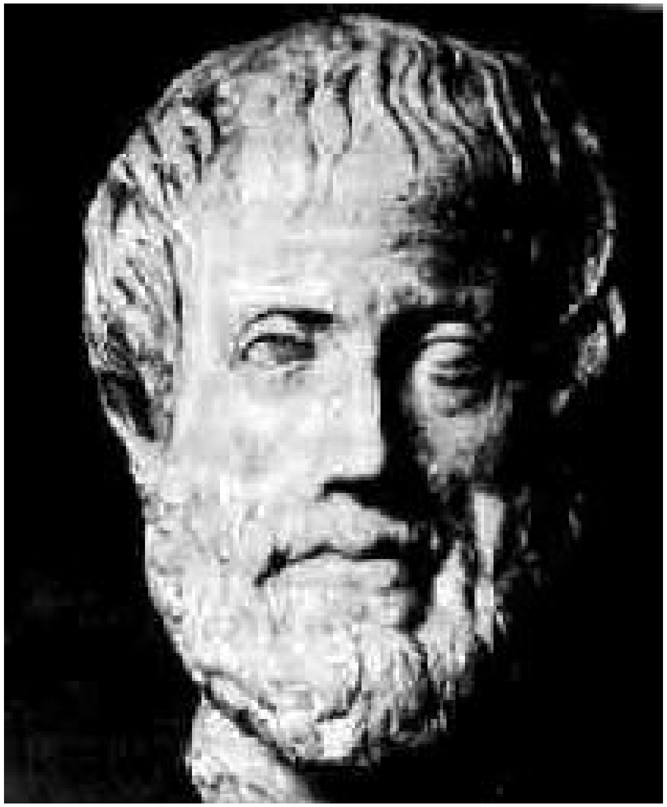
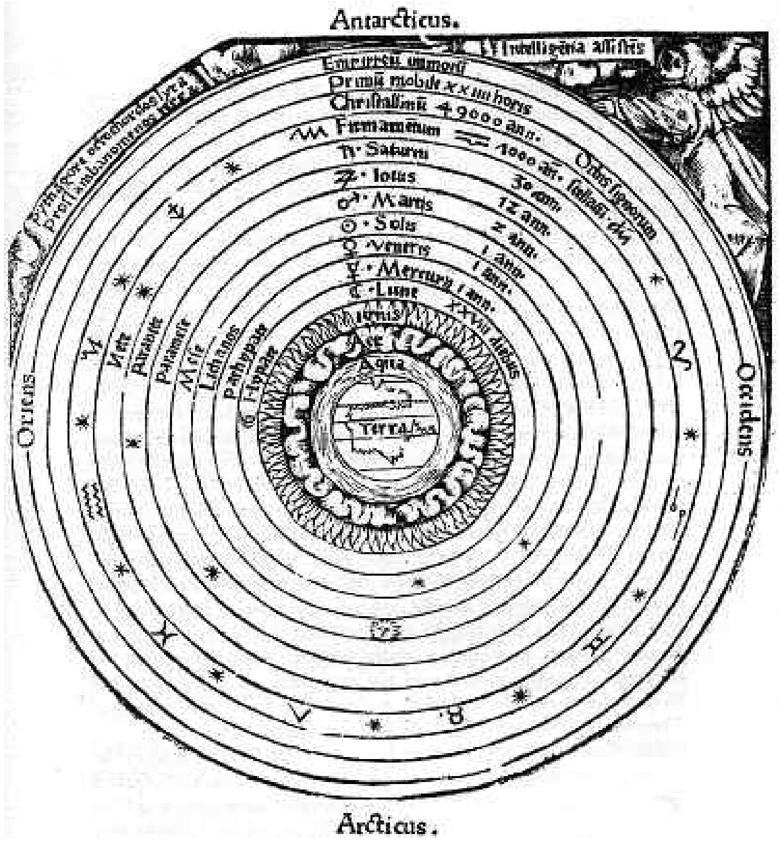
The Origin of Modern Astronomy

C&M Chap 2. (plus parallax from Chap. 1).

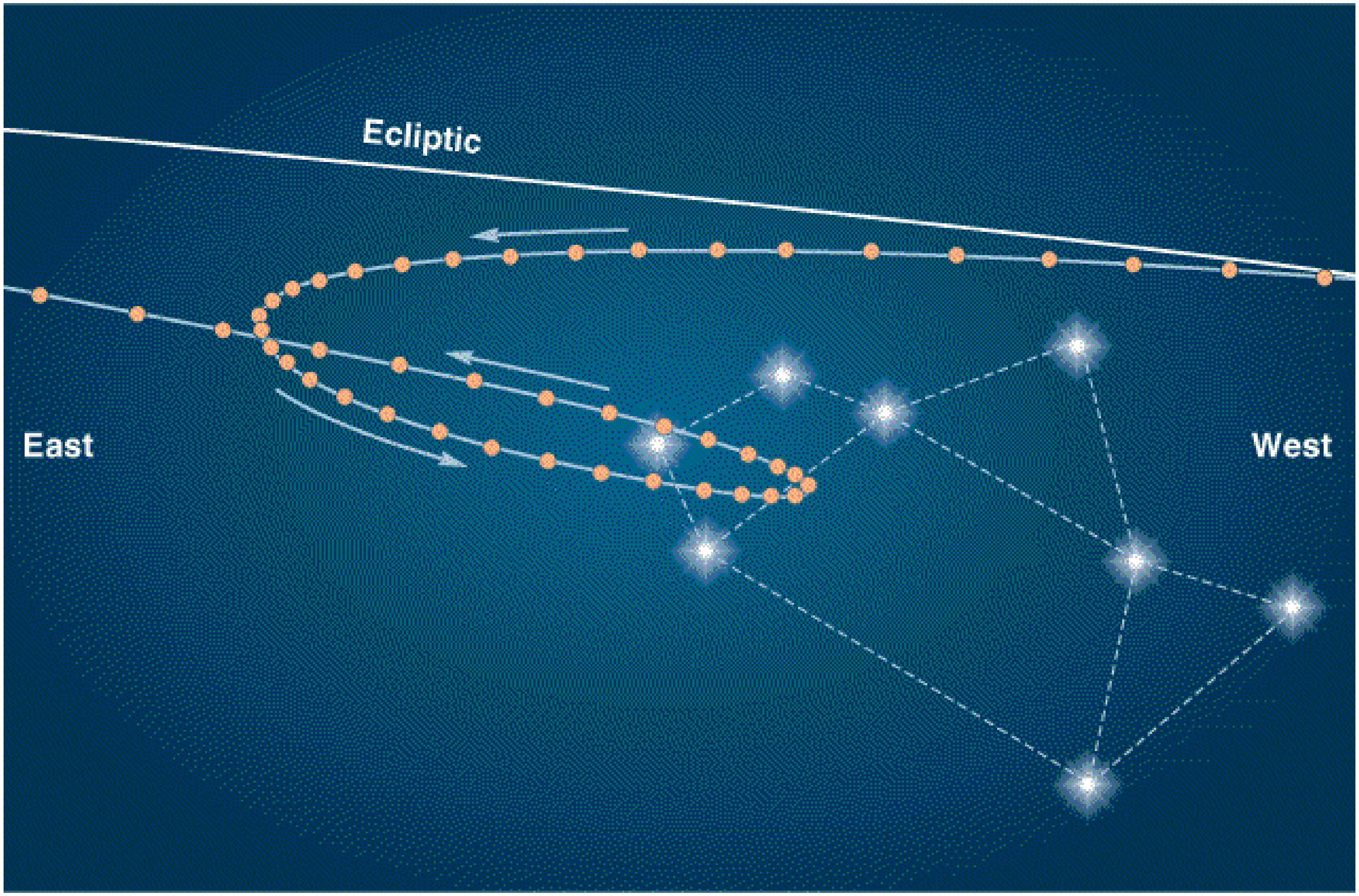
Geocentric theories

Aristotle (~400BC)



