6 The blackbody temperature of a planet

The temperature of a planet depends upon a number of factors

1. The power received by the Sun.
2. The balance between the power absorbed and reflected by the planet.
3. The energy retained in the planet's atmosphere (the greenhouse effect).
4. The internal energy of the planet (e.g. due to radioactive decay, tidal heating and the contraction of the planet's core).

For the inner planets of the Solar System, the first two considerations are the most important.

We start with the power emitted by the Sun. This is described by the blackbody theory of radiation and states that the total power emitted by the Sun (we referred to this previously as the solar luminosity) is

$$P_{\odot} = L_{\odot} = (\sigma T_{\odot}^4)(4\pi R_{\odot}^2)$$

where σ is a radiation constant called the Stefan-Boltzmann constant, $T_{\odot}$ is the surface temperature of the Sun and $R_{\odot}$ is the solar radius.

The energy absorbed by the Earth is equal to the flux of solar radiation at the Earth's orbital radius (D) multipled by the projected area of the Earth ($\pi R_{\oplus}^2$), i.e.

$$P_{\oplus,abs} = (1-\alpha)\ P_{\odot} \times \left(\frac{\pi R_{\oplus}^2}{4\pi D^2}\right).$$

Note that we have corrected the power absorbed by the Earth by the factor $1-\alpha$. The symbol α represents the Earth's albedo or reflectivity. The factor $1-\alpha$ therefore represents the fraction of incident radiation absorbed by the Earth.

What happens to the solar energy absorbed by the Earth? The surface temperature of the Earth is constant with time – it is said to be in thermal equilibrium. Therefore, the Earth must re-radiate all of the energy that it absorbs. The Earth emits radiation according to the same blackbody radiation law as the Sun, the only difference is that we use the surface temperature and surface area of the Earth instead of the Sun.

$$P_{\oplus,em} = (\sigma T_{\oplus}^4)(4\pi R_{\oplus}^2)$$

Taking the condition that the energy absorbed is equal to the energy emitted, we can write

$$P_{\oplus,abs} \quad = \quad P_{\oplus,em}$$

$$
\begin{aligned}
(1-\alpha)\;(\sigma T_{\odot}^4)(4\pi R_{\odot}^2)\times\left(\frac{\pi R_{\oplus}^2}{4\pi D^2}\right) &= (\sigma T_{\oplus}^4)(4\pi R_{\oplus}^2) \\
T_{\oplus} &= T_{\odot}\left(\frac{(1-\alpha)^{1/2}\;R_{\odot}}{2D}\right)^{1/2}.
\end{aligned}
$$

If we now insert the values appropriate for the Sun and Earth into this equation we obtain

$$
T_{\oplus} = 5778\text{ K}\times\left(\frac{(1-0.367)^{1/2}\times 6.96\times 10^{8}\text{ m}}{2\times 1.496\times 10^{11}\text{m}}\right)^{1/2} = 249\text{ K}.
$$

Note that this value does change with time as the amount of solar radiation incident upon the Earth varies (the Earth's orbit is slightly elliptical rather than perfectly circular) and the surface albedo changes seasonally.

The calculation does not take into account the additional energy stored in the Earth's atmosphere (the greenhouse effect). The Earth has a moderate greenhouse effect which increases the surface temperature by some 40 K over the blackbody temperature.