Progress in Multi-Dimensional Stellar Evolution

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Outline

- Astronomical Perspective
  - Challenges to Stellar Evolution: Convection, Double Diffusion, Boundary Layer Dynamics, Rotation, etc.
  - Goals: (a) Projection of multi-dimensional physics into a 1D model, (b) Inputs for supernovae simulation

- The Computational Approach

- Multi-Dimensional Models: Overview

- Turbulent, Compressible Convection

- Dynamics of Stable Layers – Internal Waves

- Boundary Processes - Entrainment

- Conclusions and Applications
Multi-Dimensional Stellar Evolution: Tools

Reactive Euler Equations: PROMPI

\[ \frac{\partial Q}{\partial t} + \nabla \cdot \Phi = S, \]  
\[ \text{with the state vector} \]
\[ Q = \begin{bmatrix} \rho \\ \rho u \\ \rho E \\ \rho X_i \end{bmatrix}, \]  
\[ \text{the flux vector} \]
\[ \Phi = \begin{bmatrix} \rho u \\ \rho uu + p \\ (\rho E + p)u + F_r \\ \rho X_i u \end{bmatrix}, \]  
\[ \text{and the source vector} \]
\[ S = \begin{bmatrix} 0 \\ \rho u \cdot g + \rho e_{net} \end{bmatrix}. \]  
where \( E = e_{int} + e_{ke} \) is the total energy per gram and \( \rho, p, u, g, \) and \( T \) are the density, pressure, velocity, gravitational force field and temperature.

Core Solver
- Fully compressible PPM Hydrodynamics, multi-species advection based on PROMETHEUS
- Cartesian, Spherical, Cylindrical Coordinates
- Domain decomposition for Parallel Computing (Message Passing with MPI)

Added Physics (Operator Splitting)
- Shellular Gravity
- Helmholtz Equation of State (Timmes & Swesty, 2000)
- General Nuclear Reaction Network
- Plasma Neutrino Cooling Rates (Itoh et al.)
- Radiative Transfer: Diffusive, OPAL Opacities (Implicit/Explicit Solver)

Initial Value Problem
- Initialization Routines: HSE Integrator, Star Mapper

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Initial Models

1D Stellar Evolution: TYCHO

- Open source project: http://chandra.as.arizona.edu/~dave/tycho-intro.html
- 1D (spherically symmetric)
- Hydrostatic, Hydrodynamic, Implicit and Explicit Solvers
- Diffusive Mixing Routines: Late Stage Convection, Semiconvection

- State of the art procedures and microphysics:
  - Opacities: OPAL and Alexander & Fergusson
  - General Nuclear Reaction Network: $N_{\text{species}} \sim 300$
    - Screening Corrections
    - Rates from Rauscher & Thielleman
    - New Urca rates: Martinez-Pinedo, Langanke

- Test bed for “non-standard” physics:
  - MLT+: (e.g., plume models, two-stream formalism, turbulence models)
  - Wave Physics
Initial Models

- **23 M\textsubscript{Sun} Carbon and Oxygen Shell Burning**

### Nuclear Reaction Network

<table>
<thead>
<tr>
<th>Element</th>
<th>Charge</th>
<th>Atomic Weight</th>
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<tbody>
<tr>
<td>Helium</td>
<td>2</td>
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<tr>
<td>Carbon</td>
<td>6</td>
<td>12</td>
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<tr>
<td>Oxygen</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Neon</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Sodium</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>Magnesium</td>
<td>12</td>
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<td>14</td>
<td>28</td>
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<tr>
<td>Phosphorus</td>
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<td>31</td>
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<tr>
<td>Sulfur</td>
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<td>32, 34</td>
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<td>Chlorine</td>
<td>17</td>
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<td>Argon</td>
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</tr>
<tr>
<td>Nickel</td>
<td>28</td>
<td>56</td>
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**Note.** — Network also includes electrons, protons, and neutrons.
Initial Models

