# A580: Topics in Extragalactic Astronomy 

Stellar Populations, Chemical Enrichment, and Galaxy Formation

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## Outline

- Part I (Nov. 3rd - Monday): crash course on star formation, stellar structure and evolution, nucleosynthesis, simple stellar populations
- Part II (Nov. 6th - Thursday): population synthesis models, photometric and spectroscopic stellar population diagnostics, galaxy formation, SN rates in galaxies, star-formation histories, etc.


Galaxy I Zwicky 18
Hubble Space Telescope • ACSIWFC

## Why do we care?

- quantum effects (i.e. tunneling through Coulomb barrier) power stars, which emit EM radiation
- stars are fundamental sources of chemical element enrichment in the Universe
- stellar populations are building blocks of galaxies, i.e. their genetic code, and govern their star formation and evolution processes
- without stars there would be no life*!



## Star Formation in a nutshell

- The gas density in a molecular cloud is about $\rho_{\text {cloud }} \approx 10^{5-12}$ particles $\mathrm{cm}^{-3}$, while the mean density of a typical solar-type star has $\rho_{\text {star }} \approx 10^{30} \mathrm{~cm}^{-3}$
- instability in giant gas cloud leads to contraction and fragmentation
- Tcore and Pcore rise, collapse slower $\Rightarrow$ hydrostatic equilibrium! $\dagger_{K H} \propto \rho^{-1 / 2}$
- star formation ends on the zero-age main sequence (ZAMS), after thermal equilibrium is established, and ppchain H -fusion begins at $\mathrm{T}_{\text {core }} \approx 10^{7} \mathrm{~K}$

- proto-stellar cores develop wind and polar jets that clear residual gas; formation of debris disks and planets



Starburst galaxy Heinze 2-10

## NGC 602 in the Small Magellanic Cloud



Hernalage


Omega Centauri - NGC 5 I39


Hertage
NASA, ESA, and The Hubble Heritage Team (STScI/AURA) • Hubble Space Telescope ACS • STScl-PRC08-14


Spiral Galaxy NGC 300 • ANGST


Hubble Space Telescope • ACS/WFC


## ACS Virgo Cluster Survey

Hubble Space Telescope ACS/WFC
(1)


## Basic Stellar Structure

Assuming spherical symmetry in all parameters, that depend only on distance from the center of the star, one can write

$$
m_{r}=\int_{0}^{r} 4 \pi \rho \hat{r}^{2} d \hat{r}
$$

differentiation yields

$$
\frac{d m_{r}}{d r}=4 \pi \rho r^{2}
$$

the continuity of matter equation, or with $m_{r}$ as dependent variable

$$
\frac{d r}{d m_{r}}=\frac{1}{4 \pi \rho r^{2}}
$$

## Basic Stellar Structure

Determine the motion of infinitesimal cylindrical volume element along the radial axis dr with the base surface dA and constant density

$$
d m=\rho d r d A
$$

We consider gravity and pressure as the only forces - no rotation! All non-radial force vectors cancel out and we get for the force equilibrium

$$
P(r+d r) d A=P(r) d A+\frac{d P}{d r} d r d A
$$

and rewriting as the equation of motion

$$
\frac{d^{2} r}{d t^{2}} d m=-g(r) d m-\frac{d P}{d r} \frac{d m}{\rho}
$$

where $g(r)=\frac{G m_{r}}{r^{2}}$.