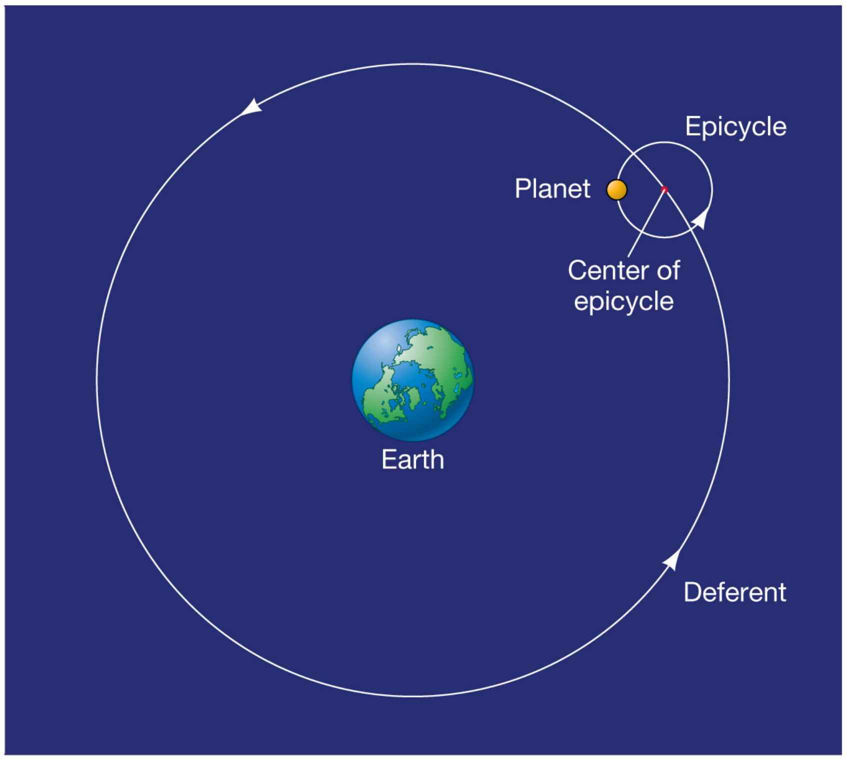


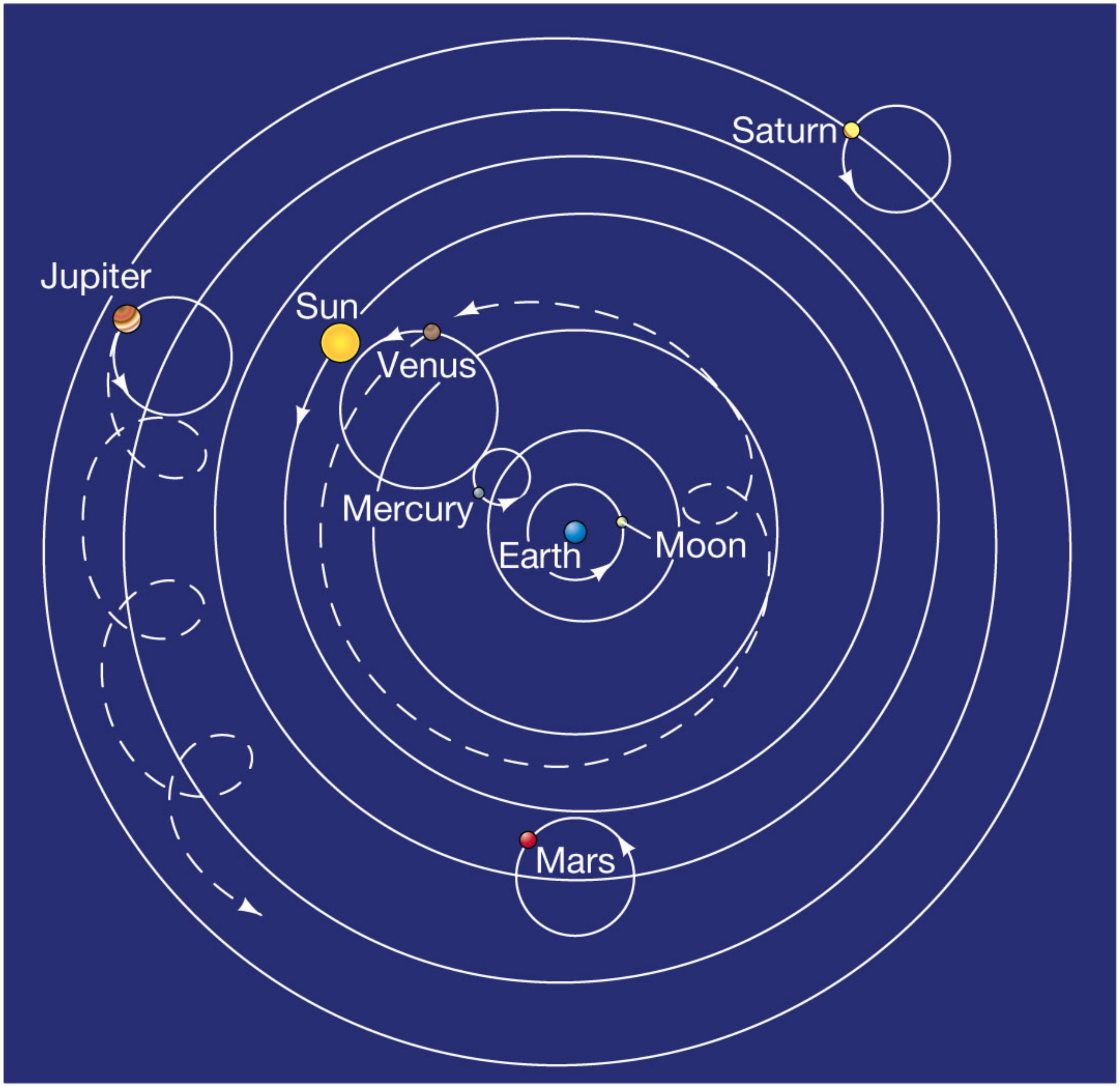
Studying the motion of planets, such as their retrograde motion was a major problem for the geocentric system.



Retrograde motions:

Ptolemy (~200AD) devised a complicated systems of epicycles.



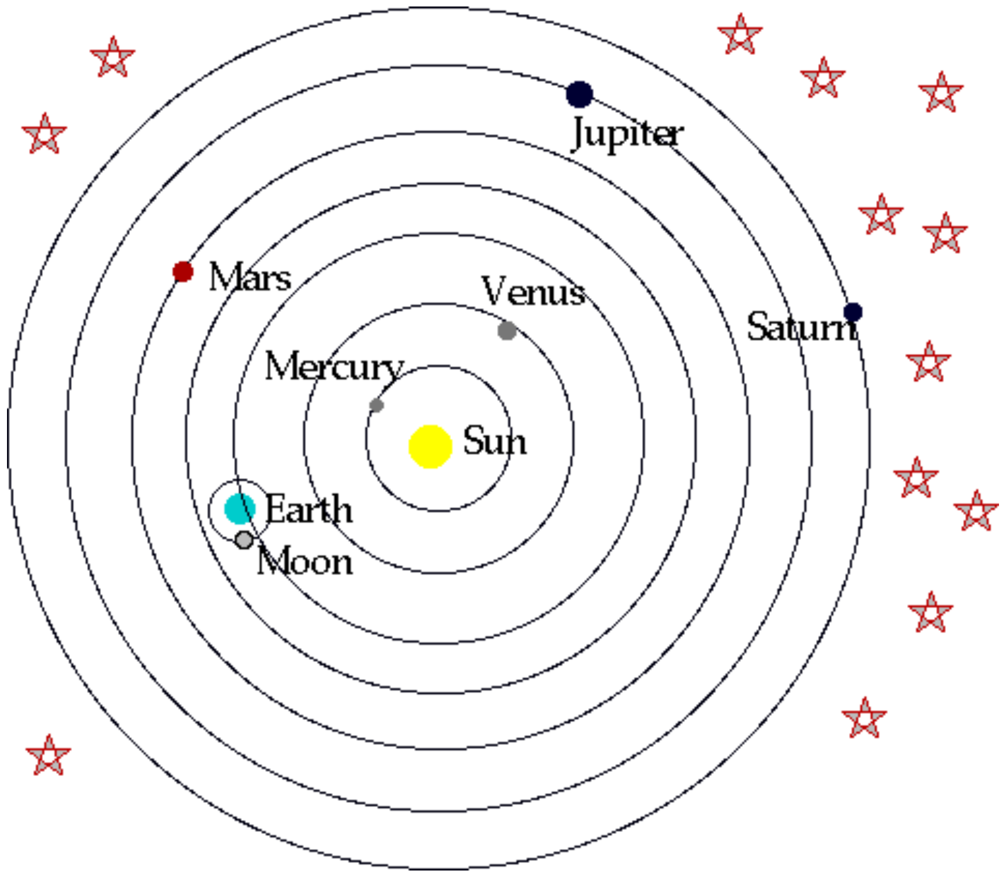

Saturn
Jupiter
Sun
Venus
Mercury
Earth
Moon
Mars

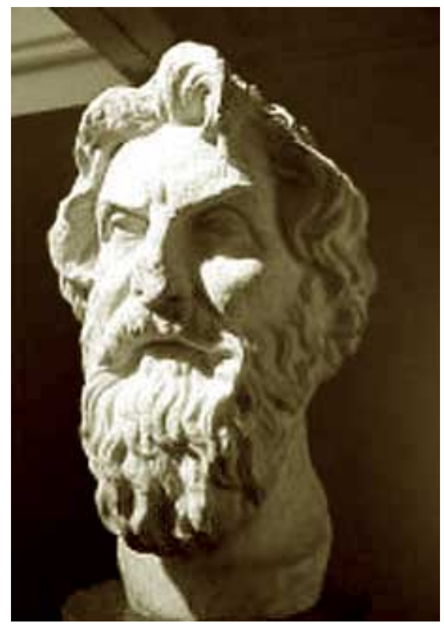
Geocentric Solar System

Created with Starry Night™

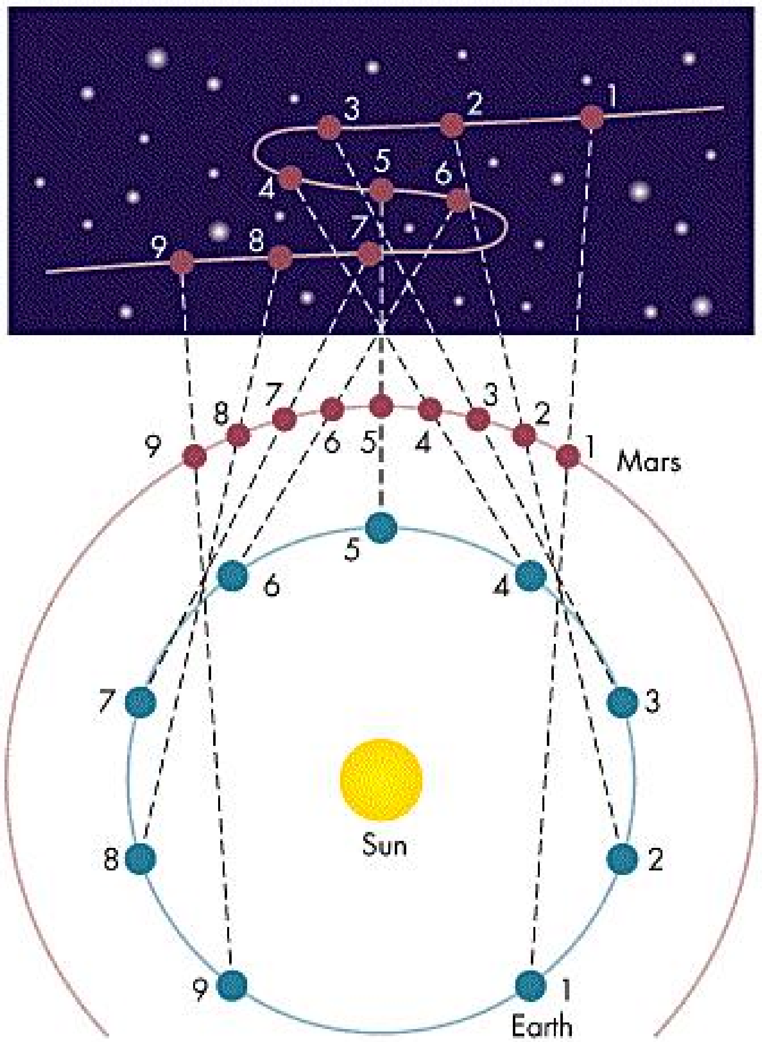
Heliocentric models

First proposed by Aristarchus of Samus ~ 300BC, but ignored for 2000 years.



