ASTR580: Assignment question from Chris Pritchet

Background: Read Hogg (astro-ph/9905116). This explains how to compute luminosity distance for a Universe with a pure cosmological constant – a simple numerical integration (see Eqn. 14 and 15), with some scaling factors out front to go from D_C to D_L.

Assume a flat Universe: $k = 0$, $\Omega_k = 0$, $D_M = D_C$. This is reasonably well-supported by WMAP results, though only at the level of ± 0.01 for Ω_{Tot}. Also assume $H_0 = 70$ km s^{-1} Mpc^{-1} and $\Omega_M = 0.3$, where relevant. You can also assume that supernovae have $M_B^{max} = -19.2$. Ignore k-corrections.

1. **Adding the effects of w to Equation 14:** A cosmological constant corresponds to equation-of-state parameter $w = dP/d\rho = -1$. Read Dragan and Huterer (astro-ph/0012510).

   (a) Argue from the form of their Equation 1 that the last term in Hogg's Equation 14 should be replaced with $\Omega_X(1+z)^{3(1+w)}$ if w is a constant, and $w \neq -1$.

   (b) Also verify this by algebraic manipulation of the following expression

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{[1 + \Omega_X((1+z)^{3w} - 1)]^{-1/2}}{(1+z)^{3/2}} \, \mathrm{d}z \qquad (1)$$

2. **Computing and plotting:** Compute and plot luminosity distance (better: distance modulus $m - M = 5\log D_L - 5$) versus redshift over the range $w = -0.5$ to $w = -1.5$; do the same thing for an Einstein-de Sitter model ($\Omega_M = 1, \Omega_X = 0$), and a Universe with no cosmological constant and matter density as observed ($\Omega_M = 0.3, \Omega_X = 0$ - the only case in this assignment where $\Omega_k \neq 0$).

   • You will have to numerically integrate Hogg Equation 15, modified for $w \neq -1$.

   • The two models with $\Omega_X = 0$ have analytical solutions for D_L (Mattig's formula); it would be interesting for you to verify that your numerical integration gives the right answer for these cases!

   • Since the none of the models are that different, you should also make a plot showing everything relative to say an Einstein-de Sitter model - to amplify the differences.

3. **Answer the following questions:**

   (a) Roughly estimate the redshift range of maximum sensitivity for the measurement of w.

   (b) Roughly estimate how many supernovae are needed to derive w to an accuracy of ± 0.05. You can assume that supernovae are standard candles with an intrinisic scatter of ± 0.1 mag.

(c) How would this change if you could figure out a way of reducing the intrinsic scatter by a factor of two?

(d) What problems would arise if you wanted to measure a time-variable $w(z)$?

(e) What do you think would be the most significant problems (read "systematic errors") that would arise in attempting a measurement of w to this accuracy? (Hint: for a top-level problem, think about the photometric accuracy required to carry out this measurement.)

4. **Extra Credit:** If you feel truly ambitious, try a full-blown simulation, distributing supernovae according to the volume element out to different limiting redshifts. Then try fitting w. You could add to this simulation any or all of the following: incompleteness as you approach the high z limit; uncertainty in Ω_M; uncertainty in Ω_{Tot}; different high redshift and low redshift samples.