ASTR 508
Cosmology

Problem Set #2
Professor: Julio Navarro
Spring 2016

Note: This problem set is an assignment and should be returned to me for grading by Feb 25.

1. Consider two flat model universes, (a) a matter-dominated one and (b) another where matter and cosmological constant make up 30% and 70%, respectively, of the critical density.

   a. Compute and plot the angle subtended by a “standard” rod of 1 Mpc/h diameter as a function of redshift for each of the two model universes. Cover the range z=(0,1000) in a log-log plot. (5 points)
   b. At what redshift is the angle minimum for each model universe? Give a physical interpretation for why there is a minimum as well as for the significance of differences in the minimum value for the two model universes. (5 points)
   c. Compute the “particle horizon” as a function of time for each of the two model universes. Is this greater than c^*t_0? If so, how is this possible? Interpret. (5 points)
   d. Compare this with the proper (physical) distance to a source whose light, emitted at t, reaches us at $t=t_0$. Does this proper distance have a maximum? If so, when does it occur? Discuss this in relation to your answer in point (b) above. (5 points)
   e. Compare the apparent brightness of a “standard” candle in the two model universes as a function of redshift. Are standard candles always brighter or dimmer in one of the two model universes? Interpret the redshift dependence of the comparison. Ignore K-corrections. (5 points)
   f. Assume that a “cosmological chronometer” is found. This provides the age of the universe at present, which is found to equal exactly the inverse of Hubble’s constant. Is this consistent with a matter-dominated flat universe? What does this imply for the ratio of energies in the form of matter and dark energy? Compute and interpret. (5 points)

2. Consider that the universe is uniformly filled with “standard candles” with mean number density n_s and (constant) luminosity L_s.

   a. Show than in a flat static universe where sources have an infinite lifetime the flux received by an observer diverges (5 points)
   b. Use the FRW metric in an expanding universe and assume that the candles (whose comoving number density is fixed) turn on at some characteristic time t_s. Assume a flat, matter-dominated universe and compare the expanding case to the static one. Derive the suppression factor of background light as a function of t_s/t_0. Explain how this solves Olber’s paradox. (10 points)

3. One may estimate the total mass (“virial mass”, in the usual jargon) of a galaxy cluster by measuring the line-of-sight (rms) velocity dispersion of its galaxies (σ_{los}) and

its "half-mass" radius (r_h), via $M_{vir}=(C/G)*\sigma_{los}^2*r_h$, where C is a dimensionless constant and G is Newton's gravitational constant. Assuming two galaxy clusters, one at $z_1=0.25$ and another at $z_2=1$, in a flat Universe with $\Omega_m=0.25$, $\Omega_\Lambda=0.75$, h=0.72:

a) what would be the ratio of the (rms) dispersion in measured redshifts of the members of each cluster if their half mass radii subtend the same angle on the sky and the virial mass of both clusters is the same? (5 points)
b) what would be the virial mass ratio if both clusters have the same cluster member redshift dispersion and their half-mass radii span the same angle on the sky? (5 points)
c) derive the relation between redshift dispersion and angular size if both clusters have the same virial mass. (5 points)

4. Derive the proper (physical) distance at the present time (t=t0) for galaxies as a function of their observed redshift, z. Assume same cosmological model as in (3). (10 points)

5. Assuming the same cosmological model as in (3), and if the comoving number density of a particular kind of galaxy remains constant in time, $n(z)=n_0$,

a) how many fewer/more galaxies would you count, in a fixed solid angle $d\omega$ and per unit redshift interval dz, at z=2 compared with at z=1? (5 points)
b) graph the number count of galaxies per unit solid angle and per unit redshift as a function of redshift, and interpret its shape. (5 points)

6. The temperature of the cosmic microwave background photons scales like the inverse of the scale factor, $T_{CMB}(z)=T_0*(1+z)$. Assuming that, at the time of recombination (z~1000) there is an ideal gas (decoupled from the CMB) that has the same temperature as the CMB:

a) what would be the gas temperature today, if T_0=2.7 K? (5 points)
b) how does the pressure of the CMB change with z? (5 points)
c) how does the pressure of the gas change with z? Interpret this in terms of our usual assumption that a matter-dominated universe may be approximated by a pressure-less "dust" fluid. (10 points)