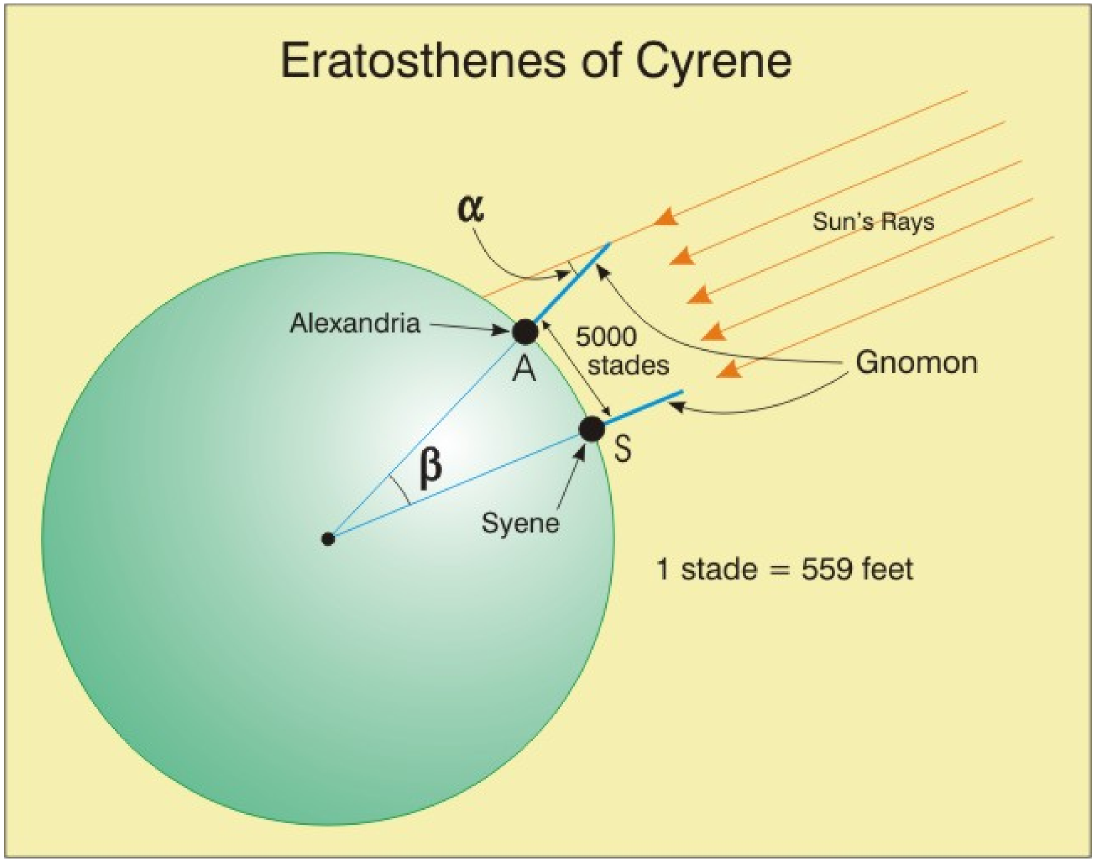


Retrograde motions of planets such as Mars naturally explained by heliocentric model.



First measurement of Earth's size

~ 200BC



β measured to be 7°.

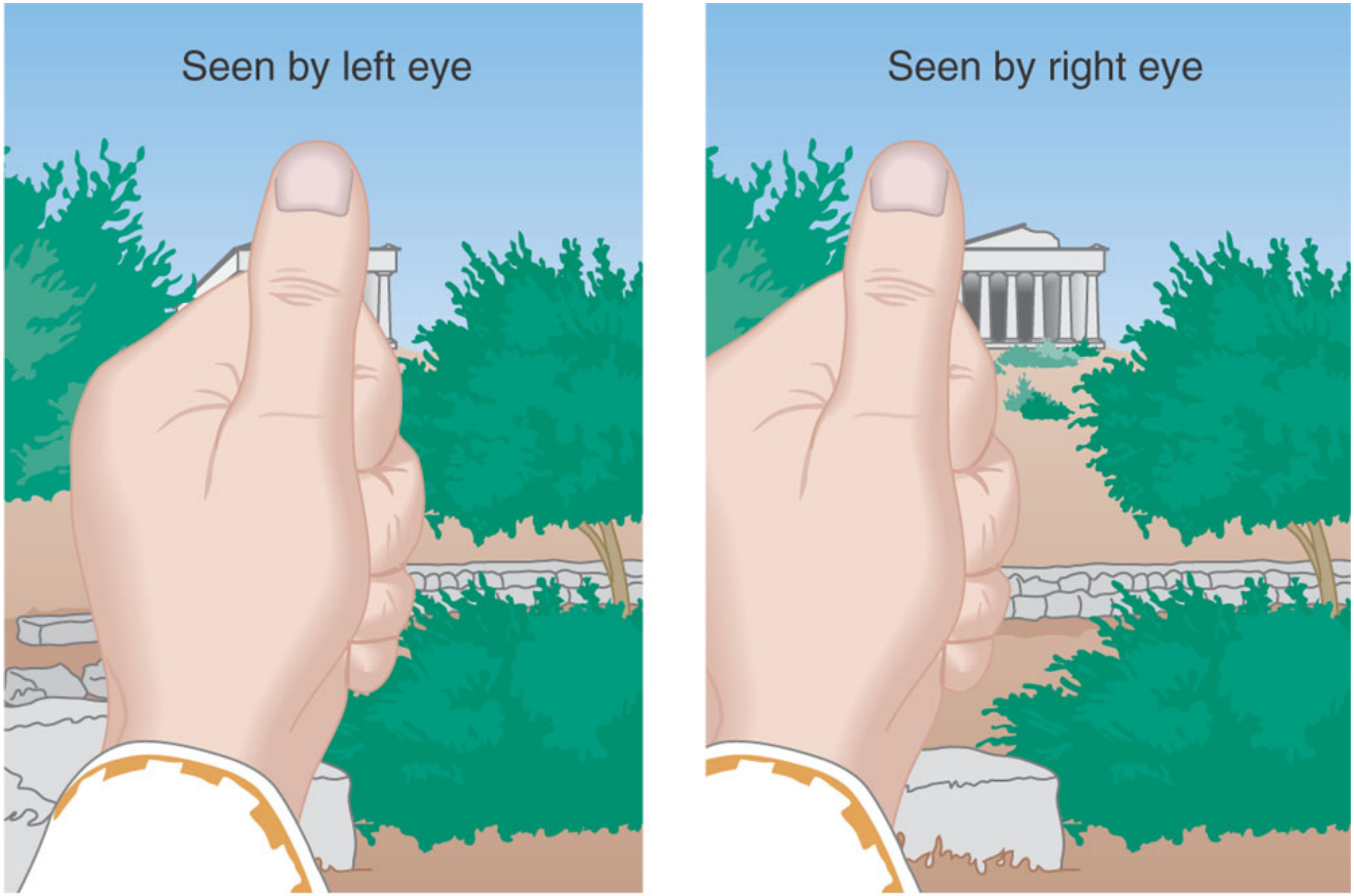
Distance between cities = 5000 stadia (1 stadium ~ 0.16km).

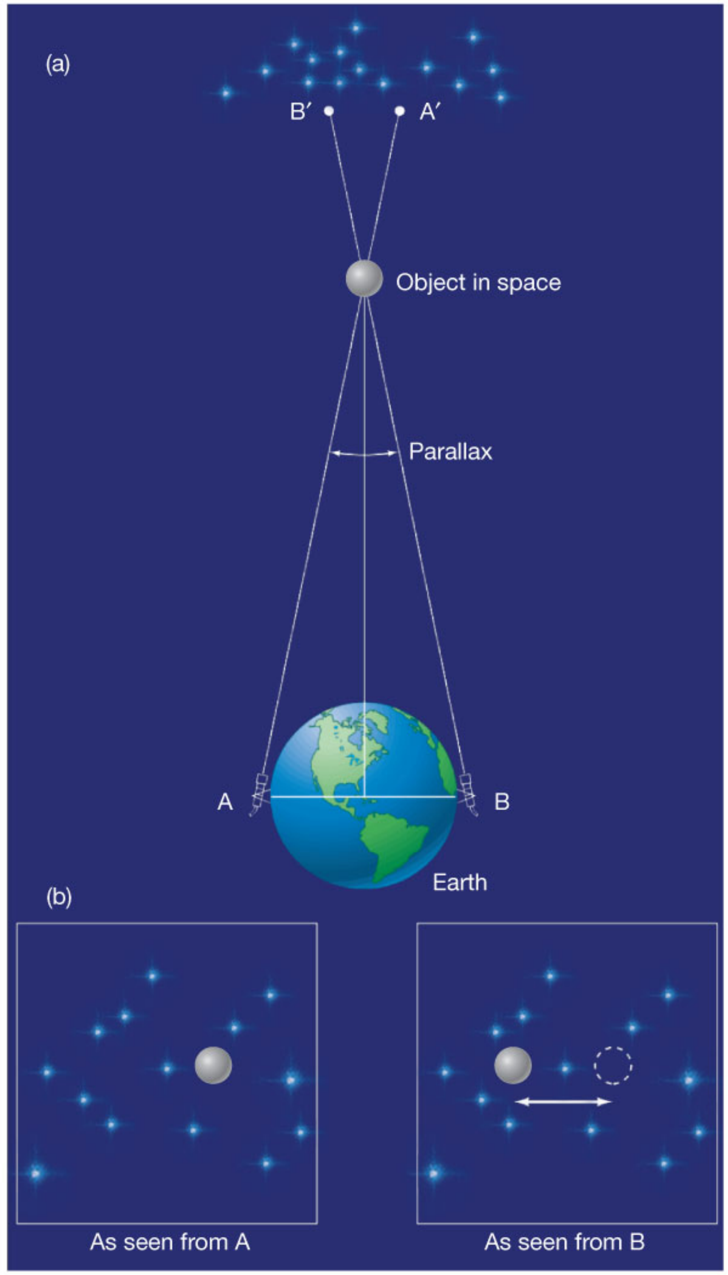
Circumference = 360/7 x 5000 ~ 250,000 stadia.

