1 Introduction

A good working definition of a galaxy is a population of stars, gas and dust bound gravitationally within a dark matter halo.

Set Christianson text as reading on the early history of galaxy studies and Hubble's confirmation that they are stellar systems external to our own Galaxy.

1.1 Morphology: the two-dimensional physical appearance of galaxies

The first systematic attempts to study galaxies focussed upon understanding the range of shapes they presented.

Hubble - though not the first - presented what is arguably the most complete yet simple classification of galaxies based upon their visible appearance: the Hubble sequence.

The sequence is an interesting mix of quantitative and qualitative criteria: smooth, elliptical galaxies showing no evidence of a disk are labelled with a number of 0 to 7 with the number computed as 10 x (1 - b/a), where b and a are the minor and major axis lengths.

An S0 or lenticular galaxy is a bulge dominated galaxy with a smooth, purely stellar disk.

Spiral galaxies are classified by more qualitative criteria such as the prominence and tightness of spiral arms and the presence of a central bar.

Hubble's system is simple yet subjective - in that it is dependent upon human visual classification. However, as numerous supporters of visual classification have remarked, the eye is exceptionally adept at recognizing and classifying two dimensional structures.

Attempts have been made to quote Hubble's scheme as a numerical sequence, such as T-types. Physical quantities such as relative fractions of bulge and disk light do correlate with T-type. However, T-types remain a subjective, relative classification scheme, i.e. there is no absolute "elliptical" or "spiral" reference. There is no absolute "ground truth" for purely visual classification schemes.

Numerous "machine-based" or automated morphological classification schemes have been proposed.

One such is the asymmetry and concentration (or AC) scheme of Abraham et al. (1994).

Ultimately all classification schemes, whether human or machine based, are fallible. However, machine based schemes are certainly more repeatable, i.e. you and I can run the same classification script even though our by-eye results may not agree.

A notable development in this field though is the advent of Galaxy Zoo (Lintott et al. 2008): a large human based classification of the morphology of SDSS galaxies. The size of the data set and hard work of the classifiers has allowed serious attempts to be made to understand the biases involved in human classification.

At the end of this discussion one broad result should be emphasized - what may be termed the morphology-colour relation: elliptical galaxies are typically redder in colour than spirals. This phenomenon is also known as colour-bimodality.

Simply put, the forces that shape the morphology of a galaxy must also influence its star formation history. For, as we shall see later, the integrated star formation history is the prime driver of its integrated colour.

1.2 Surface brightness profiles

One could think of this as one-dimensional morphology where the intensity per unit surface area is expressed via an analytic function.

Surface brightness data is generated by averaging along elliptical isophotes to generate 1D elliptically averaged light profiles.

Generally, features such as spiral arms and dust lanes average out in this process (though as will be seen some ripples and bumps can remain).

Spiral galaxy disks display surface brightness profiles that are well fit by exponential intensity profiles whereas elliptical galaxies are fit by a de Vaucouleurs (1948) or $R^{1/4}$ intensity profile (see maths notes).

In practice many galaxies display a surface brightness profile that is best described by a superposition of these two intensity profiles. This is achieved by fitting a galaxy with each profile applied over a selected range of radii, e.g. $R^{1/4}$ over inner radii and exponential over outer radii. Such an approach is referred to as bulge-disk decomposition.

A closely related topic is the idea of the total size of a galaxy.

The detectable extent of a galaxy on a detector is determined by the radius at which the surface brightness approaches the effective surface brightness of the combined sky plus detector noise. This isophotal area can be thought of as a lower limit on the size of a galaxy.

How bright is a galaxy?

One could simply integrate the relevant surface brightness profile to infinite radius. However, this a) requires obtaining an accurate determination of the SB profile and b) assuming that the profile can be reasonably extrapolated beyond the isophotal radius.

An alternative, practical approach is to determine a radius based upon a statistical analysis of the light distribution in the galaxy itself. This has the advantage that the radius is model independent and can be computed for galaxies where only a few tens (or less) of pixels are detected above the isophotal threshold. This is the idea behind the Kron (1980) and Petrosian (1976) radii.

1.2.1 Surface brightness maths notes

Surface brightness is quoted in $L_\odot \mathrm{pc}^{-2}$, i.e. $1 L_\odot \mathrm{pc}^{-2} = (3.86 \times 10^{24}\mathrm{W})/(3.09 \times 10^{16}\mathrm{m})^2 = 4.05 \times 10^{-7}\mathrm{Wm}^{-2}$.