## Basic Stellar Structure

Dividing by dm and some algebra we obtain

$$
\frac{d P}{d r}=-\frac{G m_{r} \rho}{r^{2}}-\rho \frac{d^{2} r}{d t^{2}}
$$

and with the condition

$$
\frac{d^{2} r}{d t^{2}}=0
$$

we get the equation of hydrostatic equilibrium:

$$
\frac{d P}{d r}=-\frac{G m_{r} \rho}{r^{2}}
$$

or with $d m_{r}=4 \pi \rho r^{2} d r$ we get

$$
\frac{d P}{d m_{r}}=-\frac{G m_{r}}{4 \pi r^{4}}
$$

## Basic Stellar Structure

The energy production in a spherical shell of thickness dr, located at radius $r$ has the local luminosity

$$
d L_{r}=4 \pi r^{2} \rho \epsilon d r
$$

where $\epsilon$ is the coefficient of energy production per time and mass. Derivation by radius gives the equation of conservation of energy

$$
\frac{d L_{r}}{d r}=4 \pi r^{2} \rho \epsilon
$$

or more general using $\epsilon=\epsilon_{n}-\epsilon_{\nu}+\epsilon_{g}$

$$
\frac{d L_{r}}{d m_{r}}=\epsilon_{n}-\epsilon_{\nu}-c_{P}\left(\frac{d T}{d t}-\nabla_{\mathrm{ad}} \frac{T}{P} \frac{d P}{d t}\right)
$$

## Basic Stellar Structure

The energy produced inside a star can be transported by either random motion of the constituent photons (radiative transfer), free electrons (conduction) and by large-scale motions (convection) $\Rightarrow$ energy transport.

Consider flux of photons that, through scattering processes with the gas particles, deposits the momentum $d p=d \Phi / c$ in a unity volume element.
With $d p=-d P_{\text {rad }}$ we can write

$$
d P_{\mathrm{rad}}=-\frac{d \Phi}{c}=-\frac{\Phi}{c} \frac{d r}{l}
$$

where $l$ is the mean free path of the photons. Including the opacity coefficient $\kappa_{\text {rad }} \rho \equiv 1 / l$ we get

$$
\frac{d P_{\mathrm{rad}}}{d r}=-\frac{\kappa_{\mathrm{rad}} \rho}{c} \Phi
$$

## Basic Stellar Structure

With the condition of thermodynamical equilibrium where $\Phi=-\frac{4 \sigma T^{4}}{3 c \rho}$ and $a=4 \sigma / c$ we obtain the relation $P_{\mathrm{rad}}=a T^{4} / 3$ that leads us to

$$
\frac{d P_{\mathrm{rad}}}{d r}=\frac{4}{3} a T^{3} \frac{d T}{d r}
$$

Hence, equation

$$
\frac{d P_{\mathrm{rad}}}{d r}=-\frac{\kappa_{\mathrm{rad}} \rho}{c} \Phi
$$

becomes

$$
\frac{d T}{d r}=-\frac{3 \kappa_{\mathrm{rad}} \rho}{4 a c T^{3}} \Phi
$$

which is the radiative transfer equation - special case: $\frac{d T}{d r}=-\frac{3 k_{\mathrm{rad}} \rho}{4 a c T^{3}} \frac{L_{r}}{4 \pi r^{2}}$

## Basic Stellar Structure

The energy flux of a non-degenerate electron gas through a unity volume can be written as

$$
\Phi_{e} \propto-N_{e} v_{e} l \frac{d E}{d r}
$$

and with $d E \propto k_{B} d T$ we obtain

$$
\Phi_{e} \propto-k_{B} N_{e} v_{e} l \frac{d T}{d r}
$$

Energy transport by electrons is very inefficient with respect to radiation for non-degenerate electrons, so that $\Phi=\Phi_{\mathrm{rad}}+\Phi_{\mathrm{e}}$ and

$$
\frac{d T}{d r}=-\frac{3 \kappa \rho}{4 a c T^{3}} \Phi
$$

where $\frac{1}{\kappa}=\frac{1}{\kappa_{\text {rad }}}+\frac{1}{\kappa_{\mathrm{e}}}$, and if needed higher order terms.

