8 Computing the masses of extra-solar planets

Let's consider a planet in a circular orbit around a distant star. For the moment we consider the convenient case where we view the plane of the planet's orbit from an edge on viewpoint (it keeps the maths simple at this stage).

The star-planet system is a two-body system and, via Newton's laws, we require that the linear momentum in the system is conserved (recall that momentum is mass × velocity). We may therefore write

$$m_{star}v_{star} = m_{planet}v_{planet}$$

We wish to compute the planet mass and we therefore rearrange this expression to obtain:

$$m_{planet} = \frac{m_{star}v_{star}}{v_{planet}}$$

Which quantities are the unknowns here? The colour or spectrum of the star reveals its spectral type. We can infer the mass from the spectral type – via stellar models and by comparing to stars of know spectral type in binary systems (where we can also compute the mass). The velocity of the star is given by the min/max velocity of the Doppler curve. Therefore, what about the velocity of planet?

Consider the planet moving about a circular orbit of radius a and period p. The velocity of the planet is

$$v_{planet} = \frac{2\pi a}{p}$$

with the orbital period equal to the period of the Doppler curve. However, we still require the orbital radius a.

To find this we return to Newton's original expression of Kepler's law of planetary orbits (i.e. $p^2 = a^3$). Newton wrote

$$p^2 = \frac{4\pi^2}{G(m_{star} + m_{planet})}a^3$$

where $G = 6.67 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$ is the gravitational constant. This expression can be simplified by assuming that $m_{planet} \ll m_{star}$, i.e.

$$\begin{aligned} p^2 &= \frac{4\pi^2}{Gm_{star}}a^3 \\ \text{or} \quad a &= \sqrt[3]{\frac{Gm_{star}p^2}{4\pi^2}}. \end{aligned}$$

Recall that we know the mass of the star and the orbital period. Rather than write a complicated expression, we will keep using the symbol a in the following formula, i.e.

$$m_{planet} = \frac{m_{star} v_{star} p}{2\pi a}.$$

We are now in a position to compute the mass of an extra-solar planet, so let's insert the appropriate values for 51 Pegasi b:

$$\begin{aligned} m_{planet} &= \frac{(2.12 \times 10^{30}\text{kg}) \times (57\text{ms}^{-1}) \times (3.65 \times 10^{5}\text{s})}{2\pi \times 7.82 \times 10^{9}\text{m}} \\ &\approx 9 \times 10^{26}\text{kg}, \end{aligned}$$

which is about one-half the mass of Jupiter.

8.1 Doppler measurements provide lower limits on the planet mass – why?

We previously considered the convenient case of an edge on planetary orbit. Let's now consider a planetary system inclined at some inclination angle i to the observer. The angle is defined in Figure 1 such that $i = 90°$ is the previous edge-on case. Measuring the inclination angle is only possible in Doppler systems that also transit. In other cases we cannot measure the inclination angle.

The orbital velocity of the star is confined to the orbital plane. When the observer is viewing at an angle with respect to the orbital plane the measured velocity, called the radial velocity, is related to the orbital velocity by the sine of the inclination angle, i.e.

$$v_{rad} = v_{star} \times \sin i.$$

Viewing an edge on system imples $i = 90°$, and $\sin 90° = 1$ with $v_{rad} = v_{star}$. For any case where $i < 90°$ we must have $v_{rad} < v_{star}$. As $m_{planet} \propto v_{star}$ we can now see that for $i < 90°$ we must have $m_{planet,obs} < m_{planet}$.

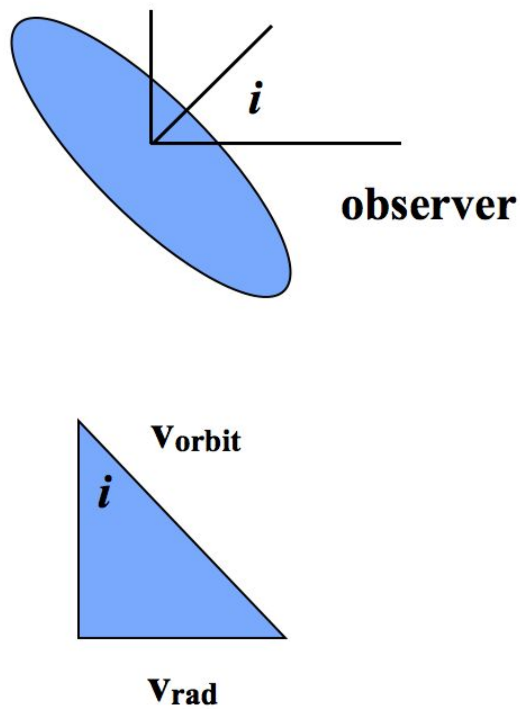

Figure 1: Computing the radial component of an inclined orbital velocity