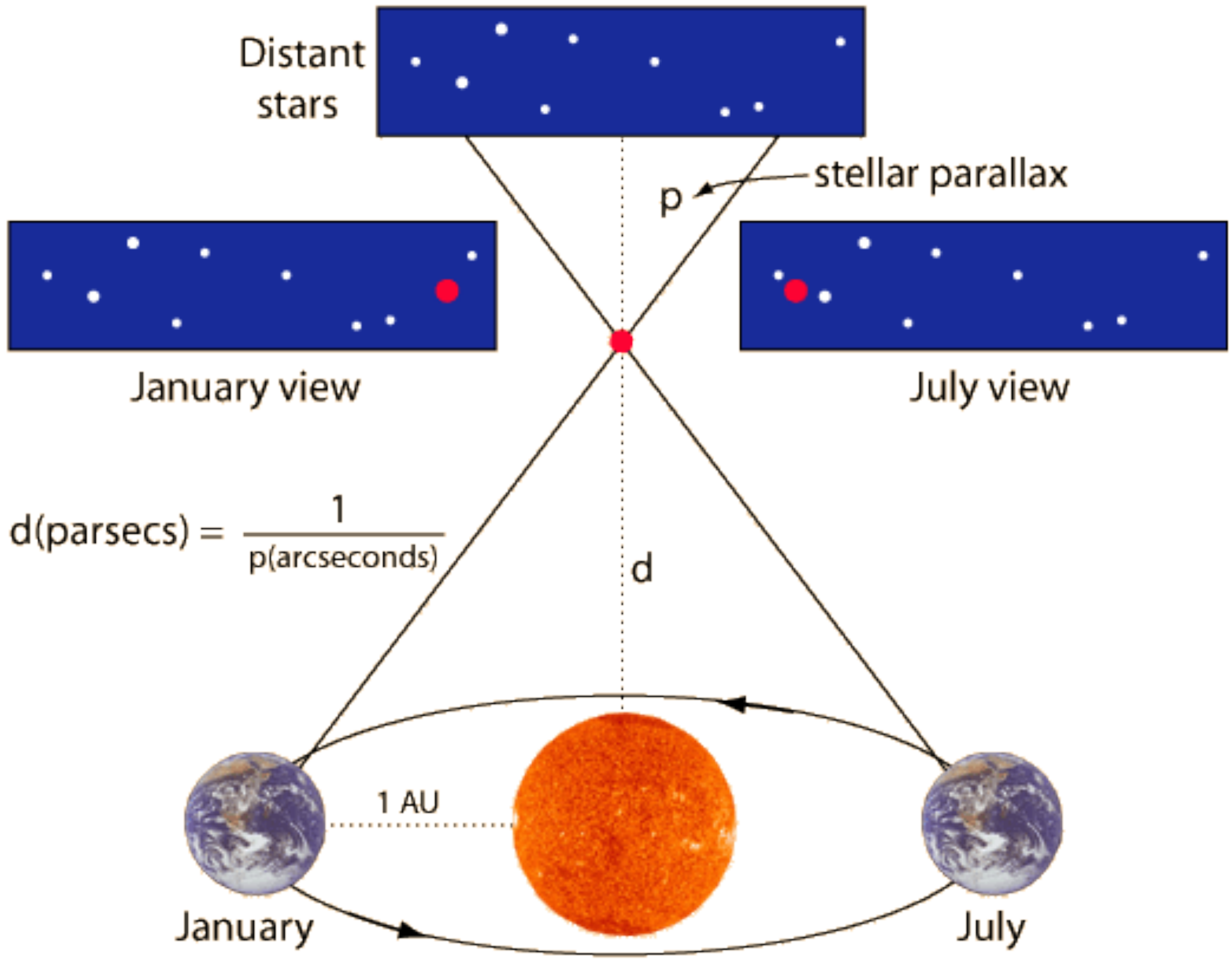
Earth's radius r = c/2π   r ~ 40,000 stadia = 6366 km.

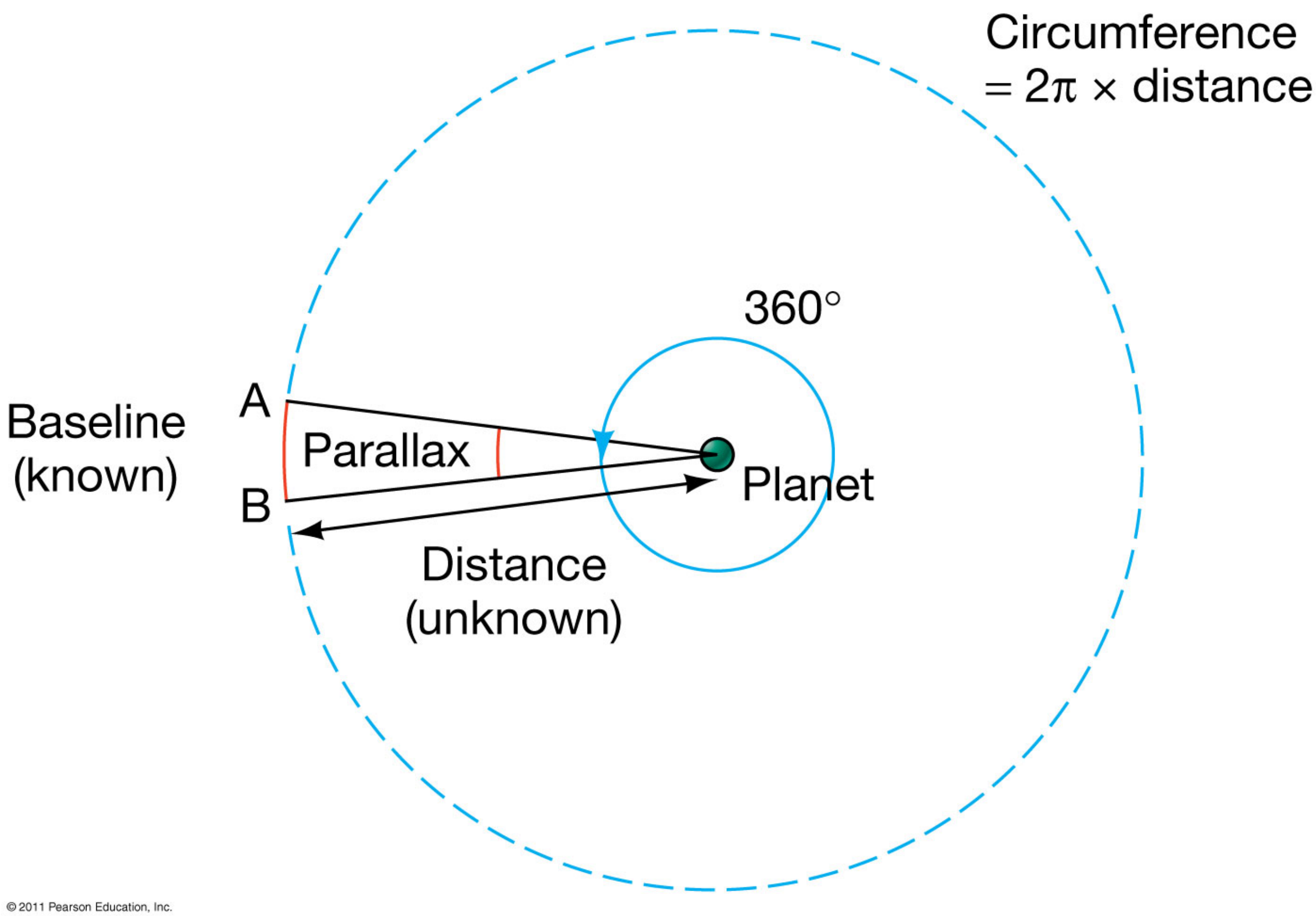
Modern value = 6378 km - very accurate!

Measuring distances within the solar system: parallax




(a)
B′
A′
Object in space
Parallax
A
B
Earth
(b)
As seen from A
As seen from B


Distant
stars
p
stellar parallax
January view
July view
d(parsecs) = 1 / p(arcseconds)
d
1 AU
January
July

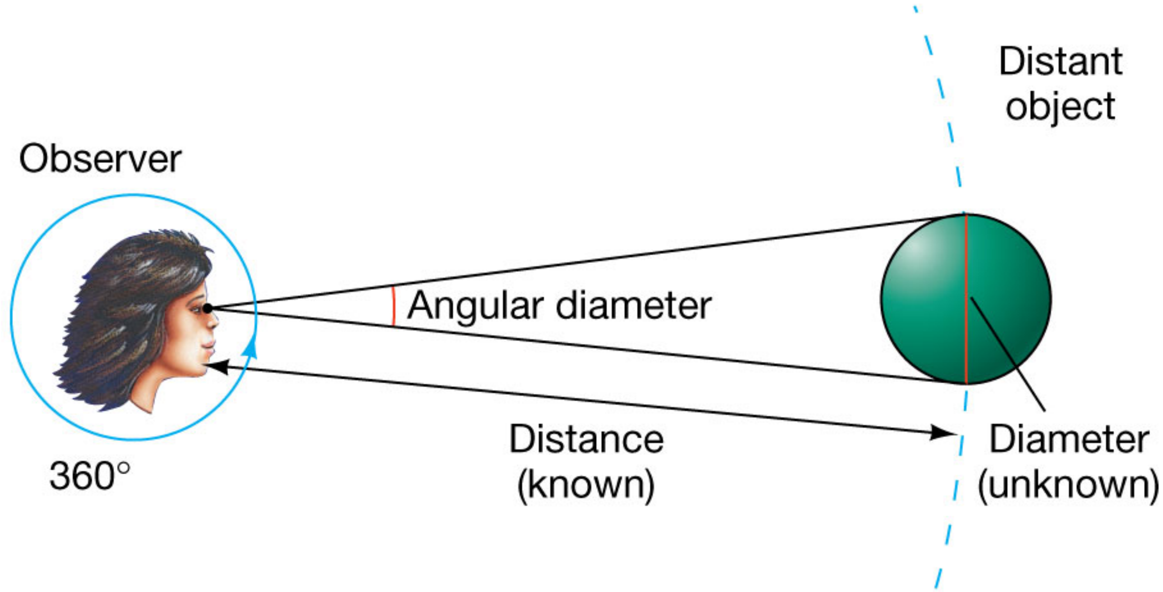


$$\frac{\text{baseline (AB)}}{2\pi \times \text{distance}} = \frac{\text{parallax}}{360^\circ}$$

Example: Two observers on opposite sides of Earth observe Venus and measure an angular parallax of 1 arcmin. Taking Earth's diameter to be 13,000 km, what is the distance to Venus?

$$\frac{\text{baseline (AB)}}{2\pi \text{ x distance}} = \frac{\text{parallax}}{360°}$$

$$\text{Distance} = \frac{13{,}000 \text{ x } 360}{2\pi \text{ x } (1/60)} = 4.6 \text{ x } 10^7 \text{ km} = 46 \text{ million km}$$