Computational Domains

A.          B.          C.          D.          E.
## Initial Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>ob.3d.B</th>
<th>ob.3d.a</th>
<th>ob.2d.c</th>
<th>ob.2d.C</th>
<th>ob.2d.e</th>
<th>msc.3d.B</th>
<th>1D-MLT</th>
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<tr>
<td>$r_{in}, r_{out}$</td>
<td>$(10^7 \text{ cm})$</td>
<td>0.3, 1.0</td>
<td>0.3, 1.0</td>
<td>0.3, 1.0</td>
<td>0.3, 1.0</td>
<td>0.3, 5.0</td>
<td>90, 250</td>
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<tr>
<td>$\Delta \theta, \Delta \phi$</td>
<td>(deg.)</td>
<td>30, 30</td>
<td>7, 7</td>
<td>90, 0</td>
<td>90, 0</td>
<td>90, 0</td>
<td>30, 30</td>
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<td>Grid Zoning</td>
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<td>400×100×100</td>
<td>400×25×25</td>
<td>400×320</td>
<td>800×640</td>
<td>800×320</td>
<td>400×100×100</td>
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</tr>
<tr>
<td>$t_{max}$</td>
<td>(s)</td>
<td>650</td>
<td>400</td>
<td>574</td>
<td>450</td>
<td>2,400</td>
<td>10^6</td>
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</tr>
<tr>
<td>$\sigma_v$</td>
<td>(cm/s)</td>
<td>5×10^6</td>
<td>1.5×10^7</td>
<td>1.3×10^7</td>
<td>1.5×10^7</td>
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<tr>
<td>$v_{conv}$</td>
<td>(cm/s)</td>
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<td>$L_H$</td>
<td>(cm)</td>
<td>2×10^7</td>
<td></td>
<td>8×10^7</td>
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<td>(cm^2/s)</td>
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<td>$\tau_{visc}$</td>
<td>(s)</td>
<td>10^4</td>
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<td>$\tau_{cool}$</td>
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<td>Re</td>
<td>-</td>
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<td>Ra</td>
<td>-</td>
<td>&lt; 5×10^8</td>
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<tr>
<td>Pe</td>
<td>-</td>
<td>5×10^4</td>
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<td>Pr</td>
<td>-</td>
<td>10^2</td>
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<tr>
<td>$\text{Ri}_B(1)$</td>
<td>-</td>
<td>240</td>
<td>107</td>
<td>89</td>
<td>150</td>
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<td>$\text{Ri}_B(2)$</td>
<td>-</td>
<td>24</td>
<td>11</td>
<td>9</td>
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<tr>
<td>$\text{Ri}_B(3)$</td>
<td>-</td>
<td>...</td>
<td>...</td>
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<tr>
<td>$u_e(1)$</td>
<td>(cm/s)</td>
<td>5×10^3</td>
<td></td>
<td>1.8×10^4</td>
<td>1.7×10^4</td>
<td>1.2×10^3</td>
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<tr>
<td>$u_e(2)$</td>
<td>(cm/s)</td>
<td>5×10^4</td>
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<td>9.4×10^4</td>
<td>1.1×10^5</td>
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<tr>
<td>$u_e(3)$</td>
<td>(cm/s)</td>
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<td>...</td>
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</tr>
</tbody>
</table>

**Note.** — In our coordinate system $\phi$ and $\theta$ denote the polar and azimuthal angles, respectively. All models use stress-free reflecting boundary conditions in the radial direction and periodic boundary conditions in the angular directions.
Overview

Oxygen Shell Burning
Overview

Oxygen Shell Burning
Overview

Oxygen Shell Burning

Internal Waves

Convective Boundary Adjustment

Nuclear “Flame”
Overview

Carbon and Oxygen Shell Burning

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Overview

Carbon and Oxygen Shell Burning

Model: ob.2d.e (entire domain)  Time = 164 sec

Net Energy Generation [1e+13 erg/g/s]

Overview

Onwards Toward Core Collapse...

TYCHO Model: W3, M = 23 Msun

- Carbon/Neon Burning
- Oxygen Burning
- Silicon Burning
- Collapsing Iron Core
Overview

Model: si.2d.a  Time = 345 sec

Casey Meakin & David Arnett (2006) – Steward Observatory

Iron Core Burning
Silicon Burning
Oxygen Burning
Neon/Carbon Burning

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Overview

Coupling between Silicon burning and convective flow topology
Overview

Excitation of internal waves in main sequence “core convection” simulation…
Secular Evolution

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Turbulent, Compressible Convection: Basic Properties

2D Simulation

3D Simulation
Turbulent, Compressible Convection: Basic Properties

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Turbulent, Compressible Convection: Basic Properties
Velocity and average background stratification compares well with standard mixing length theory.

\[ \nu_{mix}^2 \sim \Delta \nabla l_m^2 \]

\[ F_{mix} \sim \Delta \nabla^{3/2} l_m^2 \]
Internal Waves: Comparison with Linear Modes
Thermodynamic Perturbations

Perturbations arise from internal waves:

- Localized at convective boundaries – trace density gradients
- Perturbations due to turbulence much smaller
Internal Waves: Comparison with Linear Modes Modal Structure

Upper Stable Layer

Convection Zone

Lower Stable Layer

Lower Stable Layer

Linear Eigenfunctions

$k\omega$ diagrams for various heights in domain of model ob.3d.B.