The B-band solar magnitude is $M_{B,\odot} = 5.48$ and 1 pc at 10 pc subtends 0.1 radians (20626.5 arcseconds).

Expressing $L_\odot \mathrm{pc}^{-2}$ in magnitudes per square arcsecond gives $\mu_B = -2.5 \log(L_\odot/\mathrm{pc}^2) = \mathrm{M}_{\mathrm{B},\odot} + 5 \log(20626.5) = 27.05$ B magnitudes per square arcsecond (at 10 pc).

Surface brightness can be expressed as $L_{Total}/2\pi R_e^2$ where R_e is the half-light radius.

Using this simple definition, both spirals and ellipticals display surface brightnesses of order 100 $L_\odot \mathrm{pc}^2$ or $\mu_B = 22$.

Central values of surface brightness can differ greatly, e.g. of order a few hundred $L_\odot \mathrm{pc}^{-2}$ for spirals to $\sim 10^4$ $L_\odot \mathrm{pc}^{-2}$ for ellipticals.

Exponential (spiral) intensity profile:

$$I(R) = I_0 \exp(-R/a), \tag{1}$$

where I_0 is the central intensity and a is a scale length. Expressed in magnitudes

$$\mu(R) = \mu_0 + 1.086(R/a), \tag{2}$$

where $1.086 = 2.5 \log e$.

de Vaucouleurs (elliptical) intensity profile:

$$I(R) = I_0 \exp(-(R/a)^{1/4}) \tag{3}$$

$$\mu(R) = \mu_0 + 1.086(R/a)^{1/4} \tag{4}$$

These profiles can be generalised to the Sersic profile:

$$I(R) = I_0 \exp(-(R/a)^{1/n}) \tag{5}$$

$$\mu(R) = \mu_0 + 1.086(R/a)^{1/n} \tag{6}$$

where $n \geq 0$ is a real number.

Total luminosity: $L_T = \int_0^\infty 2\pi R I(R) \mathrm{d}R$.

Half light radius: $L(R_e) = \int_0^{R_e} 2\pi R I(R) \mathrm{d}R = 0.5 L_T$.

Kron radius:

$$R_K = \frac{\int R^2 I(R) \mathrm{d}R}{\int R I(R) \mathrm{d}R}. \tag{7}$$

Typically, $L(2.5 \times R_K) \sim 90\%\ L_T$.

Petrosian radius:

$$I(R_P) = 0.2 \frac{L(R_P)}{\pi R_P^2}. \tag{8}$$

One inverts the equation to obtain R_P. Typically, $L(2 \times R_P) \sim 90\%\ L_T$.

1.3 Stellar populations

A galaxy may be thought of as a population of stars. The distribution of stars can be visualised on the Hertzprung-Russell diagram with axes of either surface temperature versus luminosity or colour versus magnitude.

The spectral energy distribution (colour) of the galaxy may then be computed as the luminosity weighted integral over the spectral energy distribution (colour) of its constituent stars, e.g.

$$\bar{T} = \frac{\sum L_i T_i}{\sum L_i}. \tag{9}$$

Simple stellar populations of single age and chemical composition are best observed in star clusters. One can then observe directly how differing stellar ages and chemical compositions determine the integrated properties of a stellar population.

From this it is a small conceptual leap to understand that the spread of stellar population ages and chemical compositions - effectively the galaxy star formation history - are the principle drivers of the observed spectral energy distribution.

In this sense, we previously noted that ellipticals are red and spirals are blue. From the above discussion we can now understand that ellipticals are composed of predominantly old stars with little ongoing star formation and spirals are composed of younger, actively forming stellar populations.

1.4 The luminosity function

The luminosity function (LF) describes the space density of galaxies per unit luminosity as a function of luminosity.

Galaxy populations are almost universally described by the Schechter (1976) function which can be thought of as a composite function consisting of a faint end power law of slope alpha and a bright exponentially declining form.

The LF can be applied across all galaxy types or split by galaxy properties (e.g. Binggeli, Sandage and Tammann 1988).

Mulitplying the LF by a single M/L ratio (see Dark Matter below) generates a very simple galaxy mass function.