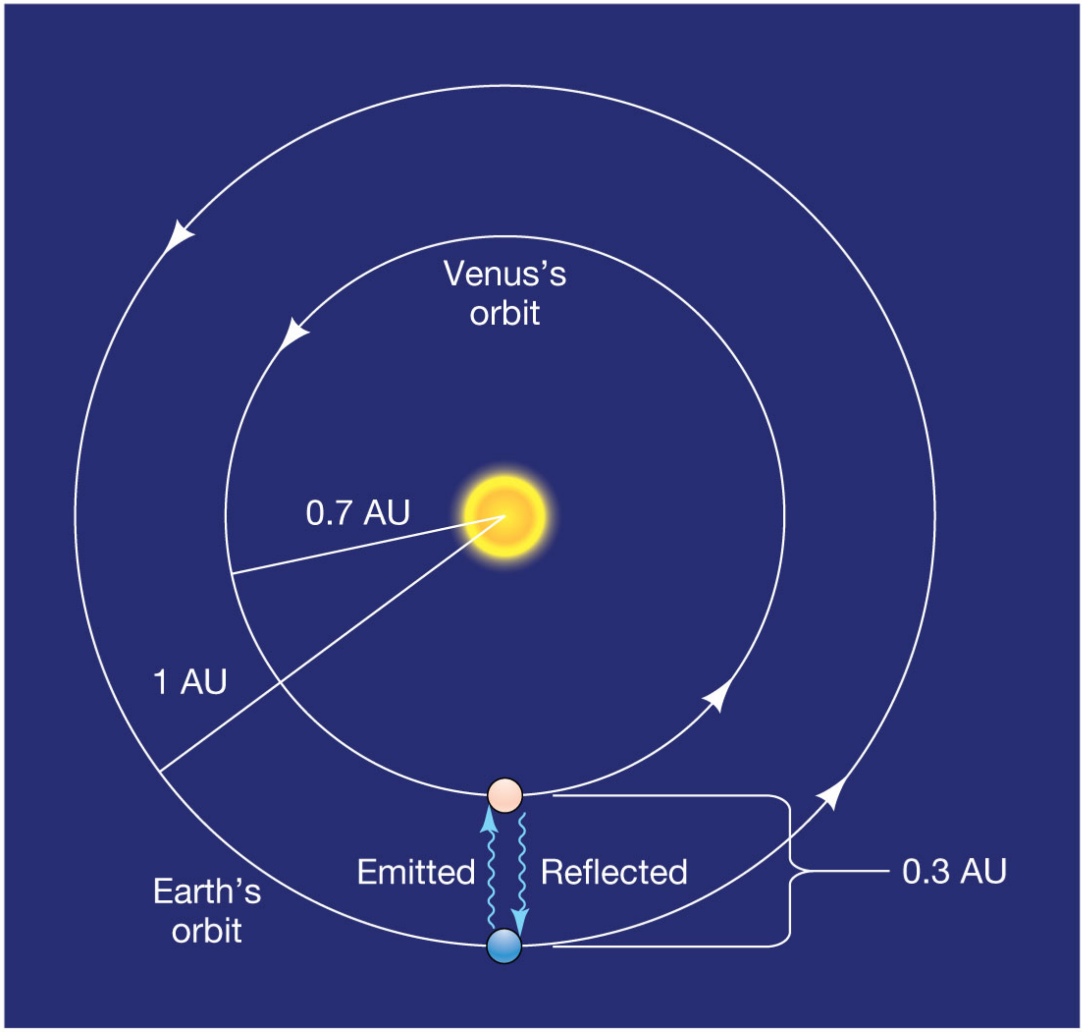


The concept of parallax is the basis for the small angle formula.

$$\frac{\text{Baseline}}{2\pi \times \text{distance}} = \frac{\text{parallax}}{360°} \rightarrow \frac{\text{Diameter}}{2\pi \times \text{distance}} = \frac{\text{angular size}}{360°}$$

$$\frac{\text{angular diameter (arcsecs)}}{206{,}265} = \frac{\text{linear diameter}}{\text{distance}}$$

Modern distance measurements use radar ranging, which measures time lag of a reflected signal.

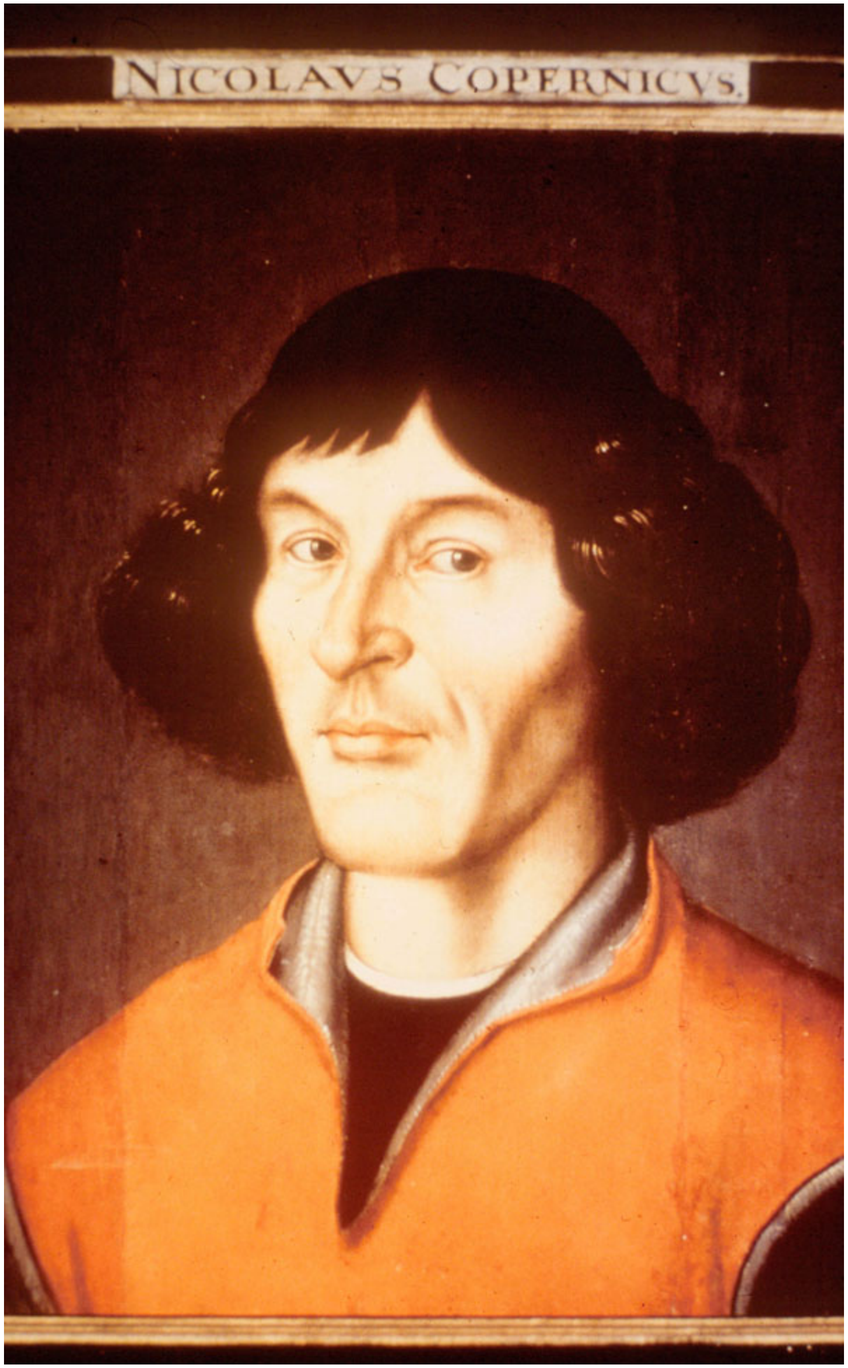


Example: How long would a radar signal take to complete the round trip to Mars when the two planets are 0.7 AU apart?

Distance travelled = 0.7 x 2 AU = 2.1 x 10^8 km

(1 AU = 1.5 x 10^8 km)

Time = 2.1 x 10^8 / 3 x 10^5
= 700 seconds.



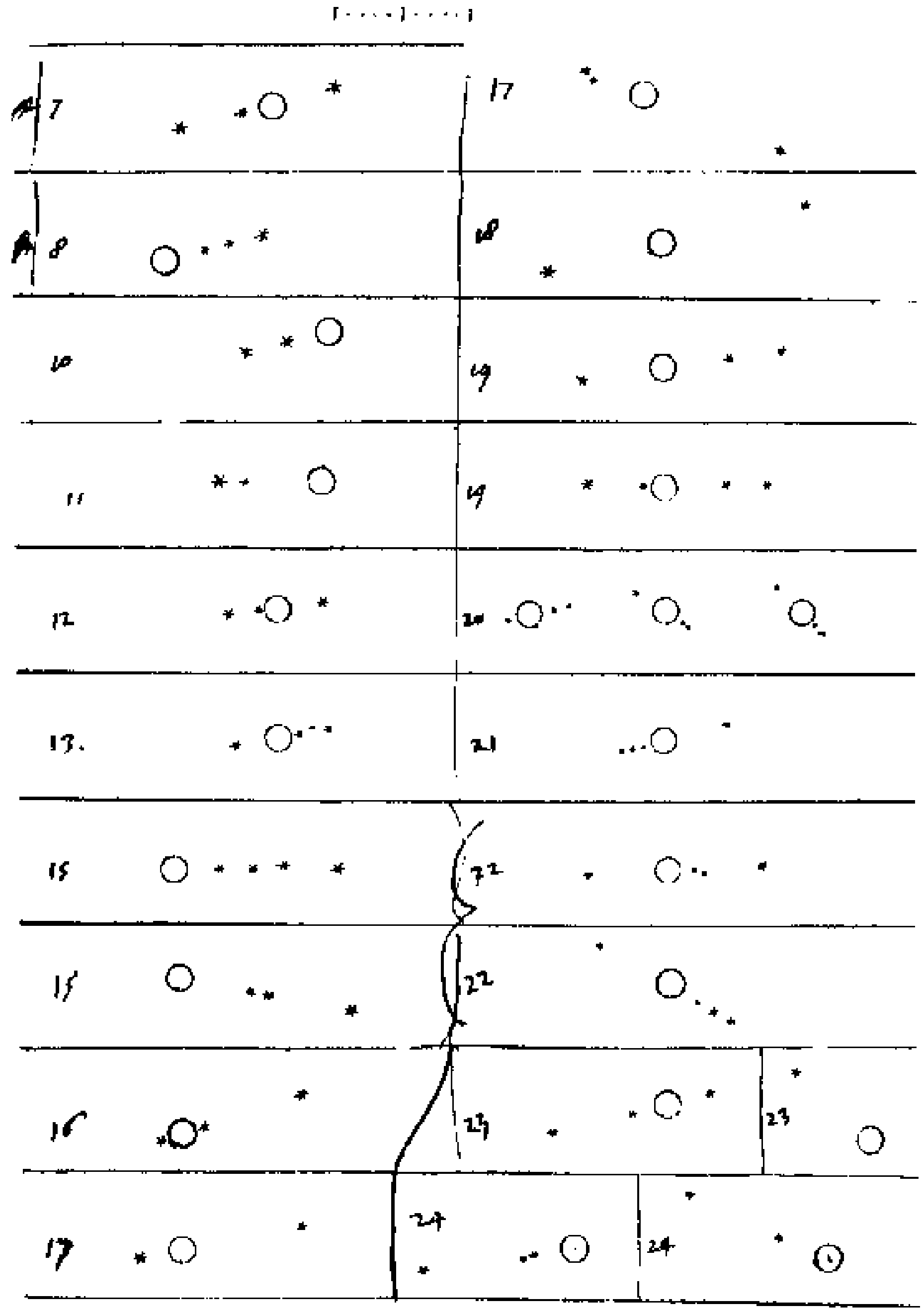
In the early 1500s, a Polish astronomer called Nicolaus Copernicus revolutionised astronomy. He was the first to have his heliocentric model widely accepted.