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Internal Waves: Comparison with Linear Modes Wave Forms
Internal Waves: Comparison with Linear Modes Amplitudes – Pressure Matching

Pressure perturbation for linear wave (Unno et al. 1989):

\[ \xi_h = \frac{1}{r \sigma^2} \left( \frac{p'}{\rho} + \Phi' \right) \]

Relationship to wave velocity amplitude:

\[ v_h = \xi_h \sigma \sim [l(l + 1)]^{1/2} \frac{p'}{r \sigma \rho} \]

Match wave pressure perturbation to turbulent ram pressure of the convection:

\[ p' = \rho \sigma v^2 \]

Pressures match at convective boundaries and we find \( v_h \sim 10^6 \) cm/s in agreement with simulation.

This matching demonstrates that convective motions drive the internal waves and not, e.g., nuclear overstability.

Must generalize to match turbulence spectrum to wave spectrum: e.g., Goldreich & Keeley (1977)

Multi-D Stellar Evolution, Casey A. Meakin, July 2006
Internal Waves: Comparison with Linear Modes Amplitudes – Pressure Matching

The displacement amplitudes are generally larger \textbf{above} a convective shell since the allowed mode frequencies are lower. By how much will depend on the turbulent ram pressure in the convection zone.

\[ \xi_h = \frac{1}{r \sigma^2} \left( \frac{\rho'}{\rho} + \Phi' \right) \]

Large asymmetries may play an important role in seeding instabilities during \textit{supernova shock breakout} and will lead to \textit{non-spherical mass accretion} during core-collapse and shock revival.

The thermodynamic perturbations are generally larger \textbf{below} a convective shell where gradients are larger. Internal waves sent into the underlying core will affect heating/cooling rates and may drive turbulent mixing.
Turbulent Entrainment in Stratified Layers: Basic Considerations

Mean Shear Flow $U(z)$

Internal Waves

Boundary Layer

Stratification $\rho(z)$

Turbulence $\sigma_H, L_H$

After Strang and Fernando (JFM, 2001)
The **Bulk Richardson** number is a measure of the potential energy in stratification relative to adjacent turbulent kinetic energy, and is written:

$$Ri_B = \frac{\Delta b L_H}{\sigma^2_H}$$

with buoyancy jump,

$$\Delta b = \int \nu_B^2 \, dr$$

and buoyancy frequency,

$$\nu_B^2 = g \left( \frac{\partial \ln \rho}{\partial r} - \frac{\partial \ln \rho}{\partial r} \bigg|_{s, \mu} \right)$$

The **Gradient Richardson** number is a measure of the potential energy in stratification relative to the kinetic energy of an adjacent shear flow:

$$Ri = \frac{\nu_B^2}{(\partial u/\partial r)^2}$$

In planar shear flow instability occurs when $Ri < 0.25$. 
Turbulent Entrainment in Stratified Layers: No Shear - *Bulk* Richardson Number Regimes

*Richardson Number Regimes*

**$Ri > Ri_c$**
- Strong stratification: “stiff” stable layer
- Mixing through molecular diffusion and shear instability

**$Ri \sim Ri_c$**
- Moderately “stiff” stratification
- Entrain *filaments*
- Turbulent convection excites internal waves: mixing through “wave-breaking”

**$Ri < Ri_c$**
- Weak stratification
- Mixing through *plume engulfement*
- Transient state for indelible mixing process (e.g., compositional mixing in most cases)

*After Carruthers & Hunt (1986), JFM*
Turbulent Entrainment in Stratified Layers: Results

Measuring the entrainment rate, $u_e$
Turbulent Entrainment in Stratified Layers: Results

Measuring the entrainment rate, $u_e$
Turbulent Entrainment in Stratified Layers: Theory

Relationship between the buoyancy profile and the “buoyancy flux” $q$.

\[
\begin{align*}
\partial_t b &= -\nabla \cdot q \quad (24) \\
\frac{\partial t}{b} &= \int N^2 dr \quad (25) \\
\partial_t b &= u_e \partial_r b = u_e N^2 \quad (26) \\
\Delta q &= \frac{\frac{\partial q}{\partial r}}{hN^2} \quad (27)
\end{align*}
\]
Turbulent Entrainment in Stratified Layers: Theory

Determining the “buoyancy flux” requires a Turbulence model.

