Math Cheat Sheet

1 Scientific Notation

This is useful for dealing with very small or very large numbers. It involves mutliplying numbers by factors of 10. E.g.

$$10^1 = 10 \qquad 10^2 = 100 \qquad 10^3 = 1000 \qquad 10^4 = 10000 \tag{1}$$

Similarly for very small numbers

$$10^{-1} = 0.1 \qquad 10^{-2} = 0.01 \qquad 10^{-3} = 0.001 \qquad 10^{-4} = 0.0001 \tag{2}$$

There is a button on your calculator to do this for you, it looks something like 10^X (it might be a shift function next to the 'log' button). If you type in '2' and then 10^X, you'll get the answer 100. For '-2' you'll get 0.01 etc.

Powers of ten are only good for multiples of 10, i.e. numbers with a '1' in. To be more flexible we use 'full scientific notation'. This is where we multiply a number by a factor of ten. E.g.

$$c = 3.0 \times 10^5 \text{ km/s} \tag{3}$$

We know that 10^5 is a 1 followed by 5 zeros, so 3 times that is just 3 followed by 5 zeros - easy! Another way of thinking of this is that you to move the decimal point five places to the right. Moving it once gives 30, twice 300, three times 3000 etc. Similarly for small numbers

$$\text{Mass of an electron} = 9.1 \times 10^{-31} \text{ kg} \tag{4}$$

Here we move the decimal point 31 times to the left. Moving it once gives 0.91, so moving it another 30 times gives 30 zeros after the decimal point, followed by a 91!

Try these examples by writing out the numbers in full

1. 4.2×10^2
2. 5.1×10^5
3. 9.5×10^{-1}
4. 0.4×10^{-3}

In order to use scientific notation in calculations, we use the 'exp' button on our calculators. For example, typing '3' followed by 'exp' and '8' is the way to type in 3×10^8. You *don't* use the multiplication button. Try typing in 3.2×10^{12} and mutliply it by 7.1×10^{-2} and check that you get 2.272×10^{11}.

2 Changing Between Units

Often we have to convert between different types of units, for example metres to kilometres, kilograms to grams. For small numbers, this is quite familiar. For example, we know that 0.5 kg is equivalent to 500 grams. Working with larger numbers is just the same. Try these examples

1. Convert 3000 km into metres
2. Convert 3000 km into centimetres
3. Convert 5×10^{12} millimetres into km

These examples are relatively straightforward because they use units that we're familiar with (km, cm, mm). In astronomy sometimes we use unfamiliar units like gigayears or parsecs. How do we deal with those? Usually we're given the conversion. For example 1 parsec (pc) = 3.08 $\times 10^{16}$ metres and 1 gigayear (Gyr) is 1 $\times 10^{9}$ (1 billion) years.

Examples: How many metres in 3 pc? How many cm in 5 pc? How many seconds in 2 Gyrs?

•1 pc $= 3.08 \times 10^{16}$ metres , so 3 pc $= 3 \times 3.08 \times 10^{16}$metres $= 9.24 \times 10^{16}$ metres

•1 pc $= 3.08 \times 10^{16}$ metres , so 5 pc $= 5 \times 3.08 \times 10^{16}$ metres $= 1.54 \times 10^{17}$metres and there are 100 cm in a metre, so $1.54 \times 10^{17} \times 100 = 1.54 \times 10^{19}$ cm

•2 Gyr $= 2 \times 10^{9}$years. 1 year $= 60 \times 60 \times 24 \times 365 = 3.15 \times 10^{7}$seconds. So $3.15 \times 10^{7} \times 2 \times 10^{9} = 6.3 \times 10^{16}$ seconds

There are many good examples at the end of chapter 1 in Seeds.

3 Re-arranging Equations

In A120, we see a few equations. I have included a little maths in this course because I find it wonderful that with a few simple calculations we can calculate the size of our galaxy, how many stars a black hole consumes each year, or even the age of the universe itself. Sometimes, these equations just need the numbers plugged in. Example: Using $E = mc^2$ calculate the energy liberate by converting 2 kilograms of material into energy.

$$E = 2 \times (3 \times 10^{8})^2 = 2 \times 9 \times 10^{16} = 1.8 \times 10^{17} \text{ Joules} \tag{5}$$

Sometimes we have to re-arrange an equation to get the number we want on its onw on the left hand side. The trick here is to do the opposite operation to the quantity you want to move. For example, if I say $A = B + C$, you have to subtract C from both sides to give $A - C = B$. Re-arrange the following equations to give A.

1. $C = B \times A$
2. $C = B - A$
3. $B = A^2$
4. $B = C^2 + A^2$
5. $D = (A + B)/C$