## Basic Stellar Structure

Together with $d m_{r}=4 \pi \rho r^{2} d r$ we can rewrite the temperature gradient

$$
\frac{d T}{d r}=-\frac{3 \kappa \rho}{4 a c T^{3}} \frac{L_{r}}{4 \pi r^{2}}
$$

as a function of dm

$$
\frac{d T}{d m_{r}}=-\frac{3 \kappa}{64 a c \pi^{2}} \frac{L_{r}}{r^{4} T^{3}}
$$

and use $\nabla \equiv d \ln T / d \ln P$ which can also be written $(d \ln T / T)(P / d \ln P)$ so that we obtain for the generalized radiative transfer equation

$$
\frac{d T}{d m_{r}}=\frac{d T}{T} \frac{P}{d P} \frac{T}{P} \frac{d P}{d m_{r}}=-\frac{T}{P} \nabla \frac{G m_{r}}{4 \pi r^{4}}
$$

where $d P / d m_{r}=-G m_{r} /\left(4 \pi r^{4}\right)$

## Basic Stellar Structure - Summary

$$
\begin{aligned}
\frac{d r}{d m_{r}} & =\frac{1}{4 \pi \rho r^{2}} \\
\frac{d P}{d m_{r}} & =-\frac{G m_{r}}{4 \pi r^{4}}
\end{aligned}
$$

continuity of matter

$$
\frac{d L_{r}}{d m_{r}}=\epsilon_{n}-\epsilon_{\nu}-c_{P}\left(\frac{d T}{d t}-\nabla_{\mathrm{ad}} \frac{T}{P} \frac{d P}{d t}\right) \text { conservation of } \mathrm{E}
$$

$$
\frac{d T}{d m_{r}}=-\frac{T}{P} \nabla \frac{G m_{r}}{4 \pi r^{4}}
$$ generalized radiative transfer

$\frac{d X_{s}}{d t} \frac{1}{A_{s}}=\sum_{w} \rho^{n_{h}+n_{k}-1} n_{p} \frac{X_{h}^{n_{h}} X_{k}^{n_{k}}}{A_{h}^{n_{n} A_{k}} A_{k}^{n_{k}}} \frac{\langle\sigma v\rangle_{h k}}{m_{H}^{n_{h}+n_{k}-1} n_{h}!n_{k}!}-\sum_{l} \rho^{n_{d}+n_{j}-1} n_{d} \frac{X_{s}^{n_{d}} X_{j}^{n_{k} j}}{A_{s}^{n_{d}} A_{j}^{n_{j}}} \frac{\langle\sigma v\rangle_{s j}}{m_{H}^{n_{d}+n_{j}-1} n_{d}!n_{j}!}$
evolution of chemical mass fractions

## Basic Stellar Structure - Summary

Equations can be solved to find $r, P, L, T$, and $P$ in terms of $m$ using the boundary conditions
$r=0, L=0$ at $m=0$ (center) $\rho=0, T=0$ at $m=1$ (surface)

Models predict a strong mass-luminosity relation for core H -burning stars in thermal and
hydrostatic equilibrium, $L \propto M^{\alpha}$


## Mass-Luminosity Relation

RT: $\frac{d T}{d m_{r}}=-\frac{T}{P} \nabla \frac{G m_{r}}{4 \pi r^{4}}$ for pure radiation we find $\frac{d T}{d m_{r}} \propto \frac{L}{T^{3} R^{4}}$
hence, we get $T^{4} \propto \frac{M L}{R^{4}}$
HSE: $\frac{d P}{d m_{r}}=-\frac{G m_{r}}{4 \pi r^{4}}$ gives us $\frac{d P}{d m_{r}} \propto \frac{M}{R^{4}}$, therefore $P \propto \frac{M^{2}}{R^{4}}$
The ideal gas equation $P \propto \rho T$ and $\rho \propto M / R^{3}$ give us $T \propto \frac{M^{2}}{R^{4}} \frac{R^{3}}{M} \propto \frac{M}{R}$
Putting it all together yields: $L \propto \frac{M^{4}}{R^{4}} \frac{R^{4}}{M}$ or $L \propto M^{3}$

## Mass-Luminosity Relation

Our sketchy derivation of $L \propto M^{3}$ is indeed a good approximation of the main sequence.