In the early 1600s, having made some of the first observations of the sky with a telescope, Galileo published 3 major findings:

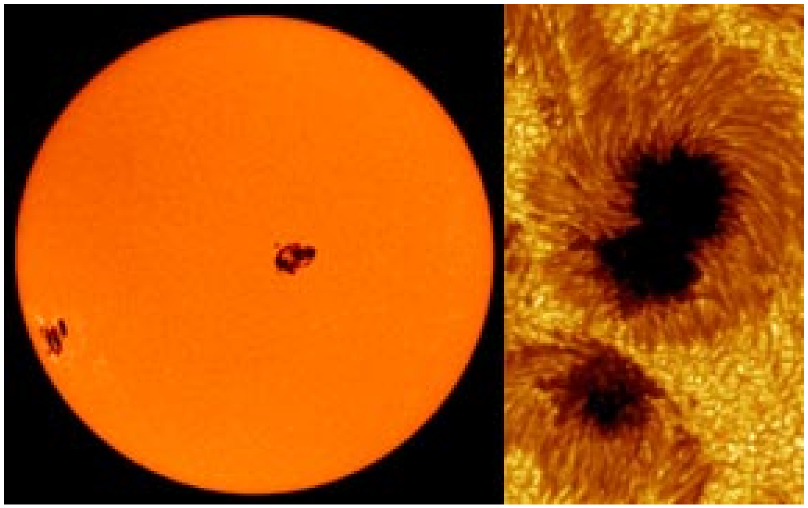
1) Jupiter has 4 moons

2) The moon has craters, "seas" and mountains

3) The Milky Way, a band of light that crosses the sky, contains thousands of individual stars.



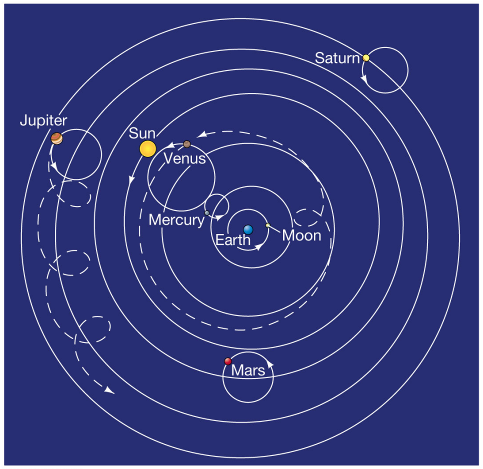
Soon after, Galileo published 2 more major findings:

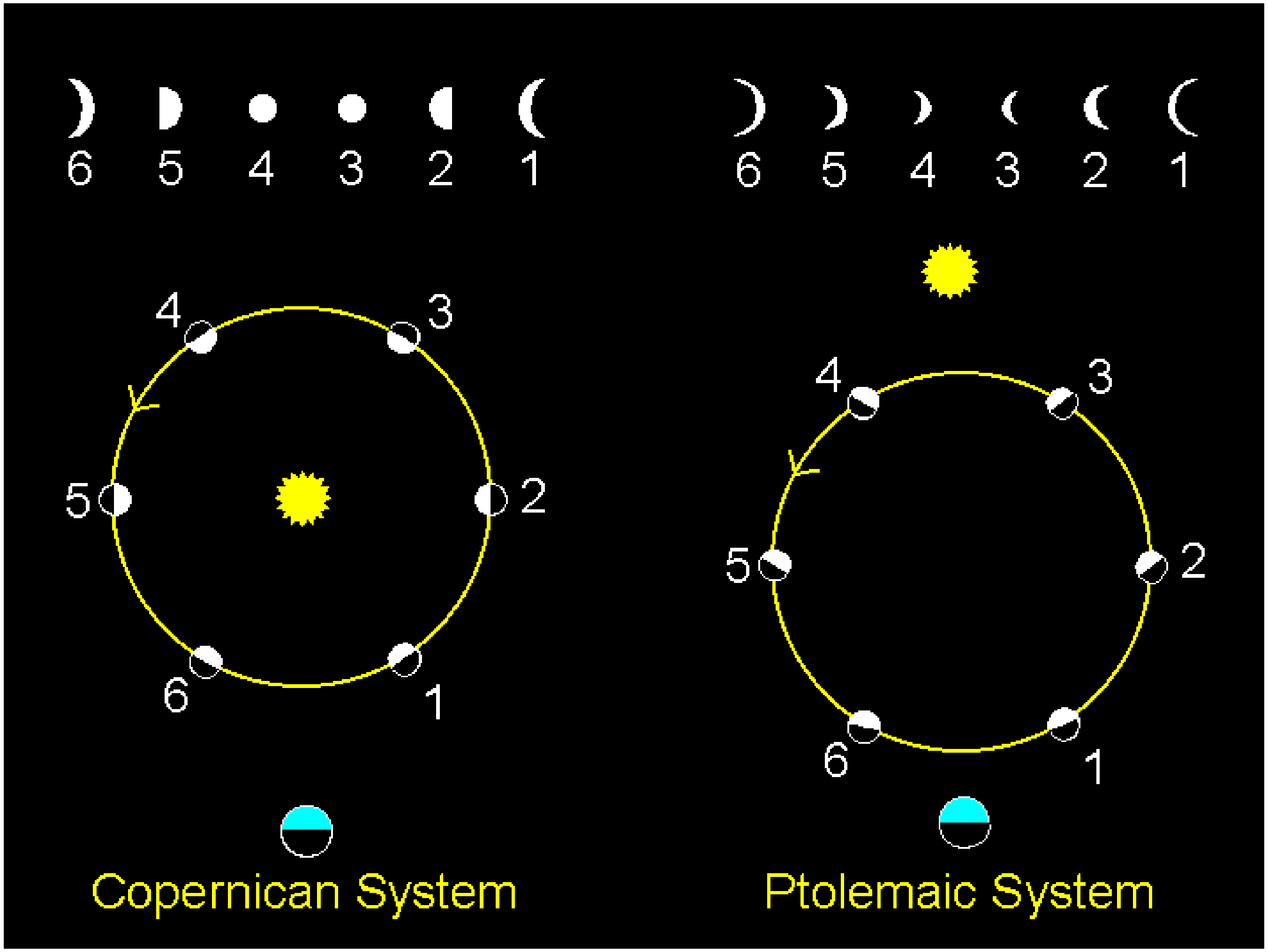


1) The sun had dark spots on its surface and was therefore imperfect (not very Greek!). These spots allowed him to determine that the sun rotated with a period of ~ 28 days.

2) Venus, just like the moon, showed phases. This was a crushing blow for the Ptolemaic model which could not explain this.

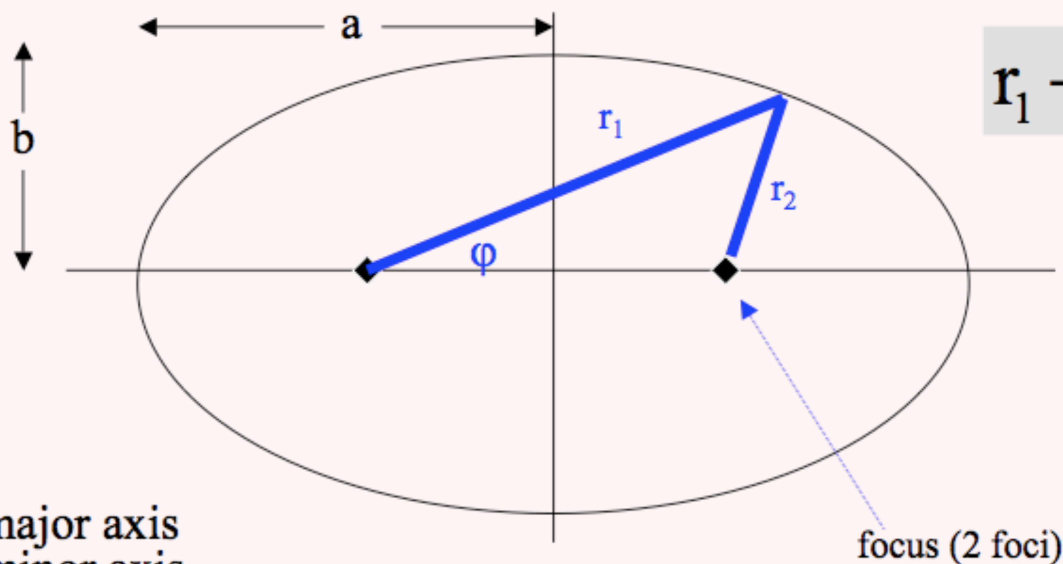
Can only get a “full” Venus when sun is in between Earth and Venus.





In the early 1600s, Tycho Brahe's observations interpreted by Johannes Kepler.