1 Turbulent Kinetic Energy Equation

The turbulent kinetic energy equation can be derived from the equation of motion,

$$\rho D_t u = -\nabla p + \rho g$$

with the material derivative,

$$D_t = \partial_t + u \cdot \nabla.$$  \hspace{1cm} (1)

It is convenient to decompose the velocity, density, and pressure fields into mean and fluctuating components:

$$u = u_0 + u'$$

$$p = p_0 + p'$$

$$\rho = \rho_0 + \rho'.$$  \hspace{1cm} (3)

The equation of motion can then be written,

$$D_t (u_0 + u') = -\frac{1}{\rho} \nabla p' + \frac{\rho' g}{\rho}$$

where we have assumed the hydrostatic equilibrium condition,

$$\nabla p_0 = -\rho_0 g.$$  \hspace{1cm} (7)

The turbulent kinetic energy evolution is found by forming the scalar product of equation (6) with the fluctuating component of the velocity,

$$u' \cdot D_t u = -u' \cdot \frac{1}{\rho} \nabla p' + q$$

where we have defined the buoyancy flux,

$$q = \frac{u' \cdot \rho' g}{\rho} = -\frac{u' \cdot \rho' g}{\rho}.$$  \hspace{1cm} (9)

The left hand side of equation 8 can be expanded to read,

$$\partial_t \frac{1}{2} |u'|^2 + u' \cdot \partial_t u_0 + u' \cdot (u_0 \cdot \nabla u_0) + u' \cdot (u' \cdot \nabla u_0) + u' \cdot (u_0 \cdot \nabla u') + u' \cdot (u' \cdot \nabla u').$$

Time averaging eliminates terms which are first order in the velocity fluctuation,

$$\bar{u'} \cdot D_t \bar{u} = \partial_t \frac{1}{2} |\bar{u}'|^2 + \bar{u'} \cdot (\bar{u}' \cdot \nabla \bar{u}_0) + \bar{u'} \cdot (\bar{u}_0 \cdot \nabla \bar{u}') + \bar{u'} \cdot (\bar{u}' \cdot \nabla \bar{u}').$$  \hspace{1cm} (11)

We now consider each of the terms with spatial derivatives separately. The first term is,
\[ u' \cdot (u' \cdot \nabla u_0) = u' \cdot \left( \begin{array}{c} u'_r \cdot \nabla u_{0(r)} \\ u'_\theta \cdot \nabla u_{0(\theta)} \\ u'_\phi \cdot \nabla u_{0(\phi)} \end{array} \right) \tag{12} \]

which can be expanded to read,

\[ u' \cdot (u' \cdot \nabla u_0) = u' \cdot \left( \begin{array}{c} u'_r \partial_r u_{0(r)} + u'_r r^{-1} \partial_\theta u_{0(r)} + u'_r (r \sin \theta)^{-1} \partial_\phi u_{0(r)} \\ u'_\theta \partial_\theta u_{0(\theta)} + u'_\theta r^{-1} \partial_\theta u_{0(\theta)} + u'_\theta (r \sin \theta)^{-1} \partial_\phi u_{0(\theta)} \\ u'_\phi \partial_\phi u_{0(\phi)} + u'_\phi r^{-1} \partial_\theta u_{0(\phi)} + u'_\phi (r \sin \theta)^{-1} \partial_\phi u_{0(\phi)} \end{array} \right) \tag{13} \]

and finally,

\[ u' \cdot (u' \cdot \nabla u_0) = \frac{u'_r u'_r \partial_r u_{0(r)}}{u'_r u'_r \partial_r u_{0(r)} + u'_r u'_r r^{-1} \partial_\theta u_{0(r)} + u'_r u'_r (r \sin \theta)^{-1} \partial_\phi u_{0(r)}} + \frac{u'_\theta u'_\theta \partial_\theta u_{0(\theta)}}{u'_\theta u'_\theta \partial_\theta u_{0(\theta)} + u'_\theta u'_\theta r^{-1} \partial_\theta u_{0(\theta)} + u'_\theta u'_\theta (r \sin \theta)^{-1} \partial_\phi u_{0(\theta)}} + \frac{u'_\phi u'_\phi \partial_\phi u_{0(\phi)}}{u'_\phi u'_\phi \partial_\phi u_{0(\phi)} + u'_\phi u'_\phi r^{-1} \partial_\theta u_{0(\phi)} + u'_\phi u'_\phi (r \sin \theta)^{-1} \partial_\phi u_{0(\phi)}} \tag{14} \]