Empirical data show that:
$L \propto M^{3.6} \rightarrow 2-20 M_{\odot}$
$L \propto M^{4.5} \rightarrow 0.5-2 M_{\odot}$
$L \propto M^{2.6} \rightarrow 0.2-0.5 M_{\odot}$
a better rule of thumb is the approximation

$$
L \propto M^{3.5}
$$



## Main-Sequence Lifetimes

The lifetime of a star on the main-sequence $\tau_{\mathrm{MS}}$ is proportional to

$$
\tau_{\mathrm{MS}} \propto \frac{\text { Fuel Supply }}{\text { Fuel Consumption Rate }}
$$

The fuel supply scales with mass and the consumption rate is proportional to the luminosity of the star, hence

$$
\tau_{\mathrm{MS}}=\frac{f \epsilon c^{2} M}{L} \propto \frac{M}{L} \propto \frac{M}{M^{3.5}} \propto M^{-2.5}
$$

When a star of 1 solar mass burns approx. $10 \%$ of its total hydrogen supply, it leaves the main-sequence, after $\tau_{\mathrm{MS}} \approx 10^{10}$ years, which is much longer than its Kelvin-Helmholtz timescale $\tau_{\mathrm{KH}} \approx G M^{2} /(L R) \approx 10^{7}$.

## Main-Sequence Lifetimes





## Evolution of Stars

1 Msol star - on main sequence

$\mathrm{K}=1 \mathrm{Z}=0.02$




## Evolution of Stars

1 Msol star - on sub-giant branch


## Evolution of Stars

1 Msol star - on red-giant branch



## Evolution of Stars

1 Msol star - core He flash at the tip of the RGB



## Evolution of Stars

1 Msol star - overview


## Evolution of Stars

1 Msol star - core He burning and H-fusion in envelope


## What powers stars on the main sequence?

Stars on the main sequence fuse hydrogen into helium via the PP-chain: $0.7 \%$ of each original proton is turned into energy, total 26.7 MeV per PPI cycle $\rightarrow$ most efficient nuclear process per nucleon ( 6.55 MeV ).


Relative importance (branching ratio) of PPI and PPII chains depends on H -burning conditions ( $T, \rho$, abundances). Transition from PPI to PPII occurs at $T \approx 1.3 \cdot 10^{7} \mathrm{~K}$.

Above $T \approx 3 \cdot 10^{7} K$ the PPIII chain dominates over the other two, but another process takes over in this case.

## What powers stars on the main sequence?

The majority of stars is born with some heavy elements, most of which are $C, N$, and $O$, which can act as catalysts for hydrogen fusion into helium through the CNO-cycle at $T \approx 3 \cdot 10^{7} \mathrm{~K}$ and above.

Also known as: CNO-bi-cycle, CNOF cycle, or CN+NO cycle, which becomes dominant at masses higher than $\sim 1.3 M_{\odot}$.


The slowest reaction is the p capture by ${ }^{14} \mathrm{~N}$. Hence most of ${ }^{12} \mathrm{C}$ is converted to ${ }^{14} \mathrm{~N}$.

The abundance in CNO-cycle processed matter leads to $C \downarrow, N \uparrow, O \downarrow$

## What powers stars after the M-S phase?

The simplest helium burning reaction is the triple- $\alpha$ cycle which occurs in stars above $\sim 0.5 M \odot$ and operates at $T \approx 1.2 \cdot 10^{8} \mathrm{~K}$ and above.


Since the first reaction is endothermic ( 92 keV ), the half-life time of ${ }^{8} \mathrm{Be}$ is very short $2.6 \cdot 10^{-16} s$ and, thus, is very sensitive to central $\rho$ and $T$.