Why does the crude galaxy mass function deviate from the expectation of a dark matter mass function at the high- and low-mass ends?

This is similar to the question of what determines the maximum and minimum mass of a galaxy? Answers include SN driven mass loss and AGN moderated feedback.

1.4.1 Schechter function maths notes

Schechter function:

$$\phi(L)\,dL = \phi^*(L/L^*)^\alpha \exp[-L/L^*]\,d(L/L^*) \tag{10}$$

where ϕ^* is a characteristic density, L^* is a characteristic luminosity and α is a faint-end slope.

The Sloan Digital Sky Survey provides the most reliable knowledge of the galaxy luminosity function (Blanton et al. 2003):

$\phi^* = 1.49 \times 10^{-2}$gals h^{3}Mpc^{-3}.

$M^* - 5\log h = -20.44$ in the $^{0.1}r$-band.

$L^*_{^{0.1}r} = 1.2 \times 10^{10} L_\odot$.

$\alpha = -1.05$.

The total density of galaxies is:

$$\Phi = \int_0^\infty \phi(L')\,dL' = \Gamma(\alpha+1)\,\phi^*, \tag{11}$$

where $\Gamma(a)$ is the Gamma function. Note that this integral diverges for $\alpha \le -1$, in which case the alternative form should be used

$$\Phi(>L) = \int_L^\infty \phi(L')\,dL' = \Gamma(\alpha+1, L/L^*)\,\phi^*, \tag{12}$$

where $\Gamma(a,b)$ is the incomplete Gamma function.

The luminosity density of galaxies is

$$\mathcal{L} = \int_0^\infty L\ \phi(L)\ \mathrm{d}L = \phi^* L^* \Gamma(\alpha+2). \tag{13}$$

Taking $\alpha = -1$ gives $\Gamma(\alpha+2) = \Gamma(1) = 1$ and $\mathcal{L} = \phi^* L^* = 1.8 \times 10^8 L_\odot \mathrm{Mpc}^{-3}$.

1.5 Redshift surveys

Large galaxy surveys typically consist of wide field imaging in order to measure the positions and brightnesses of galaxies, together with spectroscopy to measure their redshifts.

Early pioneering work was performed in the early 1980s by the CfA redshift survey of some 3,500 galaxies. The current state of the art is the Sloan Digital Sky Survey (SDSS) having imaged one quarter of the sky (10,000 deg2) in five optical bands and obtained spectra of one million galaxies.

Redshift surveys are used to construct the galaxy luminosity function:

1. Measure apparent magnitudes and redshifts.

2. Convert to absolute magnitudes: $M_\lambda = m_\lambda - 25 - 5\log d_L(z) - K_\lambda(z) + A_\lambda$.

   Where $d_L(z)$ is the luminosity distance versus redshift relation for the appropriate cosmological model, $K(z)$ is the k-correction (see below) and A is the Galactic extinction. The subscript λ refers to a generic band.

3. k-corrections:

$$\begin{aligned} K_\lambda(z) &\equiv M(z) - M(z=0) \\ &= -2.5\log\left[\frac{\int_0^\infty S(\lambda/1+z)R(\lambda)\,\mathrm{d}(\lambda/1+z)}{\int_0^\infty S(\lambda)R(\lambda)\,\mathrm{d}\lambda}\right] \\ &= 2.5\log(1+z) - 2.5\log\left[\frac{\int_0^\infty S(\lambda/1+z)R(\lambda)\,\mathrm{d}\lambda}{\int_0^\infty S(\lambda)R(\lambda)\,\mathrm{d}\lambda}\right] \end{aligned} \tag{14}$$

   For a spectrally flat SED, the second term equals zero.

4. Compute the LF using an appropriate estimator, e.g. V/V_{acc}:

   Accessible volume: $V_{acc} = \mathrm{d}\Omega_{survey}\int_0^\infty(\mathrm{d}V_c/\mathrm{d}z)W(z)\mathrm{d}z$.

   Where $\mathrm{d}\Omega_{survey}$ is the survey solid angle, $\mathrm{d}V_c/\mathrm{d}z$ is the comoving volume element and $W(z)$ is a selection function.

   Density $\phi_i = 1/V_{acc,i}$.

   LF constructed by summing the ϕ_i in a series of magnitude bins. Schechter function parameters obtained by a χ^2 fit to the binned LF data.
   Enclosed volume: $V_{enc} = \mathrm{d}\Omega_{survey}\int_0^z(\mathrm{d}V_c/\mathrm{d}z)W(z)\mathrm{d}z$.