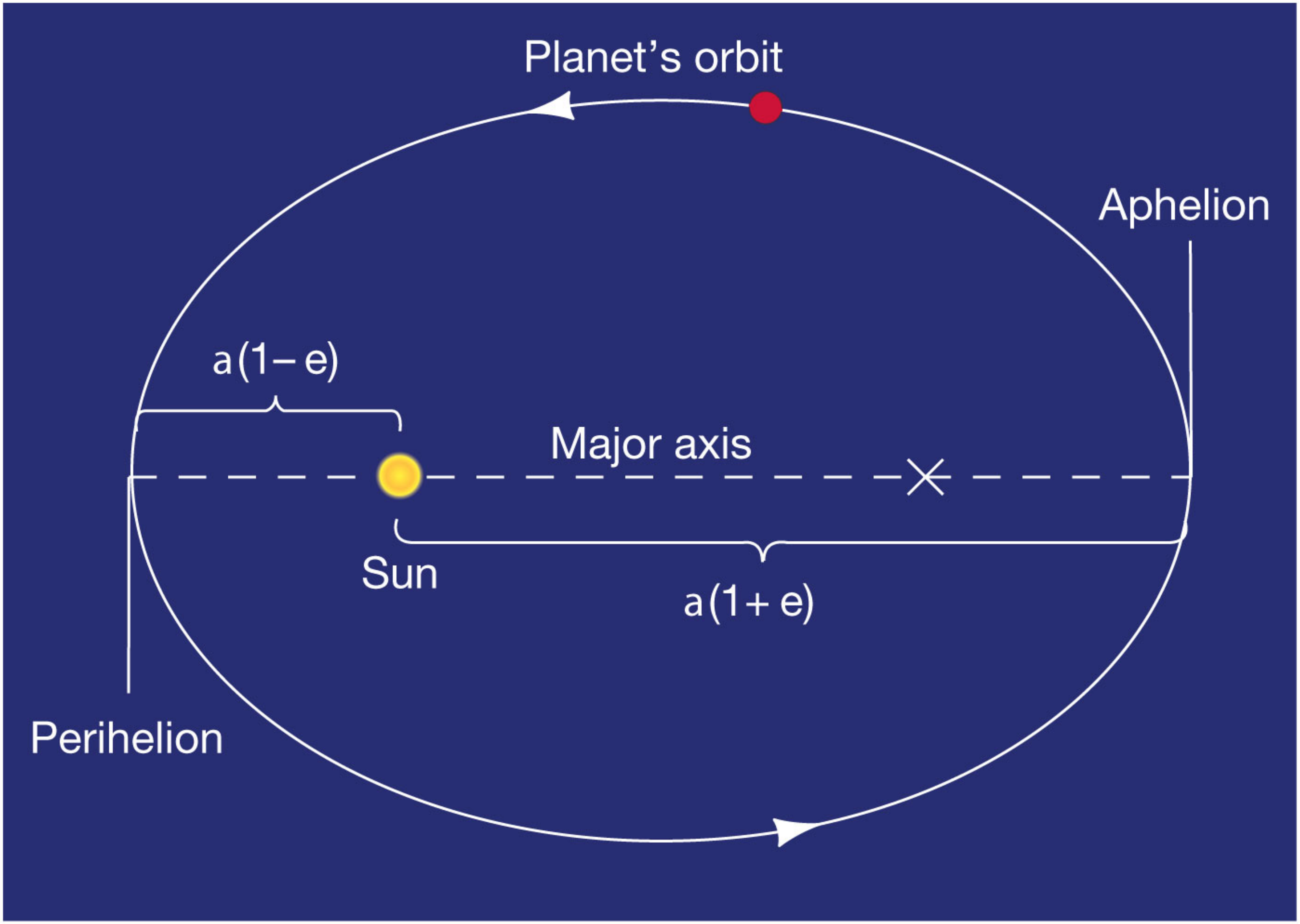
Ellipses - 1



$$r_1 + r_2 = 2a$$

a=semi-major axis
b=semi-minor axis

Ellipse in x-y: $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$


Planet's orbit
Aphelion
a(1– e)
Major axis
Sun
a(1+ e)
Perihelion

e	b/a	ae/a
0	1	0
0.1	0.995	0.1
0.2	0.98	0.2
0.5	0.88	0.5

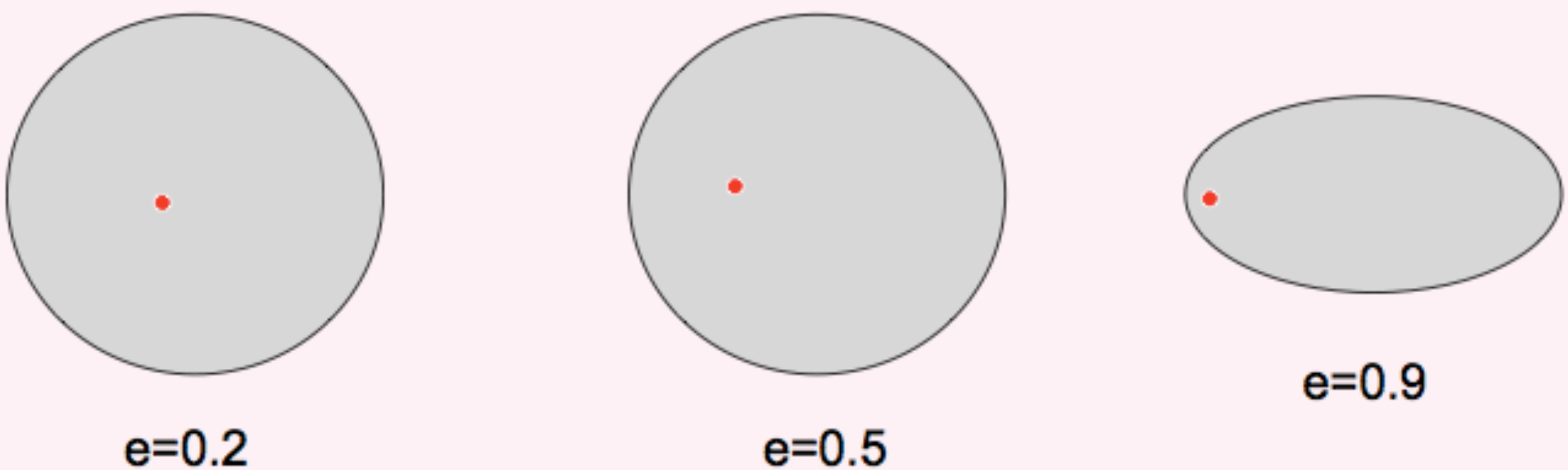
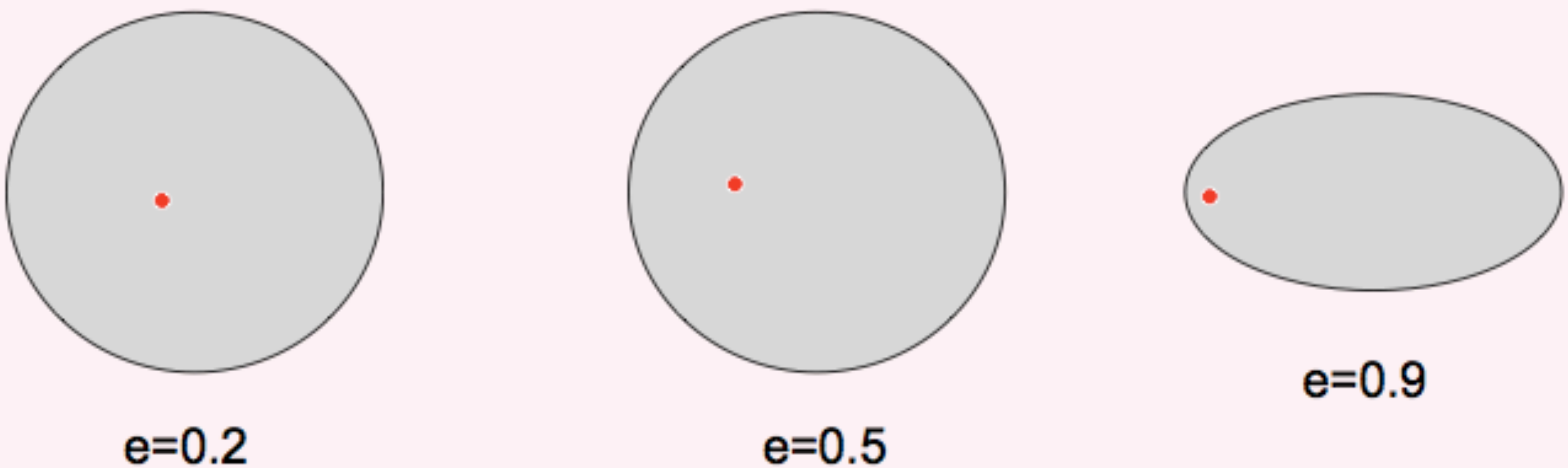
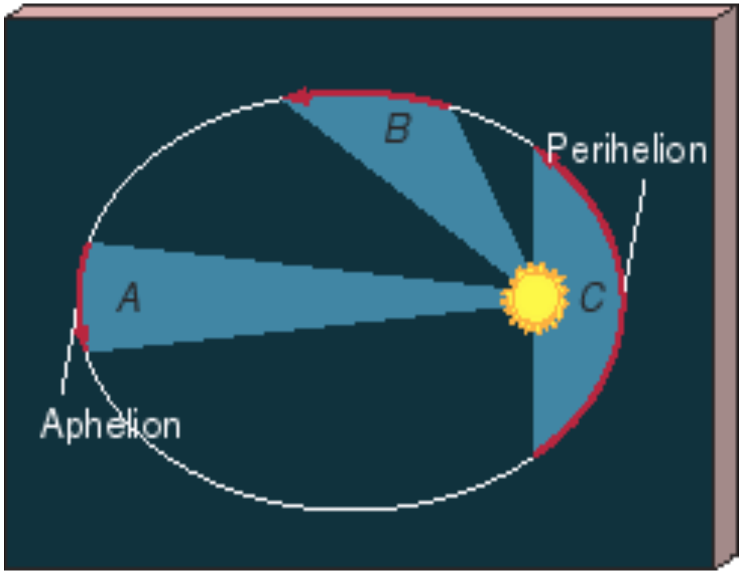


TABLE 3A Planetary Orbital Data							
Planet	**Semi-Major Axis**		**Eccentricity**	**Perihelion**		**Aphelion**	
	(AU)	(10^6 km)	(e)	(AU)	(10^6 km)	(AU)	(10^6 km)
Mercury	0.39	57.9	0.206	0.31	46.0	0.47	69.8
Venus	0.72	108.2	0.007	0.72	107.5	0.73	108.9
Earth	1.00	149.6	0.017	0.98	147.1	1.02	152.1
Mars	1.52	227.9	0.093	1.38	206.6	1.67	249.2
Jupiter	5.20	778.4	0.048	4.95	740.7	5.46	816
Saturn	9.54	1427	0.054	9.02	1349	10.1	1504
Uranus	19.19	2871	0.047	18.3	2736	20.1	3006
Neptune	30.07	4498	0.009	29.8	4460	30.3	4537



Kepler’s laws

1. Planets move in elliptical orbits with the sun at one focus.
2. Equal areas are swept out in equal times during an orbit
3. The square of a planet’s period is proportional to the semi-major axis cubed: $P^2 \propto a^3$



A more general form of Kepler's 3rd law can be applied to the orbit of any body around a mass, not just the sun:

$$P^2 = \frac{a^3}{M}$$

Where M is the mass of the central body in solar masses (ie equal to 1 if we're talking about orbits around the sun), a is the semi-major axis in AU and P is the period in years.

Example: Jupiter's semi-major axis is 5.2 AU, calculate its orbital period.

Answer: The central body in this problem is the sun, whose mass is (by definition) 1 solar mass, so
$P^2 = 5.2^3 / 1 = 140.6$ ---> $P = 11.6$ years

Example: If the period of Saturn (semi-major axis 9.54 AU) is 29.5 years, what is the period of Mercury (a=0.39 AU)?

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$P^2 \propto a^3$

$$\frac{P_s^2}{a_s^3} = \frac{P_m^2}{a_m^3}$$

$$P_m^2 = \frac{(29.5)^2}{(9.54)^3}(0.39)^3 = 0.059$$

$P_m = 0.24$ years = 89 days.