This process yields one order of magnitude lower energy return per nucleon than the pp and CNO cycles. The He-burning phase is about 100 times shorter than the M-S stellar lifetime.

Also involved ${ }^{12} \mathrm{C}(\alpha, \gamma)^{16} \mathrm{O}$ and less likely ${ }^{16} \mathrm{O}(\alpha, \gamma){ }^{20} \mathrm{Ne}$.

## $\epsilon \propto T^{\eta}$



Large exponents, decreasing per-nucleon efficiencies, and dropping stellar core densities imply short lifetimes for higher-order fusion processes!

## The $r$ - and s-process

Interactions between nuclei and free neutrons (neutron capture) - the neutrons are produced during $\mathrm{C}, \mathrm{O}$ and Si burning and are not limited by the Coulomb barrier. Only limitation: neutron density!

$$
\begin{aligned}
& I(A, Z)+n \rightarrow I_{1}(A+1, Z) \\
& I_{1}(A+1, Z)+n \rightarrow I_{2}(A+2, Z) \\
& I_{2}(A+2, Z)+n \rightarrow I_{3}(A+3, Z) \quad . . \text { etc }
\end{aligned}
$$

If a radioactive isotope is formed after absorption of $N$ neutrons, it undergoes $\beta$-decay, creating a new element:

$$
I_{N}(A+N, Z) \rightarrow J(A+N, Z+1)+e^{-}+\bar{\nu}
$$

If new element is stable, it will resume neutron capture, otherwise may undergo series of $\beta$-decays

$$
\begin{aligned}
J(A+N, Z+1) & \rightarrow K(A+N, Z+2)+e^{-}+\bar{\nu} \\
K(A+N, Z+2) & \rightarrow \mathrm{L}(A+N, Z+3)+e^{-}+\bar{\nu} \quad \ldots \text { etc. }
\end{aligned}
$$

## The $r$ - and s-process

Rapid ( $r$ ) neutron capture: nucleus absorbs many neutrons prior to $\beta$ decay, requires high $n$ fluxes + energies: $->$ AGB star and SN envelopes


Slow (s) neutron capture: nucleus absorbs neutron ... then ... later ... nucleus undergoes $\beta$-decay


## The $r$ - and s-process


r-process departs from valley of stability
s-process crawls along the valley of stability

## What powers stars?

Common feature is release of energy by consumption of nuclear fuel. Rates of energy release vary enormously up to $\epsilon \propto T^{120}$ !!

| Nuclear <br> Fuel | Process | $\mathrm{T}_{\text {threshold }}$ <br> $10^{6} \mathrm{~K}$ | Products | Energy per nucleon (Mev) |
| :--- | :--- | :--- | :--- | :--- |
| H | PP | $\sim 4$ | He | 6.55 |
| H | CNO | 15 | He | 6.25 |
| He | $3 \alpha$ | 100 | $\mathrm{C}, \mathrm{O}$ | 0.61 |
| C | $\mathrm{C}+\mathrm{C}$ | 600 | $\mathrm{O}, \mathrm{Ne}, \mathrm{Ma}, \mathrm{Mg}$ | 0.54 |
| O | $\mathrm{O}+\mathrm{O}$ | 1000 | $\mathrm{Mg}, \mathrm{S}, \mathrm{P}, \mathrm{Si}$ | $\sim 0.3$ |
| Si | Nuc eq. | 3000 | $\mathrm{Co}, \mathrm{Fe}, \mathrm{Ni}$ | $<0.18$ |

Two Si nuclei can fuse to create ${ }^{56} \mathrm{Fe} \rightarrow$ the end of the fusion chain!

Nuclear processes can also absorb energy from the radiation field, which can lead to fission of heavier elements.