   $\langle V_{enc}/V_{acc}\rangle = 0.5$ for a non-evolving population.

1.6 Dark matter

Dark matter in galaxies was first revealed by the study of their rotation curves, i.e. a resolved measurement of circular velocity as a function of radius.

The rotation curve provides a dynamical measurement of the projected galaxy mass.

Observed rotation curves were flat at large radii - a result inconsistent with a mass profile obtained by scaling the galaxy light profile by a suitable stellar mass to light ratio.

The data called for large amounts (up to 40x more than the stellar mass) of dark matter at large galactic radii - the dark matter halo.

The incidence of flat rotation curves is ubiquitous across spiral galaxies of differing morphological type. This indicates that the presence of a dark matter halo is common to all spiral galaxies.

Elliptical galaxies also exist in dark matter halos. The clearest evidence for dark matter in elliptical galaxies is provided by gravitational lensing whereby the dark halo distorts the light from a background galaxy.

1.6.1 Mass to light ratios

The B-band luminosity of the Sun is $L_{\odot,B} = 4.7 \times 10^{25}$W. The mass of the Sun is $M_{\odot} = 2 \times 10^{30}$kg. Therefore the B-band mass-to-light ratio of the Sun is

$$\frac{M_{\odot}}{L_{\odot,B}} \approx 4.25 \times 10^{4} \text{ kg W}^{-1}. \tag{15}$$

The mass-to-light ratio of the Sun provides a basic unit, i.e. one solar mass of stars liberating one solar mass of B-band luminosity possesses $\langle M/L_B \rangle = 1$.

Over the spectral sequence of stars (OBAFGKM) the mass-to-light ratio varies from $\langle M/L_B \rangle \sim 10^{-3}$ for the bright, massive O stars to $\langle M/L_B \rangle \sim 10^{3}$ for faint, low mass M stars. The average value of the stellar mass-to-light ratio with 1 kpc of the Sun (the Solar neighbourbood) is $\langle M/L_B \rangle = 4$.

From the earlier LF analysis the total luminosity density of stars is $j_{\star,B} = L_B/V = 1.2 \times 10^{8}\ \mathrm{L_{\odot,B}Mpc^{-3}}$.

Assuming that the mean stellar mass-to-light ratio in the solar neighbourhood is representative of this larger volume then the mean stellar mass density is $\rho_{\star} = \langle M/L_B \rangle\, j_{\star,B} \approx 5 \times 10^{8} \mathrm{M_{\odot}Mpc^{-3}}$.

If we go further and assume that the stellar mass density in the local volume is representative of the universe as a whole then the normalised stellar mass density of the universe is

$$\Omega_{\star,0} = \frac{\rho_{\star,0}}{\rho_{c,0}} \approx \frac{5 \times 10^8 \mathrm{M}_\odot \mathrm{Mpc}^{-3}}{1.4 \times 10^{11} \mathrm{M}_\odot \mathrm{Mpc}^{-3}} \approx 0.004. \tag{16}$$

So normal stars make up only 0.5% of the density required to generate a spatially flat universe. Consideration of BBN and CMB analyses indicates that $\Omega_b = 0.04$ and we immediately confront the fact that most of the baryons in the universe do not exist in the form of "normal" stars. As we shall see later, additional baryons in galaxies exist as low-mass stars such as brown dwarfs, stellar remnants such as faint white dwarfs and diffuse clouds of hot gas – the warm/hot interstellar medium – WHIM. On the scale of galaxy clusters, the dominant structures in the universe, we shall see that most of the baryons exist in the form of a X-ray emitting plasma.

1.6.2 Rotation curves

The Milky Way galaxy is a spiral galaxy. The Sun moves around the Galactic centre on an approximately circular orbit of radius $R = 8.5$ kpc and velocity $v = 220$ kms^{-1}. If the orbit is stable then the outward centripetal acceleration is balanced by the inward acceleration due to gravity, i.e.

$$\frac{v^2}{R} = \frac{GM(R)}{R^2} \tag{17}$$

$$\text{or} \quad v = \sqrt{\frac{GM(R)}{R}}, \tag{18}$$

where $M(R)$ is the mass of the galaxy contained within a radius R.

