2 Distance dimming

Consider a 100W light bulb. If you cup your hands around the bulb – very close to it but not touching it – your hands will absorb 100W (100 Joules per second) of radiant energy[1]. This is called the **luminosity** of the bulb, i.e. $L = 100$ W.

What happens if you now put your hand 1m away from the bulb, how much energy does you hand now absorb?

The light from the bulb spreads out in all directions, in fact it is uniformly distributed on the surface of a sphere centered on the bulb. The surface area (SA) of the sphere is SA $= 4\pi r^2$, where r is the distance from the bulb.

We refer to f as the apparent flux received from the bulb at a distance r. We write

$$f = \frac{L}{SA} = \frac{L}{4\pi r^2} \text{ (Wm}^{-2}\text{)}$$

Therefore, the flux received at 1m is

$$f = \frac{100 \text{ W}}{4\pi(1\text{m})^2} = 8 \text{ Wm}^{-2}.$$

Now your hand is smaller than 1m^2, it is more like 0.1m $\times$ 0.15m $=$ 0.015m^2 in area. Therefore the power received by your hand is

$$\begin{aligned} P_{hand} &= f_{bulb@1m} \times SA_{hand} \\ &= 8 \text{ Wm}^{-2} \times 0.015 \text{ m}^2 \\ &= 0.12 \text{ W}. \end{aligned}$$

What about a less down to Earth example? The luminosity of the Sun is $L_\odot = 3.8 \times 10^{26}$W. The Earth orbits the Sun at a radius of $r = 1.5 \times 10^{11}$m. The solar flux received at Earth is then,

$$\begin{aligned} f_{Earth} &= \frac{L_\odot}{4\pi r^2} \\ &= \frac{3.8 \times 10^{26}\text{W}}{4\pi \times (1.5 \times 10^{11}\text{m})^2} \\ &= 1345 \text{ Wm}^{-2}. \end{aligned}$$

[1]Note that we discount the heat energy produced as a by-product of the process that generates the light.

Can we then compute how much electrical power is required by the International Space Station (ISS)? Let's consider the maximum power available via solar panels. Each solar array has an area of 375m^2. Typically the ISS uses 3 out of 4 arrays at any one time. The power received by a given array is

$$
\begin{aligned}
P &= f_{Earth} \times SA \\
&= 1345 \text{ Wm}^{-2} \times 375\text{m}^2 \\
&= 5 \times 10^5\text{W} \quad \text{or} \quad 500\text{kW}.
\end{aligned}
$$

This is the maximum power available. About 30% of the available solar power is converted into electricity. Other considerations include non-perfect alignment of the panel to the Sun (perpendicular is best) and the fact that the ISS spends part of each orbit in the Earth's shadow and must use battery power (which have to be recharged in daylight). Typically the ISS uses 32 kW at any one time.