These terms represent the production of turbulent energy by the Reynolds stresses \(-\rho u'_i u'_j\) working against the mean flow. This term simplifies in the present model because only the radial component of the mean velocity is expected to be significant, \((u_0 = (u_{0(r)}, 0, 0))\) and to depend only on radial coordinate \((u_{0(r)} = u_{0(r)}(r))\),

\[ u' \cdot (u' \cdot \nabla u_0) = u'_r u'_r \cdot \nabla u_{0(r)} = u'_r u'_r \partial_r u_{0(r)}. \tag{15} \]

Next we consider the term,

\[ u' \cdot (u_0 \cdot \nabla u') = u'_r u_{0(r)} \partial_r u'_{0(r)} + u'_r u_{0(\theta)} r^{-1} \partial_\theta u'_{0(r)} + u'_r u_{0(\phi)} (r \sin \theta)^{-1} \partial_\phi u'_{0(\phi)} + \]

\[ u'_\theta u_{0(\theta)} \partial_\theta u'_{0(\theta)} + u'_\theta u_{0(\theta)} r^{-1} \partial_\theta u'_{0(\theta)} + u'_\theta u_{0(\phi)} (r \sin \theta)^{-1} \partial_\phi u'_{0(\phi)} + \]

\[ u'_\phi u_{0(\phi)} \partial_\phi u'_{0(\phi)} + u'_\phi u_{0(\theta)} r^{-1} \partial_\theta u'_{0(\phi)} + u'_\phi u_{0(\phi)} (r \sin \theta)^{-1} \partial_\phi u'_{0(\phi)} \tag{16} \]

which can be compactly written,

\[ u' \cdot (u_0 \cdot \nabla u') = u_0 \cdot \nabla \frac{1}{2} |u'|^2 \tag{17} \]

The last term can be simplified in the same manner as the previous one,

\[ u' \cdot (u' \cdot \nabla u') = u' \cdot \nabla \frac{1}{2} |u'|^2 \tag{18} \]

and the final expression for the evolution of the turbulent kinetic energy is

\[ \left( \partial_t + u_{0(r)} \partial_r \right) \frac{1}{2} |u'|^2 = -\nabla \cdot u'_r \partial_r u_{0(r)} - u' \cdot \left( \nabla \frac{1}{2} |u'|^2 + \frac{1}{\rho} \nabla p \right) + \eta - \epsilon. \tag{19} \]
Turbulent Entrainment in Stratified Layers: Theory

Other physics, including internal wave transport, and turbulent kinetic energy flux.

We rewrite this equation as,

\[
\left( \frac{\partial}{\partial t} + u_0(\tau) \frac{\partial}{\partial \tau} \right) K = -u'_r u'_\tau \frac{\partial}{\partial \tau} u_0(\tau) - u' \cdot \left( \nabla K + \frac{1}{\rho} \nabla p' \right) + \frac{\epsilon}{\bar{q}} \quad (20)
\]

where we have defined the turbulent kinetic energy (TKE) per unit mass,

\[
K = \frac{1}{2} |u'|^2. \quad (21)
\]

The advection term on the right hand side of equation 20 can be split into two terms that have simple physical interpretations,

\[
u' \cdot \left( \nabla K + \frac{1}{\rho} \nabla p' \right) = \nabla \cdot F_K + \frac{1}{\rho} \nabla \cdot F_p - \nabla \cdot u' \left( \frac{p'}{\rho} + K \right). \quad (22)
\]

The first term on the right hand side is the divergence of the energy flux transported by the velocity fluctuations. We have defined the turbulent energy flux as,

\[
F_K = u'K \quad F_p = u'p'. \quad (23)
\]
Turbulent Entrainment in Stratified Layers: 
“Empirical” Entrainment Law

\[ E \equiv \frac{u_e}{\sigma_H} = a_1 Ri_B^{-n} \]

Best fit curve:
\[ a_1 = 0.287 \]
\[ n = 1.10 \]
Conclusions and Applications

- 3D stellar convection is consistent with MLT scalings - 2D is **NOT** a good surrogate for full 3D simulation.

- Stable layer dynamics well described by linear wave equation – amplitudes consistent with wave pressure matching turbulent ram pressure of convection – results in large thermodynamic perturbations and geometric distortions of convective boundaries.

- Simulated convective boundary mixing well described by a universal entrainment law with important implications for pre-supernova core evolution \((a_1 \sim 1, n \sim 1)\).

- A theory of entrainment requires a turbulence model in order to calculate the “buoyancy flux”.

- Astrophysical implications: (a) Projection to 1D model to assess astrophysical implications with TYCHO (Arnett), (b) Using multi-dimensional models as inputs to core-collapse simulation (Burrows et al.; Fryer et al.).

- Proof of principle 3D calculation with rich dynamics – basis for: (a) rotating, (b) magnetic, and (c) larger domain models.