A flat rotation curve indicates $M(R) \propto R$.

What density profile does this imply? If we write that the density $\rho = M/V \propto R/R^3 \propto R^{-2}$, which is isothermal.

From Equation 18 one can write the mass enclosed within a radius R as

$$M(R) = \frac{v^2 R}{G} = 9.6 \times 10^{10} \mathrm{M}_\odot \left(\frac{v}{220 \mathrm{kms}^{-1}}\right)^2 \left(\frac{R}{8.5 \mathrm{kpc}}\right), \tag{19}$$

where the values scale the mass enclosed to that within the Sun's location within the Galaxy. If we assume a maximum radius for the halo of matter surrounding the Galaxy and a assymptotic velocity value we can determine the halo mass, i.e. taking $v = 220$ kms^{-1} and $R_{halo} = 100$ kpc we obtain $M_{halo} = 1.1 \times 10^{12} \mathrm{M}_\odot$. The B-band luminosity of the Galaxy is $L_{MW,B} = 2.3 \times 10^{10} \mathrm{L}_{\odot,\mathrm{B}}$ and therefore

$$\langle M/L_B \rangle_{MW} \approx 50 \langle M/L_B \rangle_\odot \left(\frac{R_{halo}}{100 \text{ kpc}} \right). \tag{20}$$

So how big is the halo of the Milky Way? Assuming the outermost globular clusters and satellites such as the Large and Small Magellanic Clouds are bound to the Milky Way then the halo extends to 75 kpc and $\langle M/L_B \rangle_{MW} \approx 40 \langle M/L_B \rangle_\odot$. If the Milky Way extends some four times further, i.e. to 300 kpc and halfway to M31, then we obtain $\langle M/L_B \rangle_{MW} \approx 150 \langle M/L_B \rangle_\odot$. The implies that the Milky Way contains some 10 to 40 times more mass than can be accounted for by its visible stars. Should the Milky Way be typical in this respect then we can write the normalised mass density in galaxies as

$$\Omega_{gal} = (10 \rightarrow 40) \Omega_\star \approx 0.04 \rightarrow 0.16. \tag{21}$$

Although this simple prediction contains many assumptions we are therefore forced to contemplate the fact that some fraction of the dark halos surrounding galaxies is composed of non-baryonic dark matter.

1.7 Structural overview

The visible portion of a galaxy is located within a much larger, more massive dark matter halo. Globular clusters and diffuse clouds of neutral hydrogen orbit in the galactic halo. These provide important clues as to the formation of galaxies that we will discuss later.

The nucleus of the Milky Way galaxy is believed to contain a supermassive black hole of order 3e6 solar masses.

The black hole is associated with the radio sources Sgr A* and is the presence of an unseen massive object is revealed via stellar orbits.

Flaring in X-rays and IR observations is thought to reveal the signature of material falling into the BH near to the event horizon.

1.8 Active galactic nuclei

The black hole at the centre of the Milky Way is essentially quiescent.

Black holes are now thought to be a common feature in the nucleus of all massive galaxies (perhaps even all galaxies).

Active (as opposed to quiescent) galactic nuclei are associated with highly energetic processes originating in the central nucleus. Observed features include radio jets and synchrotron lobes and beamed X-ray and gamma ray emission.

The energetics and timescales of the emission is consistent with the infall of material onto a supermassive black hole.

The optical spectra of can AGN display both broad and narrow high-ionisation lines thought to be due to intense UV radiation emitted by the BH accretion disk.

AGN as a class cover objects such as quasars, blazars, Seyfert and radio galaxies.

1.9 Large scale structure

Galaxies do not exist in isolation.

Field galaxies are not strictly isolated. They represent a mean volume density that displays fluctuations as function of varying position.

Galaxies also exist within larger structures, i.e. gravitationally bound dark matter halos. These are referred to as groups (few to tens of bright galaxies) and clusters of galaxies (hundreds to thousands of bright galaxies).

Groups and clusters of galaxies often display evidence of galaxy interactions, e.g. tidal tails and mergers.

They also display different population trends compared to the field, e.g. blue fractions and dwarf-to-giant ratios, that we will discuss later.

Beyond the scale of individual clusters one begins to see the "cosmic web" of large scale structure. The large scale distribution is isotropic. However, the statistical properties of the distribution ("clustering") arises from basic physics and the cosmological model.