Derivation of Kepler's 3rd law

- If centripetal force =gravitational force:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}, \quad v = 2\pi r/P$$

$$\therefore \frac{4\pi^2 r^2}{r\,P^2} = \frac{GM}{r^2}$$

$$\therefore P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \quad \text{or} \quad P^2 \propto a^3$$

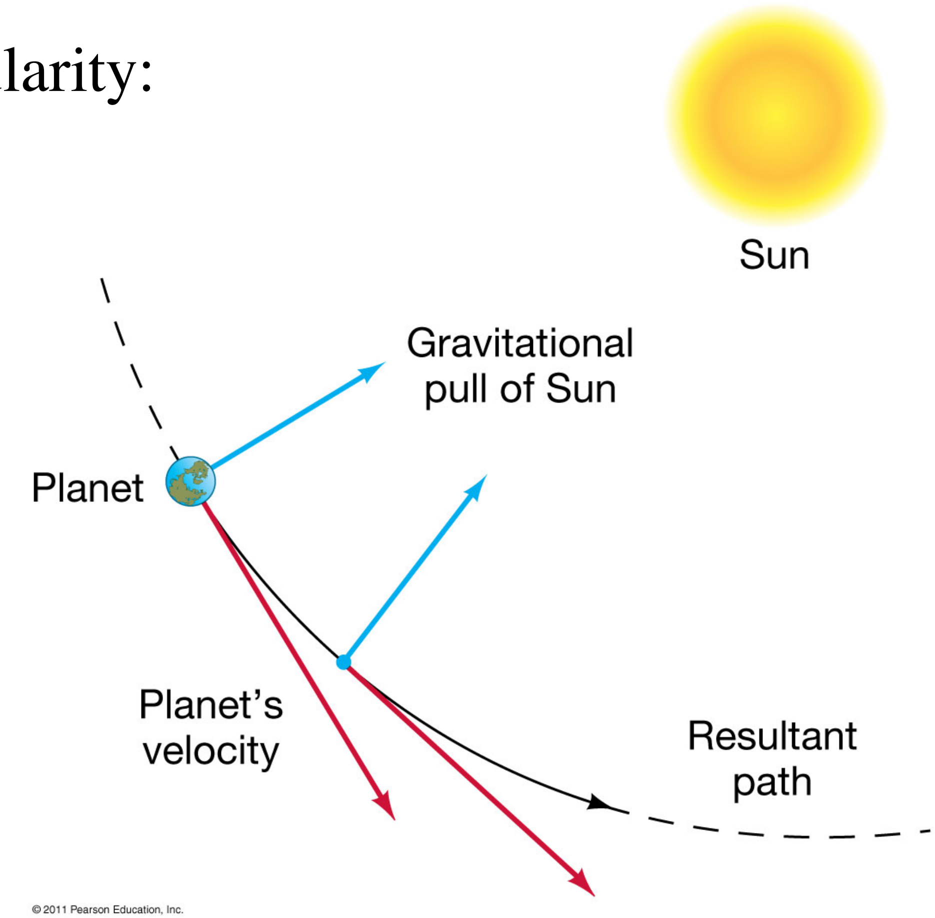
for elliptical orbits

P in yr, a in AU, M in solar masses:

$$P^2 = a^3 / M$$

where $M = m_1 + m_2 \cong M_{sun}$ for the solar system

Assuming circularity:



Speed in an orbit comes from equating gravitational and centripetal forces:

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \quad \text{or} \quad v^2 = \frac{GM}{R}$$

Can also approach the velocities by equating energies.

Energy Considerations

- Kinetic and potential energy of a mass m:

$$K = \tfrac{1}{2} m v^2 \quad \text{and} \quad U = -\frac{GMm}{r}$$

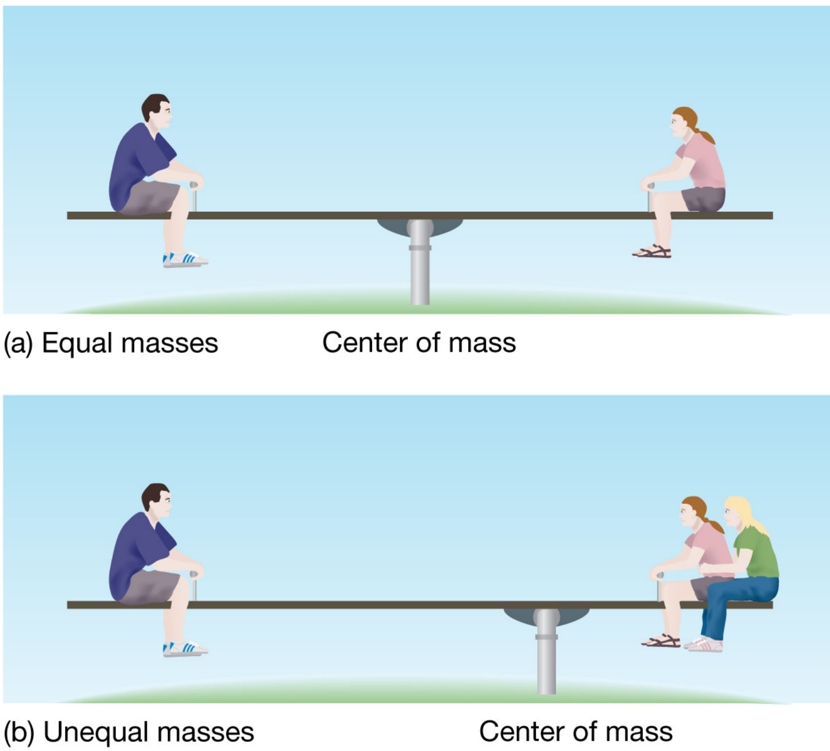
- Total energy of orbit (no proof):

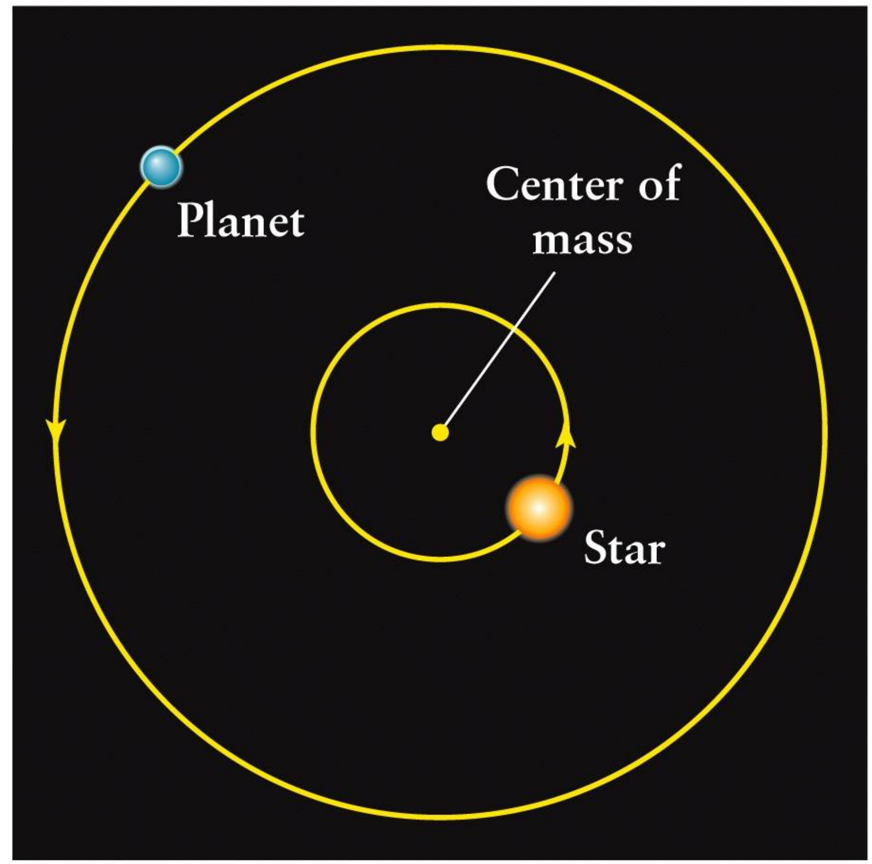
$$E_{tot} = K + U = -\frac{GMm}{2a}$$

a=semi-major axis

"the energy required to 'unbind' an object - move it to infinity"

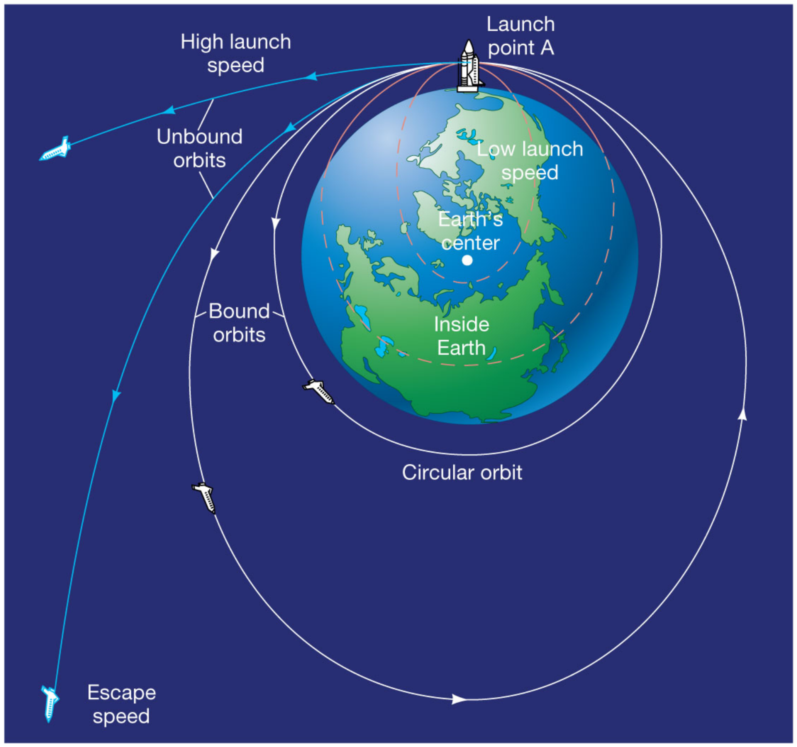
Two orbiting bodies move around a common centre of mass.





a

Escape velocity.



Speed in an orbit comes from equating gravitational and centripetal forces:

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

The escape velocity of a mass depends on its mass and the distance from the centre of mass. Equate initial and final kinetic and gravitational potential energies:

$(K+U)_I = (K+U)_f = 0$ (moving to infinity)

$$\frac{mv^2}{2} = \frac{GMm}{R}$$

$V^2_{esc} = 2\ GM / R$ (careful with units!)

Some examples of escape velocity.

$$V^2_{esc} = 2\,GM / R$$

in m/s — const. — in kg — in m

The Earth from the ground: sqrt(2 x 6.67 x 10^{-11} x 6 x 10^{24} / 6378 x 10^{3})

= 11202 m/s = 11 km/s

The Earth from a space station in a 1000 km high orbit:
sqrt(2 x 6.67 x 10^{-11} x 6 x 10^{24} / 7378 x 10^{3}) = 10416 m/s = 10 km/s

Jupiter: sqrt(2 x 6.67 x 10^{-11} x 2 x 10^{27} / 71500 x 10^{3}) = 61 km/s

The sun: sqrt(2 x 6.67 x 10^{-11} x 2 x 10^{30} / 7 x 10^{8}) = 617 km/s