1 Powers of 10 and scientific notation

Understanding powers of 10 allows us to express exceptionally large and small numbers in a concise form. A power, or exponent, applied to 10 simply tells us how many times to multiply 10 by itself, e.g.

$$\begin{aligned} 10^2 &= 10 \times 10 = 100 \\ 10^6 &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000. \end{aligned}$$

Similarly, a negative power of ten tells us how many times to divide by 10, e.g.

$$\begin{aligned} 10^{-2} &= \frac{1}{10 \times 10} = \frac{1}{100} = 0.01 \\ 10^{-6} &= \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{1,000,000} = 0.000001 \end{aligned}$$

The use of powers of 10 makes multiplying and dividing by large numbers very easy, e.g.

$$10^4 \times 10^7 = 10,000 \times 10,000,000 = 100,000,000,000 = 10^{11} = 10^{4+7}$$

$$10^5 \times 10^{-3} = 10,000 \times 0.001 = 100 = 10^2 = 10^{5+(-3)}$$

$$\begin{aligned} \frac{10^5}{10^3} &= \frac{100,000}{1,000} = 100 = 10^2 = 10^{5-3} \\ \frac{10^3}{10^7} &= \frac{1,000}{10,000,000} = \frac{1}{10,000} = 10^{-4} = 10^{3-7}. \end{aligned}$$

Scientific notation allows us to use powers of 10 to write down ordinary (yet very large or small) numbers in a concise form, e.g.

$$\begin{aligned} 3,042 &= 3.042 \times 10^3 \\ 0.00012 &= 1.2 \times 10^{-4} \\ 226 \times 10^2 &= (2.26 \times 10^2) \times 10^2 = 2.26 \times 10^4. \end{aligned}$$

We can see the use of scientific notation by considering a simple question: How long does it take light to travel from the surface of the Sun to the Earth? The Earth orbits the Sun at a mean distance of 148 million km, a distance defined as one astronomical unit (1 AU). The velocity of light is 300,000 km/s and is denoted by the symbol c. The time taken is equal to the distance divided

by the velocity. Let's make sure we use the international system of units for our calculation and put everything in scientific notation

$$t = \frac{d}{v} = \frac{1.48 \times 10^{11}\text{m}}{3 \times 10^{8}\text{ms}^{-1}} \approx 0.5 \times 10^{3}\text{s} = 8\,\text{minutes and}\,20\,\text{seconds}.$$