Note: Coulomb barrier at T above O-burning very high, but below that required for Si burning, photo-disintegration of Si takes place at $3 \times 10^{9} \mathrm{~K}$, lighter nuclei are absorbed by remaining Si and leakage occurs into heavier, stable(!) Fe-group nuclei, $\mathrm{Fe}, \mathrm{Co}, \mathrm{Ni} \rightarrow$ dead end.

## What powers stars at high masses?

Stars with successively higher masses ignite higher-order nuclear reaction chains in shells at ever higher $T$ and larger $\eta$. These processes are less efficient in heating the core and are very short lived $\rightarrow$ days!
Process are generally exotherm until the formation of a ${ }^{56} \mathrm{Fe}$ core.



Burning lifetimes become shorter and of the order of KH-timescale of particular shell $\rightarrow$ freeze-out.

## SN nucleosynthesis



Internal structure of a $15 \mathrm{Msol}_{\text {sol }}$ star at Fe-core collapse


Time $\longrightarrow$

## Local Abundance of Chemical Elements

In the solar neighborhood we find a characteristic abundance pattern that is the result of stellar and supernova nucleosynthesis enrichment.



## What can we learn from the chemical compositions of stellar populations?

Stars up to $8 M_{\text {sol }}$ lock most of their nucleosynthesis product in the white dwarf or neutron star remnants. The chemical evolution of galaxies is therefore dominated by supernovae (20-30 Msol):<br>© different supernovae types (core-collapse, degenerate C/O WDs, pair-instability)<br>- different SN types have different progenitor lifetimes


(2 Pair-Instability SNe occur "instantaneously" after progenitor formation (few Myr?)

- Core-collapse SNe detonate very early after onset of SF (after a few 10 Myr)
6 Thermo-nuclear SNe (delayed by ~1-3 Gyr -> only way to "unlock" the white dwarf nycleosynthetic products!)


## Abundance Chronometer

Different supernova types and various stars inject their thermonuclear ashes into interstellar space, from which new stars are formed. The past chemical evolution history is reflected in their stellar atmospheres.


Abundance patterns can provide strong constraints for star formation timescales and enrichment mechanisms. -> galaxy formation diagnostics!

## Evolution of Simple Stellar Populations

The evolution of a simple stellar population (SSP)
SSP = stellar generation that condensed out of a giant molecular cloud at one given time with a well-defined chemical composition*
will see stars evolve onto and off the M-S in the H-R diagram at very different time as f(stellar mass*).


## Evolution of Simple Stellar Populations

Stars of a given mass evolve along evolutionary tracks at various speeds. At any given time during the evolution of an SSP we can take a snapshot of the H-R diagram and derive the "isochrone".

Isochrone: L-T relation for stars of a defined age and chemical mix.



## Evolution of Simple Stellar Populations

Stellar evolution is not only a function initial stellar mass, but changes throughout the lifetime of a star. Later stellar evolutionary phases, that include nuclear fusion are generally much shorter than M-S lifetimes.


## Evolution of Simple Stellar Populations

The stellar spectral energy distribution (SED) changes as a function of stellar mass and chemical composition. Therefore, different stellar evolutionary phases (M-S, HB, RGB, AGB, etc.) contribute specific fractions of their total luminosity as a function of wavelength.


## Evolution of Simple Stellar Populations

The differential L-T evolution of the various stellar evolutionary phases, in particular the M-S turn-of relative to the Horizontal Branch enables us to derive the evolutionary parameters for that particular SSP.



## Evolution of Simple Stellar Populations

Survey of Milky Way Globular Clusters with HST.



Rosenberg et al. (1999)

## Evolution of Simple Stellar Populations

The comparison of theoretical predictions from SSP models with measurements via horizontal and vertical method gives relative ages



Rosenberg et al. (1999)

## Evolution of Simple Stellar Populations



Rosenberg et al. (1999)
Milky Way globular clusters appear to have an age dispersion of less than $15 \%$ of their absolute age.

There seems to be a weak trend of more metal-rich GCs being younger.

